

**MODELING OF DYNAMICAL REGIMES OF TWO-MOTOR ELECTRICAL DRIVES WITH DUE REGARD FOR ELASTICITY AND GAPS OF MECHANICAL TRANSDUCERS**

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**Abstract:** In the paper a mathematical model of two-motor electrical drive with one speed controller and two thyristor converters, by taking into account elasticity's and gaps of mechanical transducers, has been constructed. By using state-space methods, the transfer functions of the object have been determined. Some recommendations for tuning of the regulators have been suggested. Dynamical characteristics that are near to the optimal ones have been obtained by applying MATLAB.

**Key words:** dynamics, two-motor electrical drive, modeling.

**Introduction**

Advanced Electric Companies (Siemens, AEG, Harland etc.) for the purpose of performance reliability in the technological processes and of improving the

quality of manufactured products for various machines suggest several constructions of control systems of two-motor electrical drives, e.g.: the systems with one speed regulator (RS) and one (common) thyristor converter (TC); the systems with individual channels of control for each of the motors, i.e. the systems with two speed regulators and individual thyristor converters for each motor; the systems with a common converter and speed regulator changing the motor's excitement, etc. The above mentioned control systems of two-motor electrical drives do not protect proportional distribution of load between the motors without auxiliary contours of regulation, i.e. without auxiliary sensors and regulators, but their presence in the control circuit complicates tuning of the whole system and reduces its reliability [5, 6].

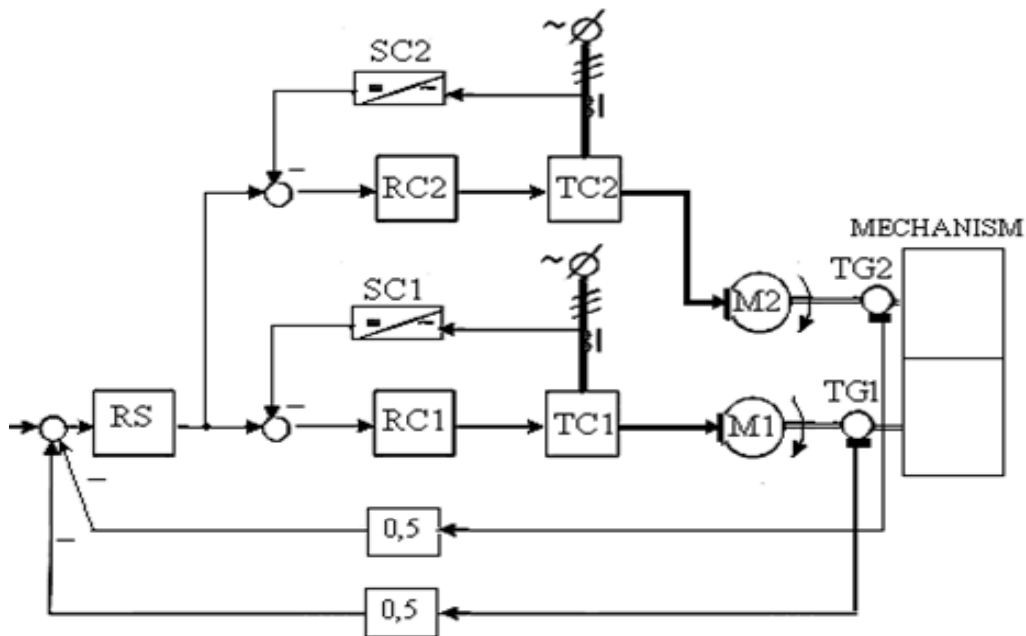


Fig. 1. Functional scheme of two-motor electrical drive with one RS and two TC

In Fig.1 we propose a scheme of two-motor electrical drive, which provides proportional distribution of loadings between the motors without any auxiliary elements and control circuits [1, 2]. Here we use the

following notations: M1 and M2 denote different motors of the electrical drive, respectively; TC1 and TC2 are abbreviations for the thyristor converters; RC1 and RC2 are the regulators of the anchor currents of the motors;

SC1 and SC2 are the sensors of the anchor currents of the motors; TG1 and TG2 are the tachogenerators at the motors (i.e. sensors of speed); RS is the regulator of speed.

The mathematical model of the electrical drive in the relative increments (with respect to their values in static), taking into account elasticity's of shafts and gaps, is described with the following equations of movement:

$$\left\{ \begin{array}{l} \mu_1 - \mu_{e1} = T_{M1} \frac{dv_1}{dt}; \\ \mu_2 - \mu_{e2} = T_{M2} \frac{dv_2}{dt}; \\ k_{L1}\mu_{e1} + k_{L2}\mu_{e2} - \mu_M = T_{MM} \frac{dv_M}{dt}; \\ \mu_{e1} = \begin{cases} \frac{1}{T_{c1}} \int (v_1 - v_M) dt + \frac{T_{d1}}{T_{c1}} (v_1 - v_M), & \text{if } \Delta\Psi_{S1} \in ]-\infty; -(1 + \Psi_{G1})[U]-1; +\infty[ \\ -1, & \text{if } \Delta\Psi_{S1} \in [-(1 + \Psi_{G1}); -1]; \end{cases} \\ \mu_{e2} = \begin{cases} \frac{1}{T_{c2}} \int (v_2 - v_M) dt + \frac{T_{d2}}{T_{c2}} (v_2 - v_M), & \text{if } \Delta\Psi_{S2} \in ]-\infty; -(1 + \Psi_{G2})[U]-1; +\infty[ \\ -1, & \text{if } \Delta\Psi_{S2} \in [-(1 + \Psi_{G2}); -1] \end{cases} \end{array} \right. \quad (1)$$

where:  $\mu_1, \mu_2, \mu_{e1}, \mu_{e2}, \mu_M, v_1, v_2$  and  $v_M$  are the relative increments of the torques of the motors, of the moments of elasticity of the mechanical transducers (long connecting shafts) connecting the motors and the mechanism, as well as the angular speeds of inertial masses (namely, of the motors and the mechanism);

$k_{L1} = \frac{M_{M1}}{M_{MC}}$  and  $k_{L2} = \frac{M_{M2}}{M_{MC}}$  are the load factors of

each motor calculated with respect to the total static load of the drive;  $M_{MC}$  is the total static moment of resistance of the drive;  $T_{M1}, T_{M2}$  and  $T_{MM}$  are the mechanical constants of time of the inertial masses of the drive;  $T_{d1}$  and  $T_{d2}$  are the time constants, that characterize an attenuation process of elastic fluctuations due to viscous friction;  $T_{c1}$  and  $T_{c2}$  are the time

constants, characterizing processes of torsional deformation of the long shafts;  $\Delta\Psi_{S1} = \frac{\Delta\varphi_1}{\varphi_{c1}}$  and

$\Delta\Psi_{S2} = \frac{\Delta\varphi_2}{\varphi_{c2}}$  are the relative torsional angles of the

long connecting shafts;  $\varphi_{c1} = \frac{M_{M1c}}{c_1}$  and  $\varphi_{c2} = \frac{M_{M2c}}{c_2}$

are the torsional angles of the long shafts arising from the action of the moments of resistance  $M_{M1c}$  and  $M_{M2c}$ ;  $c_1$  and  $c_2$  are the stiffness ratios of the long shafts between the motors and the mechanism;  $\Delta\varphi_1 = \varphi_1 - \varphi_M$  and  $\Delta\varphi_2 = \varphi_2 - \varphi_M$  are the torsional angles of the connecting shafts;  $\varphi_{G1}$  and  $\varphi_{G2}$  are the reduced gaps of the mechanical transducers.

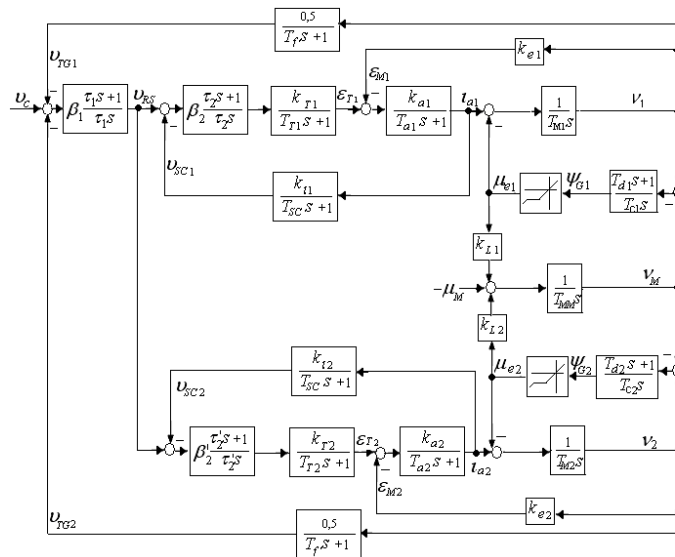


Fig. 2. Structural scheme of elastic two-motor thyristor electrical drive taking into consideration the gaps in kinematic pairs

In Fig.2 you can see the following notations:  $\beta_1$  and  $\tau_1$  are the dynamic gain and time constant of RS;  $\beta_2, \tau_2, \beta'_2, \tau'_2$  are the parameters of the RC<sub>1</sub> and RC<sub>2</sub>;  $k_{T1}, T_{T1}, k_{T2}, T_{T2}$  are the gains and time constants of TC<sub>1</sub> and TC<sub>2</sub>;  $k_{d1}, k_{d2}, T_{sc}$  are the gains and time constants of SC1 and SC2;  $k_{a1}, T_{a1}, k_{a2}, T_{a2}$  are the coefficients of transducers and time constants of the anchor circuits of the motors M<sub>1</sub> and M<sub>2</sub>;  $T_f$  is the time constant of the filters after TG1 and TG2;  $\Psi_{S1}$  и  $\Psi_{S2}$  are the gaps of mechanical connections;  $\nu_c$  is the relative increment of control signal of the drive system;  $\nu_{TG1}$  and  $\nu_{TG2}$  are the relative increments of output voltages from TG1 and TG2;  $\nu_{SC1}$  and  $\nu_{SC2}$  are the relative increments of output voltages from SC1 and SC2;  $\varepsilon_{T1}$  and  $\varepsilon_{T2}$  are the relative increments of electromotives from TC1 and TC2;  $\varepsilon_{M1}$  and  $\varepsilon_{M2}$  are the relative increments of electromotives from the motors;  $l_{a1}$  and  $l_{a2}$  are the relative increments of the anchor currents (i.e. running the torques) of the motors.

To obtain the transfer functions from the signal  $\mu_1$  to  $\nu_1$  as well as from  $\mu_2$  to  $\nu_1$  of the mechanical part of the drive system ignoring gaps in the reducers, we write a system of equations in Cauchy's form:

$$\begin{cases} \frac{dx}{dt} = Ax + Bu \\ y = Cx, \end{cases} \quad (2)$$

where:  $x^T = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]$ ;  $x_1, x_2, x_3, x_4, x_5$  are the states of the control system, namely:  $x_1$  and  $x_2$  are the angular speeds of the motors,  $x_3$  is the angular speed of the mechanism;  $x_4$  and  $x_5$  are the moments of elasticity of the mechanical connecting shafts;  $u$  is the input signal of the object of the drive system ( $\mu_1$  and  $\mu_2$ );  $y$  is the output signal of the system (i.e.  $\nu_1$ );

$$A = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{T_{M1}} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T_{M2}} \\ 0 & 0 & 0 & \frac{k_{L1}}{T_{MM}} & \frac{k_{L2}}{T_{MM}} \\ \frac{1}{T_{c1}} & 0 & -\frac{1}{T_{c1}} & -\frac{T_{d1}}{T_{c1}} \left( \frac{1}{T_{M1}} + \frac{k_{L1}}{T_{MM}} \right) & -\frac{T_{d1} \cdot k_{L2}}{T_{c1} \cdot T_{MM}} \\ 0 & \frac{1}{T_{c2}} & -\frac{1}{T_{c2}} & -\frac{T_{d2}}{T_{c2}} \cdot \frac{k_{L1}}{T_{MM}} & -\frac{T_{d2}}{T_{c2}} \cdot \left( \frac{1}{T_{M2}} + \frac{k_{L2}}{T_{MM}} \right) \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{T_{M1}} \\ \frac{1}{T_{M2}} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

It is well known that, to obtain the transfer function of the system described by the equations (2), we need to apply the expression:

$$W(s) = C \cdot (sE - A)^{-1} \cdot B, \quad (4)$$

where:  $s = d/dt$ ;  $E$  is the identity matrix.

Substituting the expressions (3) in the (4), we define the transfer functions of the mechanical part of the drive with respect to the angular speed of the first motor as follows:

$$W_{01}(s) = \frac{\nu_1(s)}{\mu_1(s)} = k_{H1} \left[ 1 + \frac{T_1^2 s^2 (T_2^2 s^2 + 2\xi_1 T_2 s + 1)}{k_{L1} (T_{d1} s + 1)(T_{d2} s + 1)} \right] = \frac{T_{M\Sigma} s N(s)}{T_{M\Sigma} s N(s)}, \quad (5)$$

$$M_{12}(s) = \frac{\nu_1(s)}{\mu_2(s)} = \frac{k_{L2}}{T_{M\Sigma} s N(s)}, \quad (6)$$

where:

$$T_1 = \sqrt{k_{L1} T_{M2} T_{c2} + (k_{L2} T_{M2} + T_{MM}) T_{c1}};$$

$$T_2 = \frac{\sqrt{T_{M2} T_{MM} T_{c1} T_{c2}}}{T_1};$$

$$\xi_1 = \frac{k_{L1} T_{M2} T_{c2} T_{d1} + (k_{L2} T_{M2} + T_{MM}) T_{c1} T_{d2}}{2T_1^2 T_2};$$

$$N(s) = 1 + \frac{T_{M1} T_{M2} T_3 s^2 (T_4 s + 1)}{T_{M\Sigma} (T_{d1} s + 1)(T_{d2} s + 1)} \times \left[ 1 + \frac{T_{MM} T_5^2 (T_6^2 s^2 + 2\xi_2 T_6 s + 1)}{T_{M1} T_{M2} T_3 (T_4 s + 1)} \right];$$

$$T_3 = k_{L1} T_{c2} + k_{L2} T_{c1}; \quad T_4 = \frac{k_{L1} T_{c2} T_{d1} + k_{L2} T_{c1} T_{d2}}{T_3};$$

$$T_5 = \sqrt{T_{M1} T_{c1} + T_{M2} T_{c2}}; \quad T_6 = \frac{\sqrt{T_{M1} T_{M2} T_{c1} T_{c2}}}{T_5};$$

$$\xi_2 = \frac{T_{M1} T_{c1} T_{d2} + T_{M2} T_{c2} T_{d1}}{2T_5^2 T_6};$$

$T_{M\Sigma} = k_{L1}T_{M1} + k_{L2}T_{M2} + T_{MM}$  is the total mechanical time constant of the drive.

Below we use the following parameters of the electric drive system:  $P_{M1} = 300$  kW;  $P_{M2} = 100$  kW;  $k_{L1} = 0,7$ ;  $k_{L2} = 0,3$ ;  $T_{M1} = 1,5$  s;  $T_{M2} = 0,7$  s;  $T_{MM} = 10$  s;  $T_{d1} = T_{d2} = 0,002$  s;  $T_{c1} = 0,0004$  s;  $T_{c2} = 0,00035$  s.

The transfer function of the controlled object of the total system, i.e. the system comprising all the objects from the output voltage of RS to the signal of the angular speed of the first (basic) motor, with optimized contours of currents, can be written as follows:

$$W_{Obj.}(s) = \frac{v_1(s)}{U_{pc}(s)} = \frac{W_{01}(s) + M_{12}(s)}{k_1(T_{\Sigma 2}s + 1)}, \quad (7)$$

where  $U_{RS}$  is the relative increment of the output voltage of RS;  $k_1$  is the coefficient of transmission of the sensors of the anchors currents of the motors ( $k_1 = 0,1$ );  $T_{\Sigma 2}$  is the small time constant of the optimized current circuits ( $T_{\Sigma 2} = 0,01$  sec.).

By applying the above mentioned parameters of the electrical drive, we rewrite the transfer function (7) in numerical form as expression:

$$W_{Obj.}(s) = \frac{0,48s^4 + 9,84s^3 + 4150s^2 + 7000s + 300000}{s(0,00072s^5 + 0,2s^4 + 8,9s^3 + 786,6s + 15560s + 1110000)}. \quad (8)$$

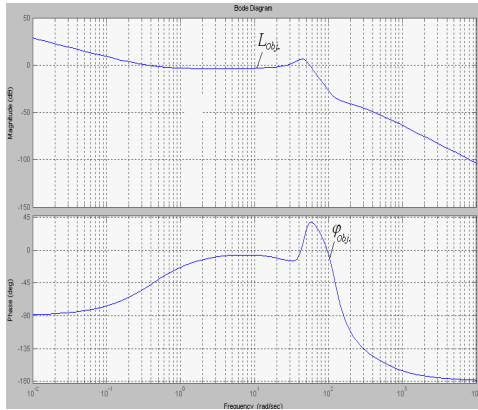


Fig. 3. Logarithmic magnitude ( $L_{Obj.}$ ) and phase ( $\varphi_{Obj.}$ ) as frequency characteristics of the object with optimized current circuits

Synthesis of RS of the drive system has been obtained by methods of frequency characteristics, i.e. Bode Plots via MATLAB.

Fig. 3 presents the logarithm magnitude ( $L_{Obj.}$ ) and the phase ( $\varphi_{Obj.}$ ) as frequency characteristics of the object with optimized current circuits for the drive system described by the transfer function (8). At the frequency

$$\omega_e = \frac{1}{\sqrt{T_{M1}T_{c1}}} = 42, \text{sec.}^{-1}$$

the magnitude has a moderate resonance peak. Consequently, the first step of approximate tuning of regulators might be carried out similarly as recommended for the drives with hard connecting shafts. For tuning of currents' regulators during computer modeling of the electrical drive system the method of "modulus optimum" has been applied, whereas the "symmetric optimum" method has been used for the regulator of speed [3;4].

To implement the characteristics of a gap (III-characteristic in Fig. 4) in the process of modeling by representation of the variables in relative increments, it has been represented by the sum of two blocks shown in Fig. 4 as II-characteristic and I-characteristic. These blocks were connected in-parallel in the drive scheme for computer modeling and the sum of their signals has been processed.

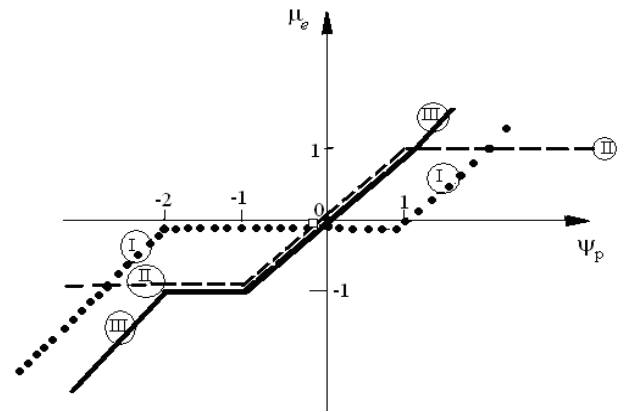


Fig. 4. Nonlinear characteristic of a gap

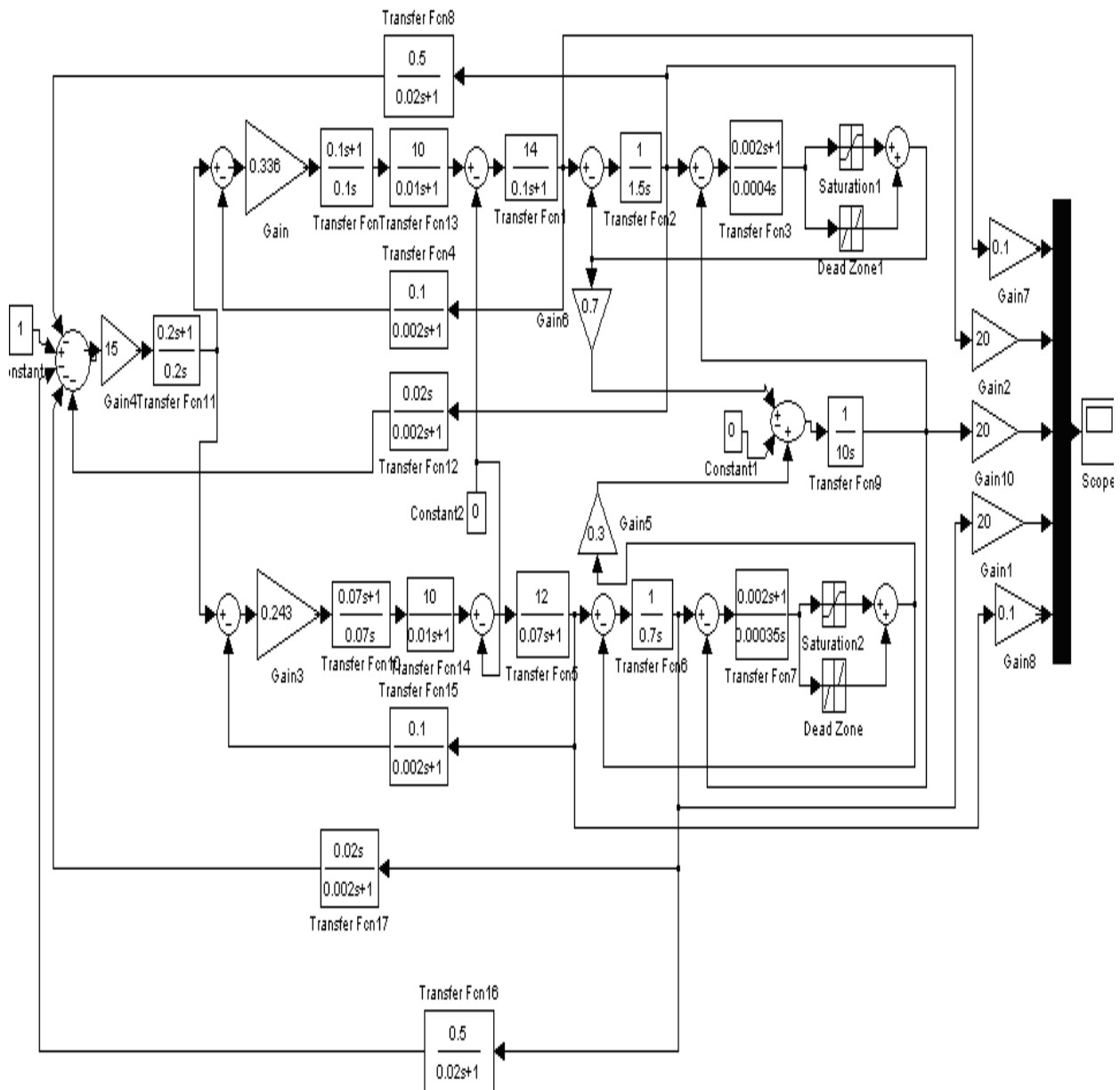


Fig. 5. Computer modeling of two-motor thyristor drive including gaps in its kinematic pairs

The results of computer modeling of the considered system including the two-motor electrical drive with gaps in its mechanical transducers (Fig. 5) show that without auxiliary variable velocity feedbacks they are highly unstable (Fig. 6,a).

Let us apply variable feedbacks for separate motors in the input of RS with the following transfer function:

$$W_{flex.feedback.}(s) = \frac{0,02s}{0,002s + 1} \quad (9)$$

The dynamic characteristics of the electric drive are substantially improved (Fig. 6, b,c). Quality parameters (performance is approximately equal to 1 s, overshoot  $\sigma = 35\%$  and dynamic decrease of the speed  $\Delta v \leq 0,15$ ) are near to optimal ones [4;6].

By means of the investigations made, we have established that for proportional distribution of load between the motors for the considered control system it is necessary to choose coefficients of transmission of the sensors of anchor currents inversely-proportional to the powers of the electrical motors.

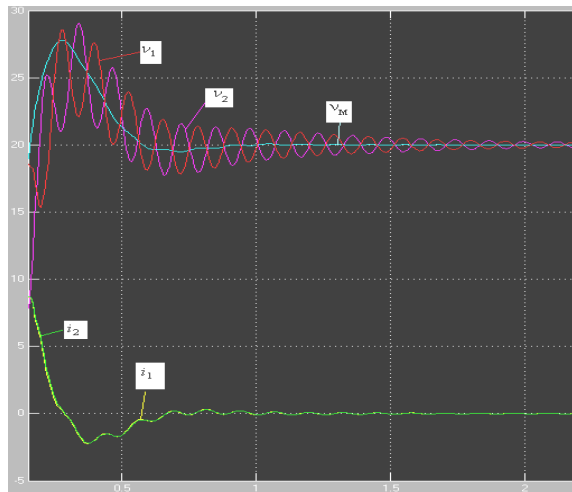


Fig. 6, a

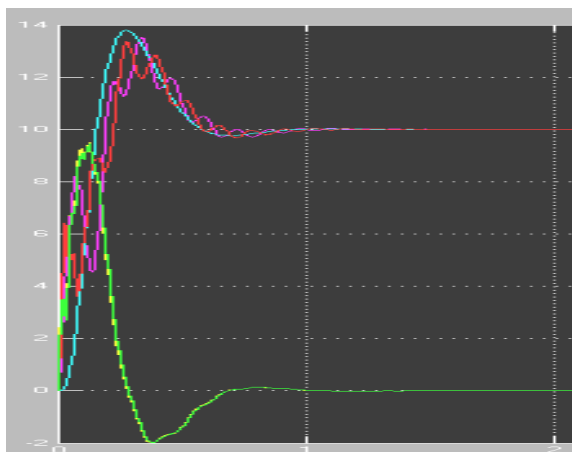


Fig. 6, b

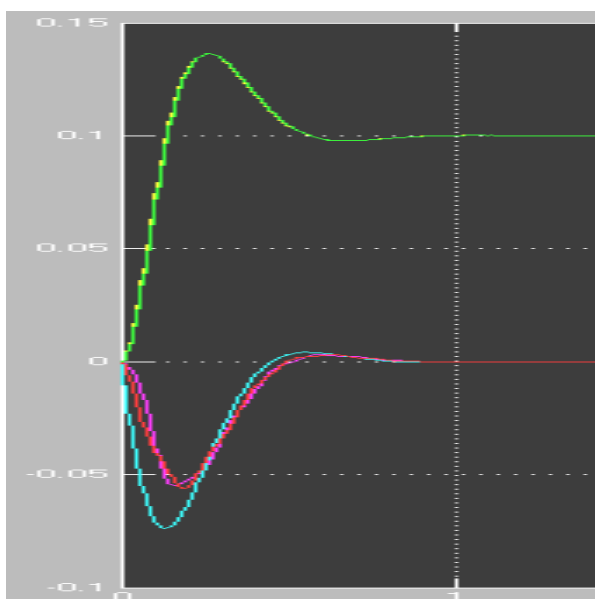


Fig. 6, c

Fig. 6 Transient processes by step influence on the system  
 a – from control signal, without corrector; b – from control  
 signal with corrector; c – from load with corrector

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## МОДЕЛЮВАННЯ ДИНАМІЧНИХ РЕЖИМІВ ДВОМОТОРНИХ ЕЛЕКТРОПРИВОДІВ З УРАХУВАННЯМ ПРУЖНОСТІ ТА ЛЮФТІВ МЕХАНІЧНИХ ПЕРЕТВОРЮВАЧІВ

Дж. М. Дочвірі, О.С. Хачапуридзе

Стаття присвячена побудові математичної моделі двомоторного електроприводу з одним регулятором швидкості і двома тиристорними перетворювачами (конвертерами) з урахуванням пружності і люфтів механічних перетворювачів. З використанням методів простору станів сформульовано передавальні функції об'єкту. Запропоновано рекомендації для налаштування регуляторів. За допомогою програми MATLAB отримано динамічні характеристики, що є близькими до оптимальних.



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