COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 4, No. 2, 2014

INDUCTION MOTOR MACROMODEL BASED ON EXPERIMENTAL DATA

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Abstract: The macromodel of transients of a three-phase induction motor in the terms of stator current, rotation rate and loading on the axis is described.

Experimantal data are averaged per an alternating current (AC) period. Regularization of the experimantal data differentiation is demonstrated using cubic smoothing splines. The number of coefficients of macromodel is minimized by reduction of the approximating polynomial. Due to the reduction of macromodel the identification became correct. Adequate behavior of the macromodel has been verified for input signals different from those for which the macromodel has been built. Experimental transients are reproduced by two continuous nonlinear macromodels of the first order with a relative mean square error less than 1%.

The received macromodel is notable for its low order and high-fidelity output signals.

Key words: mathematical macromodel, induction motor.

1. Introduction

Mathematical models of varying complexity describing the induction motor (IM) have been known for a long time [1]. They were created according to the description of the engine dynamics with the use of algebraic-differential equations of electromechanical processes.

The macromodel approach (“black box’ approach) allows us to create a model that is much simpler than the traditional ones and is not inferior to them in point of accuracy of external variables, omitting complex internal processes of the object [2].

The macromodel can be built in different mathematical forms: integral, differential, difference equations. The examples of an induction motor macromodel in difference equations are presented in [3]. A model in [3] is of the third order and reproduces transients with averaged relative square error of several percent. The proposed publication describes the macromodel of the induction motor by differential equations of a lower order and with less error reproduction of transients.

According to experimental data shown below the induction motor transient processes may be discribed by differential equations of the first order. For first-order systems with scalar input and output the overall macromodel structure looks like

$$\frac{dy}{dt} = \sum_{i,j=0}^{r} C_{ij} y^i u^j; \quad i + j \le r \tag{1}$$

where $u(t)$ is the input variable, $y(t)$ is the output variable; and the function of the right side of the equation is approximated by a power polynomial.

In the seventies such an equation was used to identify the dynamic macromodels of non-linear systems [4].

The macromodel is expected to repeat output signals. Identification of the mathematical model (1) in the quadratic metric means the determination of the vector of coefficients C for given sets of values $u(t_k)$, $y(t_k)$, k=1,...,N with the least square error of the equation (1)

$$\min_C \sum_{k=1}^{N} \left(\frac{dy(t_k)}{dt} - \sum_{i,j=0}^{r} C_{ij} y^i(t_k) u^j(t_k) \right)^2, \quad i + j \le r . \tag{2}$$

The problem (2) is reduced to solving a system of linear algebraic equations being always compatible and having a unique solution. But in general, the problem (2) is incorrect and needs regularization [2, 5].

2. Preparation of experimental data

Experimental transient characteristics of the three-phase IM A051-4A (wye-connection of stator windings, nominal power P_n=4.5 kW, supply voltage 220 V, nominal rotation rate ω_n=150.8 rad/sec) have been kindly provided by Professor Y. Paranchuk (Lviv Polytechnic, Ukraine, yparanchuk@yahoo.com). The induction motor is considered as a “black box”. As input signal the load current S of a direct-coupled DC generator, that simulates the mechanical load of the motor, has been chosen. Output signals are supply current I of one of the phases and voltage of a direct-coupled tacho-generator recalculated into rotation rate W of the rotor. The corresponding graphs are shown in Fig. 1.

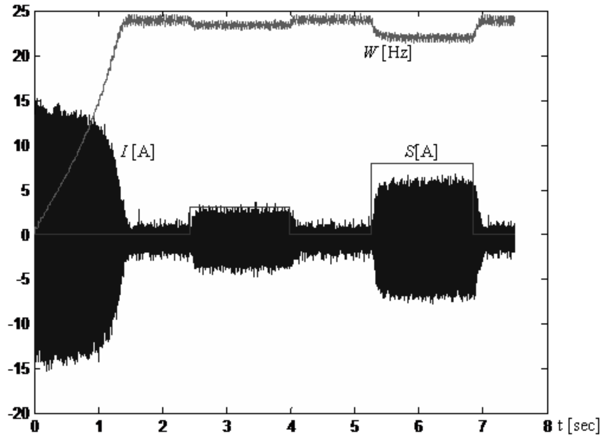

Fig. 1. Graphs of IM transients. S - load current, A; W – rotatoin rate, Hz; I - supply current, A.

The beginning of the graphs from 0 to 1.4 sec corresponds to the IM acceleration after switching on the power. Next, the generator load is switched on and off twice, first with current *S* equal to 3A, and then with current *S* equal to 8 A.

The data in Fig. 1 contain more than 13,000 time points and, therefore, are unsuitable for building a macromodel.

The macromodel is constructed for root-mean-square (rms) alternating current *Is* and averaged on the period of AC power (0.02 sec) rotation rate *Ws*. The number of time points is reduced to 357. Graphs of these signals are shown in Fig. 2.

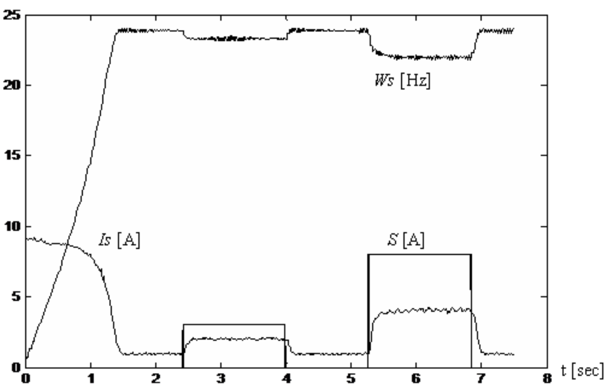

Fig. 2. Graphs of rms AC Is and the averaged rotation frequency Ws.

3. Macromodel development

The graphs in Fig. 2 show that the dynamics of the IM can be described by two independent nonlinear systems of first order. So, the macromodel equations may take the form of two differential equations for the state variables *Ws* and *Is*:

$$\frac{dIs}{dt} = \sum_{i,j=0}^{r} KI_{ij} \cdot Is^i \cdot S^j; \quad i+j \le r;$$
$$\frac{dWs}{dt} = \sum_{i,j=0}^{r} KW_{ij} \cdot Ws^i \cdot S^j; \quad i+j \le r; \qquad (3)$$

The identification of the macromodel (3) can be performed traditionally (2), as the problem of minimizing mean square residuals of the equations (3) at all 357 time points t_k where values $S(t_k)$, $Is(t_k)$, $Ws(t_k)$ are given:

$$\min_{KI} \sum_{k=1}^{357} \left(\frac{dIs(t_k)}{dt} - \sum_{i,j=0}^{r} KI_{ij} \cdot Is^i(t_k) \cdot S^j(t_k) \right)^2;$$
$$\min_{KW} \sum_{k=1}^{357} \left(\frac{dWs(t_k)}{dt} - \sum_{i,j=0}^{r} KW_{ij} \cdot Ws^i(t_k) \cdot S^j(t_k) \right)^2. \qquad (4)$$

To solve the problem (4), besides $S(t_k)$, $Is(t_k)$, $Ws(t_k)$, we must have $\frac{dIs(t_k)}{dt}$ and $\frac{dWs(t_k)}{dt}$. The derivative of the discrete functions can be calculated by many methods [6]. But, if the numerical calculation of derivatives is incorrect, it causes additional difficulties. A universal and regularized method is using smoothing splines [7]. Cubic smoothing splines are constructed for $Is(t_k)$ and $Ws(t_k)$ using 357 time points. The corresponding function *csaps* with smoothing parameter 0.99999 is implemented in MATLAB software environment. The derivatives $\frac{dIs(t_k)}{dt}$ and $\frac{dWs(t_k)}{dt}$ are calculated by analytical differentiation of splines. The graphs of interpolating splines and their derivatives are shown in Fig. 3 and Fig. 4.

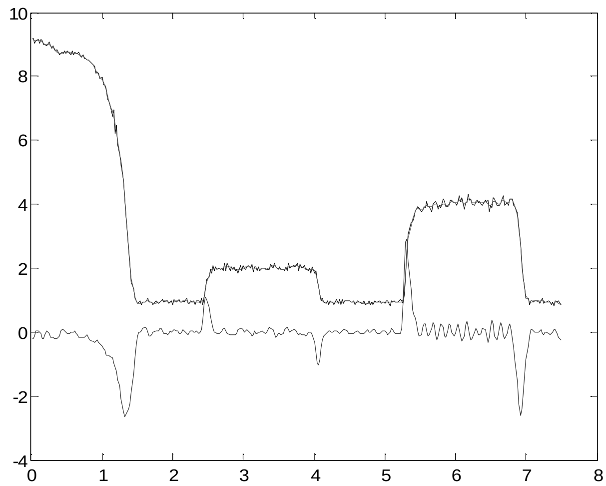

*Fig. 3. Graphs of the spline interpolating $Is(t_k)$ and its derivative $\frac{dIs(t_k)}{dt}$ *0.1, k=1,…,357.*

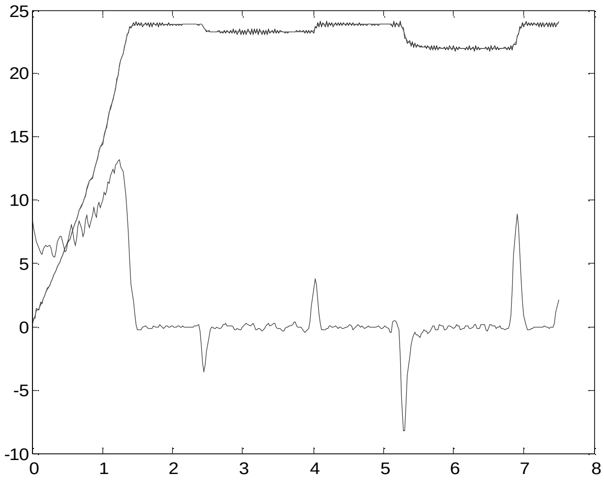

*Fig. 4. Graphs of the spline interpolating $Ws(t_k)$ and its derivative $\frac{dWs(t_k)}{dt}$ *0.5, k=1,…,357.*

So, the identification of the systems (3) means determining the coefficients KI_{ij} and KW_{ij} in the problems (4).

If r=5, the number of coefficients is 21 for every polynomial. So the problems (4) are incorrect and do not give the desired result.

The method of aproximated polynom reduction [2] may regularize the problems (4). This method determines mathematically the required coefficients for a given task, while all others should be removed.

The idea of the method consists in the twofold identification of the coefficients - for an initial problem and for a problem with small random deviations of experimental data. Coefficients with the greatest relative deviations should be remove. This method is justified in [2].

As a result of the reduction of the polynomials in (4), only 8 coefficients in the first polynomial and 7 ones in the second polynomial have been kept. Then the macromodel (3) becomes much easier:

$$\begin{aligned} \frac{dIs}{dt} &= KI_1 + KI_2 \cdot S + KI_3 \cdot Is + KI_4 \cdot S \cdot Is + \\ &+ KI_5 \cdot Is^2 + KI_6 \cdot Is^3 + KI_7 \cdot Is^4 + KI_8 \cdot Is^5; \\ \frac{dWs}{dt} &= KW_1 + KW_2 \cdot S + KW_3 \cdot Ws + KW_4 \cdot S \cdot Ws + \\ &+ KW_5 \cdot Ws^2 + KW_6 \cdot Ws^3 + KW_7 \cdot Ws^5; \end{aligned} \quad (5)$$

where the first three terms of the right parts of differential equations describe the linear parts of the macromodels, and the rest reproduce nonlinear effects.

The corresponding identification problems (6) are formulated as follows:

$$\begin{aligned} &\min_{KI} \sum_{k=1}^{357} \left(\frac{dIs(t_k)}{dt} - \sum_{i=1}^{8} KI_i \cdot S(t_k)^m \cdot Is(t_k)^n \right)^2; \\ &\min_{KW} \sum_{k=1}^{357} \left(\frac{dWs(t_k)}{dt} - \sum_{i=1}^{7} KW_i \cdot S(t_k)^m \cdot Ws(t_k)^n \right)^2. \end{aligned} \quad (6)$$

Fifteen obtained coefficients (7) are substituted in macromodel differential equations (5).

$$\begin{aligned} KI = \big(&2,3037 \cdot 10^{+1} \quad 6,5546 \cdot 10^{+0} \quad -3,2962 \cdot 10^{+1} \\ &-9,3053 \cdot 10^{-1} \quad 9,9106 \cdot 10^{+0} \quad -1,7113 \cdot 10^{+0} \\ &1,7815 \cdot 10^{-1} \quad -7,6451 \cdot 10^{-3}\big); \\ KW = \big(&1,0955 \cdot 10^{+1} \quad 3,0797 \cdot 10^{+0} \quad 2,2376 \cdot 10^{+0} \\ &-2,4924 \cdot 10^{-1} \quad -5,0003 \cdot 10^{-1} \quad 3,3550 \cdot 10^{-2} \\ &-3,0302 \cdot 10^{-5}\big). \end{aligned} \quad (7)$$

The solutions of the equations (5) with the coefficients (7) and the experimental signals $Is(t_k)$, $Ws(t_k)$ are shown in Fig. 5.

The averaged relative square error of the reconstruction is less than 1%. Thus, they are practically identical.

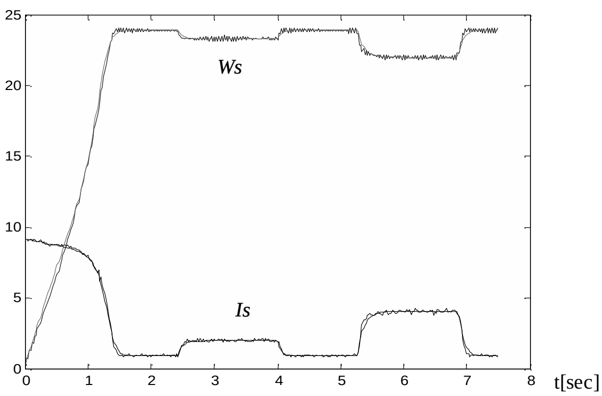

Fig. 5. Comparative graphs of IM experimental data and corresponding values calculated using the macromodel.

4. Verification of the macromodel for different input signals

Adequate behavior of the macromodel has been verifyed for input signals $S(t)$ different from those for which the macromodel was built. Fig. 6 shows comparative graphs of solutions of the macromodel equations (5) for input signals multiplied by 0.6, within the same time as for signals $S(t)$ 3A and 8A, which were the basis for building the macromodels. The same figure shows the macromodel transients with input signals $S(t)$ multiplied by 1.3. The behavior of the macromodel is qualitatively satisfactory.

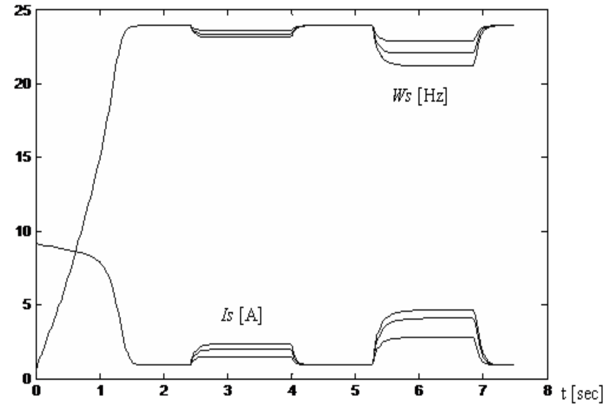

Fig. 6. The transients obtained by the means of the macromodel with input signals S(t) multiplied by 1.0, 0.6 and 1.3.

5. Conclusion

The developed IM macromodel is notable for its extremely low order and high-fidelity reproduction of the experimental transient response.

The identification of the macromodel has become satisfactory only owing to the regularization of calculating derivatives and reduction of the macromodel.

However, for macromodels with different input and output signals and/or different IM the whole identification procedure must be performed from its very beginning.

All calculations have been carried out in the MATLAB R2012a. The corresponding program is available on request through the author's e-mail box matv@ua.fm.

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МАКРОМОДЕЛЬ АСИНХРОННОГО ДВИГУНА ЗА ЕКСПЕРИМЕНТАЛЬНИМИ ДАНИМИ

Ярослав Матвійчук

Описано макромодель трифазного асинхронного двигуна, отриману для таких параметрів, як навантаження на валу, струм живлення однієї з фаз та частота обертання ротора. Експериментальні дані усереднено за період змінного струму живлення.

Продемонстровано регуляризацію обчислення похідних вихідних сигналів макромоделі за допомогою згладжуючих кубічних сплайнів. Кількість коефіцієнтів макромоделі мінімізовано за методом редукції апроксимуючого поліному. Завдяки редукції ідентифікація макромоделі стала коректною. Адекватну поведінку макромоделі перевірено на сигналах, відмінних від сигналів, за якими збудовано макромодель. Експериментальні перехідні процеси відтворено двома нелінійними макромоделями першого порядку зі се-редньоквадратичною похибкою, меншою за 1 %.

Отримана макромодель вирізняється гранично малим порядком і високою точністю відтворення ви-хідних сигналів.

Yaroslav Matviychuk – Doctor of Engineering, Professor. Born in Lviv, Ukraine. He received his Master's Degree as Radiophysicist at Department of Physics of Ivan Franko Lviv State University in 1969, PhD in 1974, DSc. in 1998. Since 2000 he has been working as Professor at Lviv Polytechnic National University, Ukraine.

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