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P. Krawiec
Poznań University of Technology (Poland)

DESIGN, MANUFACTURING AND MEASUREMENT TEETH OF NON-CIRCULAR GEARS

Вибрані проблеми проектування і виробництва нетипових машинних елементів, як, наприклад, некруглі механізми, наведені в статті. Такі механізми використовують в нерівному пробігу зубчато-ремінних передач, які дають змогу отримувати періодично-змінні кінематичні параметри. Описано переваги і обмеження на формування зубів. Також наголошено на відсутності інформації про стандарти некруглих механізмів і про методи вимірювання геометричних особливостей.

Ключові слова – некруглі механізми, зубчато-ремінна передача, виробництво

Selected designing and manufacturing problems of nontypical machine elements such as non-circular gears are presented in the paper. Such gears are used in uneven-running cog-belt transmissions, which allow to obtain periodically variable kinematical features. There are presented advantages and limitations of teeth forming. There was pointed a lack of information about standards of non-circular gears and measuring methods of geometrical features.

Keywords – non-circular gears, cog-belt transmission, manufacturing

Introduction

Issues of non-circular wheels manufacturing ranges from forties of previous century, when one of directions of variable kinematical characteristics obtaining was usage of non-circular gear wheels. This problem was presented in literature of F. Litwin [9] works. There are incorporated, actual till now, technological assumptions concerning mating conditions of non-circular gear wheels, but methods of manufacturing lost their topicality. A little bit different problem, taking into consideration a character of mating of wheel and the cogbelt, is non-circular pulleys manufacturing. Searching of accurate and cheap manufacturing methods became one of problems which solving would enable more common usage transmissions in technique of control and drive of uneven transmissions obtaining variable kinematical features such as gear ratio and speed, thanks to usage wheels with out-of-round envelopes. Model of uneven transmission is presented in drawing 1.

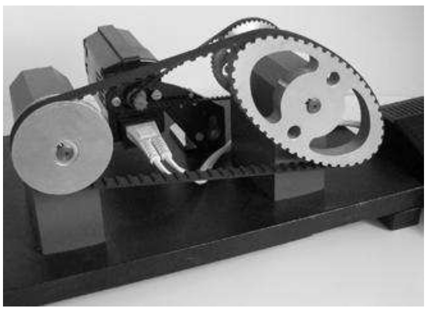

Fig.1 Uneven transmission with two wheels

In available writing there are little information about forming of non-circular pulleys toothing. Analogical, like in case of circular wheels machining, problems connected to non-circular wheels may be divided into [1], [2], [4],[5],[11]:

machining in which traditionally there can be distinguished shape, gear generating methods and point-by-point method formerly called tracing method. Traditionally tracing method consisted in taking the shape of outline (profile) from pattern-wheel on manufactured wheel using mechanical coupling. Actually in CNC machine tools carry of information concerning wheel geometrical features is done on mechatronical way by using of numerical control to classical tool. This principle is also used for milling with end mill process, for work-pieces machining on electro-erosion machines where part of the tool is taken by properly chosen wire. Analogical to this process is machining on CNC contour saw where the tool is gas, water with abradant jet or plasma.

-plastic forming,

-metal powder sintering technology.

-rapid prototyping and rapid manufacturing methods

Samples of wheels for which there were carried out experimental investigations referring to machining methods and measurements are presented in figure 2.

Fig. 2 Demonstration non-circular pulleys

Usage of Bezier curves and spline

In case of envelopes of wheels which can not be described with classical equations, their profile must be properly matched to given geometrical conditions. Then can be used Weierstrass approximation theorem. „For any continuous curve there can be shown curve (of sufficient grade), which approximates basic curve with determined accuracy. For that there should be taken control polyline of which vertices are the points of basic curve and they are distributed on it suitable densely". Proof of that theorem is connected to personage of Sergey Bernstein of which name there are named polynomials used for presenting of Bezier curves.

A Bernstein polynomial, $B_{f,s}$, of the function $f \in C_{<0,\,1>}$ and of degree n is defined by the Eq. 1: [7], [8], [14]:

$$B_{f,s}(t) := \sum_{k=0}^{s} f\left(\frac{k}{s}\right) \cdot p_{s,k}(t) \tag{1}$$

where. $p_{s,k}(t) := \binom{s}{k} \cdot t^{k} \cdot (1-t)^{s-k}$

The polynomial $p_{s,k}$ is called k-th basic Bernstein polynomial, and the set $B = [\, p_{s,0}, p_{s,1}, ..., p_{s,s}\,]$ is called s-th (standard) Bernstein base.

In practical appliances this polynomial is determined in recurrent way and for points $P_i = (x_i, y_i)$, i=1,2,…,s, there is used the following formula

$P_{i,k} = (1-t) \cdot P_{i,k-1} + t \cdot P_{i+1,k-1}$ where k = 1,2…,s , i = 0,1,…,s-k,

taking on the beginning $P_{i,0} = P_i$. This way there can be obtained

$$B_{f,n}(t) = P_{0,s} \tag{2}$$

The inconvenience of Bezier curves using is that there cannot be described conic sections, that is ellipses and circles, with them. Implementing local change i.e. displacement of control point causes changing of the whole curve. Usually in modeling process of sophisticated shapes there should be use curves of high degree. Lagrange interpolation problem is relatively easy to solve, but there is extremely difficult to select points and nodes in such a way to eliminate interpolation polynomial curve of high degree between given points (classical example of it is Runge effect).

Cutting treatment methods

Forming of non-circular pulleys teeth and also gear wheels by cutting treatment is a complex problem. It results from many reasons, among of them there can be specified: shape of wheels envelopes, non-possibility of using of classical methods of manufacturing, necessity of designing new machine tools or adaptation of classical ones, often not very much batches of the same elements. Searching the solution of that problem, one of natural conceptions is machining of wheels with usage of special modular round cutter (fig.3) on numerically controlled machines such as CNC milling centre.

Fig. 3 Special modular round cutter

In discussed sample it was milling machine DECKIEL MACHO with control system Heidenhein 350. This machine tool has got 5 controlled axes, and three of them are table feed x, y, z, fourth axis is table turn and fifth one is rotational tool head. Elaboration of control software requires determining of coordinates x, y of wheel tooth space and angle to taken zero coordinate axis of coordinate system connected to work-piece. Remaining technology parameters are chosen according to commonly known technological recommendations. In case of elliptical wheels machining, the unavoidable faults of this method is possibility of occurring of addendum surface damages on small radius and non-machining of full addendum profile.

The second of non-circular wheels machining methods consists in usage of machining with end mills set which moves on trajectory determined as a equidistant (equidistant curve) of wheel profile.

Important technological problem is obtaining proper rounding radius between tooth bearing and tooth side surface and also between tooth side surface and tooth crest profile. In considered examples rounding minimal radius at base is equal to 1.17 mm, and it requires usage of mill of diameter 3 mm. Basic problem, which occurs here, is possibility of mill deformation during machining. It may result in non-keeping perpendicularity of gear tooth form to wheel base. There is necessary obtaining of compromise taking into account kind of work-piece material and tool's strength features.

Wire electrical discharge machining

Next method of forming of wheels from figure 2 forming was wire electrical discharge machining WEDM, in which the electrode was thin wire of diameter 0,02 - 0,5 mm with brass, copper, tungsten, molybdenum or wire with coat e.g. galvanized brass [6]. Work-piece was clamped on immovable table. Whereas wire guides were moving in directions U, V and Z (upper head) and X, Y for lower head. For the sake of erosive wear, wire is rewinded from the reel to container or from the reel to reel with speeds 0,5 - 20 m/min. For assurance of wire positioning high accuracy referring to work-piece there are used special eye guides and constant tension with force 5 - 20 N. This method enables cutting of complicated shapes thanks to possibility of controlling with tool both in translational and angular directions. Technology of cutting on CNC electro-erosion machines is characterized with necessity of using small energy discharges, conditioned by small wire diameter (for avoiding of wire rupture) causes that obtained surface quality is included in range (Ra = 2,5 - 0,5 micrometer).

Cutting out with gas laser

Next proposed method of non-circular pulleys shaping is laser cutting out. Characteristic feature of laser cutting out is point-wise energy feeding and high-power cutting jet [3], [4], [10], [12], [13]. Main influence on cutting quality and high grade of keeping shape of cut work-pieces has got accurate guided cutting jet in connection to stable cutting machine with high resistance to vibrations and good repeatability. High requirements concerning cut outlines in reference to cutting surface and dimension keeping, are fulfilled by using of freely programmable numerically controlled cutting machines. For creating of heat needed to laser cutting process, there are used gas lasers CO_2. As a result of triatomic molecule CO_2 vibrations, they generate invisible laser radiation of wave length 10.6 micrometer. To obtain heat quantity required for heating on sheet plate surface or in zone of cutting gap, laser ray must by focused by proper lens or mirrors systems. In result of laser ray absorption the work-piece becomes heated to temperature needed for cutting process. It is ignition temperature during laser cutting resulted of material combustion (oxidizing) and melting temperature during laser cutting consisting in melting of metal in laser ray. Besides of classical, numerical controlled machines for cutting in directions x-y, there are more often used manipulating systems for three-dimensional laser cutting. The condition of obtaining cutting good quality, also on sharp corners and angles, is automatic cutting parameters choice. Laser power and cutting speed must be selected individually for cut outline.

Water with abradant jet cutting

In case of described above method of gas laser cutting the important limitation is heating and in consequence of it deformation of worked material [6], [10]. This fault does not occur in treatment with using of water jet. Cut material is not subjected to mechanical overloading nor to thermal impact. Obtained, in result of treatment, surface quality is satisfactory and does not need further machining. After cutting there can be obtained material of unchanged physical-chemical features (fig. 4).

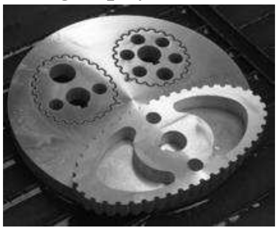

Fig. 4 Non-circular wheels cut with water with abradant jet

With water jet we can cut all grades of steel. Important problem in this method of material treatment is abradant recycling, which consists in constant reception of mass after cutting (abrasive sand, cut material small particles, water) from water plotter table, putting this mass to preliminary segregation (separation of discards of granulation below 80 microns), and next drying of the rest of mass and further segregation into clean abrasive sand of granulation 80 – 100 microns and the rest of mass. Water from recycling comes back to water table.

Methods rapid prototyping and rapid manufacturing

History of method Rapid Prototyping goes back to half of eighties previous century, when worker of MIT Charles Hull observed the phenomena of polymer coats hardening with light UV. This event was a base to elaboration of method SLA called – stereolitography. Hulls's method became the base for elaborating of some methods among of them there can be specified: printing 3D (fig.5), SLS - Laser selective sintering, SGC – Direct base hardening. Specified methods are more widely applied in such industry branches as: automotive, electro-and-machine building industries and also in medicine, in industrial art design.

Fig. 5 Non-circular wheels made by printing 3D method

Continual increment technology development caused that laser began used for welding other materials such as polyamide powders, aluminum powders, mixtures of them and also metals. In these methods there is transition of material from solid status (powder), through liquid status, again to solid status (sinter). On operating platform of the appliance, there is distributed a powder layer with special shaft, and next this layer is locally sintered with laser ray in infrared range. Laser ray is guided on powder surface according to previously properly configured information (so called bitmap), and there also comes sintering of previously put layer. Thanks to it there can be obtained the uniform solid of material. Choice of proper parameter of laser ray enables melting or sintering of powder particles in exactly determined zones. Usage of these methods is especially advisable in process of designing of new non-classical geometrical shapes which are non-circular wheels of uneven transmission (fig.6).

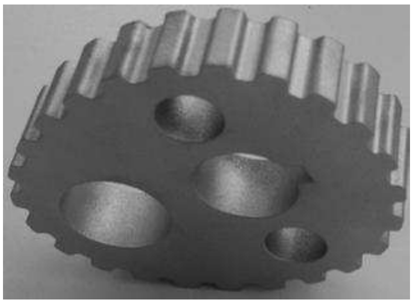

Fig. 6 Non-circular wheel made with method SLS

Noncircular wheels manufacturing methodology

Usability evaluation of mentioned treatment methods requires carrying out of wheels geometrical features measurements. Significant, till now not solved problem is lack of standard recommendations presenting permissible deviations values of geometrical features and surface quality.

So, there is necessary the elaboration of measurements methodology for these features and surface structure after treatment. For selected wheels shapes there will be carried out measurements of envelope outer profile in three parallel planes, measurement of outer profile perpendicularity to end face and measurement of teeth flank pitch line (fig.7). Measurements are carried out on coordinate measuring machine of company ZEISS.

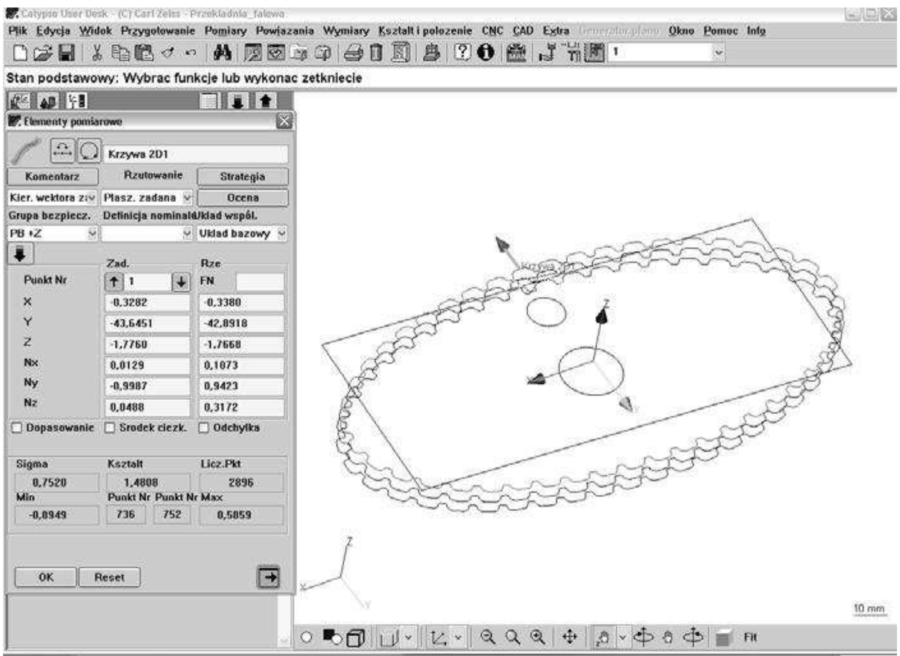

Fig. 7 Analysis of measurements results

Summary

In paper there was presented selected designing and manufacturing problems of nontypical machine elements such as non-circular gears. These gears are used in uneven-running cogbelt transmissions, which allow to obtain periodically variable kinematical features. For the description of geometrical features of envelopes of gears there were used Bezier curves and there were pointed the possibilities of applying spline curves. There were presented many machining methods of these gears, namely forming: with usage of end mill units, profile cutters, cutting out on NC electro discharge machines, NC gas laser cutting machines, water cutting with abrasive and sintered metal powders.

There were presented advantages and limitations of teeth forming. There was pointed a lack of information about standards of non-circular gears and measuring methods of geometrical features

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