$$M^{\text{odeling}}_{\text{athematical}} MC^{\text{omputing}}$$

Construction of the solution of the thermal-convective drying problem for porous solids in drying plants

Chaplya Ye. Ya.[1], Hayvas B. I.[2], Torskyy A. R.[2]

[1]*Institute of Mechanics and Applied Informatics Kazimierz Wielki University in Bydgoszcz*
1 Kopernik str., 1, 85-064, Bydgoszcz, Poland
[2]*Centre of Mathematical Modelling of Ukrainian National Academy of Sciences*
15 Dudayev str., 79005, Lviv, Ukraine

In this paper the approximate solutions of the one-dimensional linear mathematical model of porous bodies drying are obtained, which makes it possible to establish the temperature of the phase transition for arbitrary changes of temperature regimes of a drying agent, the changes of the relative moisture in the body during drying as a function of geometrical and physical parameters, the influence of the rate and relative moisture of a drying agent on the drying process in order to optimize this process.

1. Introduction

The processes of hydrothermal treatment of materials to dry are power-consuming, and the application of thermo-moist treatment is the main means to create energy saving technologies of their drying. The use of thermal regimes of drying which vary in time can significantly save the thermal energy in the process and increase the material quality parameters. When the temperature of the drying agent decreases, the temperature of the surface layers of the material decreases too. In this regard, an additional temperature gradient arises that drives the moisture movement in the material to dry. During cooling, the relative moisture of drying agent increases, thermo moist processing of the surface of the material has place due to which internal stresses are reduced. There are still undefined moments how to perform thermo moist processing and how long it should be for different materials. This requires the development of methods for studying the processes of heat and mass transfer and diagnosis of stress-strain state of materials, and determining according to the results of diagnosis the optimal values of the parameters of the drying agent [1]. To avoid material cracking is also possible in the way of decreasing of the tensile stress by reducing the differences in moisture along cross-section. This can be achieved by weakening of the regime at the beginning of drying and conducting thermo treatment in the middle and at the end of the process [2]. Problems of modeling the drying processes with taking into account changes in the state of moisture in the body over time, qualitative analysis of the formation and dynamics of the area of dried pores in terms of isothermal drying and two-phase zone are investigated in [3–6].

The aim of this paper is to construct an approximate solution of the nonlinear problem of drying the layer of the thickness

$$2L_0(-L_0 \leqslant y \leqslant L_0) \tag{1}$$

under the influence of a convection and heat unsteady flow of the drying agent in order to estimate the influence of variable in time thermal regime of the drying plant onto the process.

The evenness and symmetry of bilateral drying the material is reached by means of circulation of drying agent. We shall solve the problem as a problem with changeable in time boundary of phase transition. Mass transfer occurs with the thickness (1) of the body.

2. Problem formulation

The problem is reduced to solving the following system of equations in the dried domain.

The equation of balance of energy (the heat equation)

$$\left[\Pi(c_v\gamma_v + c_a\gamma_a) + (1-\Pi)c_s\gamma_s\right]\frac{\partial T}{\partial t} = \frac{\partial}{\partial y}\left(\lambda_c\frac{\partial T}{\partial y}\right) + W, \quad L_m \leqslant y \leqslant L_0, \quad 0 \leqslant L_m \leqslant L_0. \quad (2.1)$$

The equation of mass transfer

$$\gamma_v\frac{K_g}{\mu_g}\frac{\partial^2 P}{\partial y^2} + \frac{\partial}{\partial y}\left[D_1\left(\frac{\partial\gamma_v}{\partial y}\right)\right] = 0, \quad \gamma_a\frac{K_g}{\mu_g}\frac{\partial P}{\partial y} + D_1\left(\frac{\partial\gamma_a}{\partial y}\right) = 0. \quad (2.2)$$

The equation of energy balance on the moving boundary of phase transitions $y = L_m$

$$-\lambda_c\frac{\partial T}{\partial y}\bigg|_{y=L_m+0} = r_k\frac{K_g}{\mu_g}\frac{\partial P}{\partial y}\bigg|_{y=L_m+0}. \quad (2.3)$$

Linearized equation of state on moving boundary of phase transitions $y = L_m$

$$T_m = T_{mk} + \alpha_{mk}P_n, \quad (2.4)$$

where c_v, γ_v; c_a, γ_a is the heat capacity and the density of vapor and air, respectively; F is the source factor, Π is the porosity of the material; $D_1 = \left(D + 1.064\sqrt{\frac{RT}{M_v}}\varepsilon\right)$ is the effective coefficient of diffusion; K_g is the coefficient of gas permeability; μ_g is the coefficient of dynamic viscosity; R, T are the gas constant and the temperature; ε is the coefficient of vapour molecular flow; $T_{mk} = \frac{9T_kV_k}{8V}$, $\alpha_{mk} = \frac{3T_kV}{8V_k}$, here T_k, V_k are the critical temperature and the volume, r_k is the heat of phase transition.

In particular, in [7, 8] the temperature of saturated vapour $T_m = 83 + 16\cdot10^{-5}P_n$ is linear approximation of the state equation for the change of saturated vapour pressure P_n.

On the border $y = L_0$ we assume a heat transfer by Newton's law

$$\lambda_c\frac{\partial T}{\partial y} + \tilde{\alpha}\left[T - u(t)\right] = 0, \quad (2.5)$$

where $\tilde{\alpha}$ is the heat transfer coefficient, $u(t)$ is variable in time the temperature of drying agent, λ_c is effective thermal conductance of dried domain; and on the interface of a phase transition $y = L_m$, we consider that

$$T = T_m, \quad (2.6)$$

where T_m is unknown phase transition temperature dependent only on the vapor pressure P_n, i.e. $T_m = f(P_n)$. In addition, we use the formula

$$D_1 = D_{ij}^1 = \left(1/D_i^\infty + (1-\alpha_{ij}y_i)/D_{ij}\right)^{-1},$$

where $D_{va}^1 = D_{av}^1 = D_1$ is the effective coefficient of diffusion, $D_{va} = D_{av} = D_{ij}$ is the effective binary coefficient of diffusion in macropores, the term in the expression $D_v^\infty = D^\infty$ represents the Knudsen's effect of vapour flow in micropores.

On the surfaces of porous plate, the conditions of convective mass transfer are satisfied. Due to the symmetry of the problem, let us write down them only for one of the sides. The conditions on the surfaces $y = L_0$ and $y = L_m$ of the gas domain can be written as follows

for $y = L_0$

$$\gamma_v \frac{K}{\mu_g} \frac{\partial P}{\partial y} + D_1 \left(\frac{\partial \gamma_v}{\partial y} \right) = -j_1, \quad \gamma_a = \gamma_{a0}, \tag{2.7}$$

for $y = L_m$

$$\gamma_v = \gamma_n, \tag{2.8}$$

$$j_1 = \tilde{\beta} \left(\gamma_v - \gamma_0 \right). \tag{2.9}$$

Here $\tilde{\beta}$ is the mass transfer coefficient, γ_n is the density of saturated vapor at the given temperature of phase transition (unknown), γ_0 is the density of vapor in environment outside the porous body (in a drying agent). We introduce dimensionless variables $\tau = \frac{at}{L_0^2}$, $\kappa = \frac{y}{L_0}$, $\kappa_m = \frac{L_m}{L_0}$, where κ_m the dimensionless coordinate of the boundary of the phase transition. In particular cases the temperature of the drying agent can be represented as a Fourier series

$$u(\tau) = \alpha_0 + \sum_{n=1}^{p} \left(\alpha_n \cos \nu_n^2 \tau + \beta_n \sin \nu_n^2 \tau \right). \tag{2.10}$$

The equations of the movement of the interface

$$\frac{d\kappa_m}{dt} = -\frac{j(\kappa_m)}{\Pi \gamma_L L_0} \tag{2.11}$$

with the initial condition

$$\kappa_m = 1. \tag{2.12}$$

The system of equations (2.1), (2.12) describes a model of convective heat-drying of a plate (layer) in a drying installation with variable boundary phase transition.

The heat equation takes the form

$$\frac{\partial T}{\partial \tau}(\kappa, \tau) = \frac{\partial^2 T(\kappa, \tau)}{\partial \kappa^2} + F, \tag{2.13}$$

where $F = \dfrac{L^2 W}{ac\gamma}$ is the source factor, $a = \dfrac{\lambda_c}{\Pi[(1 - \kappa_m)(c_v \gamma_v + c_a \gamma_a)] + (1 - \Pi)c_s \gamma_s}$ and the boundary conditions on the boundary $\kappa = 1$

$$\frac{\partial T}{\partial \kappa} + H[T - u(\tau)] = 0, \quad H = \frac{\tilde{\alpha} L_0}{\lambda}, \tag{2.14}$$

and on the interface of the phase transition $\kappa = \kappa_m$

$$T = T_m. \tag{2.15}$$

We solve the problem under the following initial condition

$$T(\kappa, 0) = f(\kappa). \tag{2.16}$$

Further, to construct the solution of the problem (2.1), (2.5), and (2.7) we use the influence function (Green's) of the problem with the variable boundary of phase transition κ_m under homogeneous boundary conditions of the first and the third kind.

The influence function

$$G(\kappa, \kappa_m, \xi, \tau) = \sum_{n=1}^{\infty} \frac{\left[\frac{\sin \mu_n (\kappa - \kappa_m)}{\mu_n}\right] \left[\cos \mu_n (1 - \xi) + H \frac{\sin \mu_n (1 - \xi)}{\mu_n}\right] e^{-\mu_n^2 \tau}}{\cos \mu_n (1 - \kappa_m) \left[(\mu_n^2 + H^2)(1 - \kappa_m) + H\right]}, \qquad (2.17)$$

where μ_n are the roots of the transcendental equation

$$\operatorname{tg} \mu_n (1 - \kappa_m) = -\frac{\mu_n}{H}, \qquad (2.18)$$

is obtained from the solution of the heat conduction problem (2.13), (2.14) where the functions $u(\tau)$ and W are given implicitly using Laplace transforms, formulae of Duhamel-Neumann, and the convolution theorem under the assumption that

$$T_m(\kappa_m, \tau) = 0, \quad u(1, \tau) = 0.$$

2.1. Representation of the solution of heat conduction problem in trigonometric form

It is known that an arbitrary function defined at the interval of the length $2l$ can be expanded in a Fourier series, which is identical with the Fourier series of its periodic extension along the whole axis Ox. Let the control functions $u(\tau)$ are represented as a trigonometric series (2.6). The solution of the heat conduction problem (2.13)–(2.16) under the absence of internal sources $F = 0$ we try in the form

$$T(\kappa, \tau) = T_1(\kappa, \tau) + T^*(\kappa, \tau).$$

Here the function

$$T_1(\kappa, \tau) = \varphi_0(\kappa) + \sum_{n=1}^{p} \left[\varphi_n(\kappa) \cos \nu_n^2 \tau + \chi_n(\kappa) \sin \nu_n^2 \tau\right] \qquad (2.19)$$

is the solution of the heat equation that satisfies the boundary conditions of the problem, but does not satisfy the initial condition. T^* is the solution of the heat conduction problem that satisfies the initial condition and the homogeneous boundary conditions. Therefore, it is basic for solving at the initial period when the temperature distribution is strongly affected by the initial state, i.e. during the irregular regime.

If we substitute the solution (2.19) into the equation (2.13) and into the boundary conditions, to determine the unknown functions $\varphi_n(\kappa)$, $\chi_n(\kappa)$ $(n = 0, 1, \ldots, p; \ \nu_0 = 0)$ we obtain the system of equations [9]

$$\frac{d^2 \varphi_n}{d\kappa^2} - \nu_n^2 \chi_n = 0 \quad (n = 0, 1, \ldots, p; \quad \nu_0 = 0),$$

$$\frac{d^2 \chi_n}{d\kappa^2} + \nu_n^2 \varphi_n = 0 \quad (n = 1, \ldots, p;), \qquad (2.20)$$

and taking into account the representations (2.19) — the boundary conditions take the form

$$\frac{d\varphi_n(1)}{d\kappa} + H\varphi_n(1) = H\alpha_n, \quad \varphi_n(\kappa_m) = 0 \quad (n = 0, 1, \ldots, p); \qquad (2.21)$$

$$\frac{d\chi_n(1)}{d\kappa} + H\chi_n(1) = H\beta_n, \quad \chi_n(\kappa_m) = 0, \quad (n = 0, 1, \ldots, p). \qquad (2.22)$$

The term $T^*(\kappa, \tau)$ we determine from the heat equation

$$\frac{\partial T^*}{\partial \tau}(\kappa, \tau) = \frac{\partial^2 T^*(\kappa, \tau)}{\partial \kappa^2} \qquad (2.23)$$

with homogeneous boundary conditions

$$\frac{\partial T^*(1,\tau)}{\partial \kappa} + HT^*(1,\tau) = 0, \quad T^*(\kappa_m, \tau) = 0 \tag{2.24}$$

and the initial condition so that it satisfies the condition (2.7). $T^*(\kappa, 0) = f(\kappa) - T_0(\kappa)$, where $T_0(\kappa)$ is the solution of the problem (2.19)–(2.22) for $\tau = 0$.

From the first equation of the system (2.20) for $n = 0$ we find $\varphi_0(\kappa) = a_0 + b_0\kappa$, where

$$a_0 = \frac{T_m(1+H) - H\alpha_0\kappa_m}{[1 + H(1 - \kappa_m)]}, \quad b_0 = \frac{H(\alpha_0 - T_m)}{[1 + H(1 - \kappa_m)]}. \tag{2.25}$$

T_m is still unknown function.

For solving the system of equations (2.22) $(n = 1, \ldots, p)$, we introduce the complex function

$$\Phi_n(\kappa) = \varphi_n(\kappa) - i\chi_n(\kappa).$$

Then the associated system of equations (2.22) for functions $\varphi_n(\kappa)$, $\chi_n(\kappa)$ is reduced to separate p equations for complex functions $\Phi_n(\kappa)$

$$\frac{d^2\Phi_n}{d\kappa^2} - i\nu_n^2\Phi_n = 0 \quad (n = 1, 2, \ldots, p), \quad i = \sqrt{-1}, \tag{2.26}$$

having the solutions

$$\Phi_n(\kappa) = C_n \operatorname{ch} \sqrt{i}\nu_n\kappa + D_n \operatorname{sh} \sqrt{i}\nu_n\kappa. \tag{2.27}$$

From these expressions, after presenting complex coefficients in algebraic form $C_n = a_n + ib_n$, $D_n = c_n + id_n$, and considering that $\sqrt{i} = \frac{1+i}{\sqrt{2}}$, and denoting

$$\frac{\nu_n\kappa}{\sqrt{2}} = \tilde{\nu}_{n\kappa},$$

$$\operatorname{ch} \sqrt{i}\nu_n\kappa = \operatorname{ch} \tilde{\nu}_{n\kappa} \cos \tilde{\nu}_{n\kappa} + i \operatorname{sh} \tilde{\nu}_{n\kappa} \sin \tilde{\nu}_{n\kappa},$$

$$\operatorname{sh} \sqrt{i}\nu_n\kappa = \operatorname{sh} \tilde{\nu}_{n\kappa} \cos \tilde{\nu}_{n\kappa} + i \operatorname{ch} \tilde{\nu}_{n\kappa} \sin \tilde{\nu}_{n\kappa}; \tag{2.28}$$

we find the functions $\varphi_n(\kappa)$ and $\chi_n(\kappa)$:

$$\varphi_n(\kappa) = a_n Y_1(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa}) - b_n Y_2(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa}) + c_n Y_3(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa}) - d_n Y_4(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa}), \tag{2.29}$$

$$\chi_n(\kappa) = -a_n Y_2(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa}) - b_n Y_1(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa}) - c_n Y_4(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa}) - d_n Y_3(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa}), \quad (n = 1, 2, \ldots, p). \tag{2.30}$$

Here $Y_m(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa})$ $(m = 1, 2, 3, 4)$ are hyperbole-trigonometric functions of the form

$$Y_1(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{kn}) = \operatorname{ch} \tilde{\nu}_{n\kappa} \cos \tilde{\nu}_{k\kappa}, \quad Y_2(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{k\kappa}) = \operatorname{sh} \tilde{\nu}_{n\kappa} \sin \tilde{\nu}_{k\kappa}, \tag{2.31}$$

$$Y_3(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{k\kappa}) = \operatorname{sh} \tilde{\nu}_{n\kappa} \cos \tilde{\nu}_{k\kappa}, \quad Y_4(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{k\kappa}) = \operatorname{ch} \tilde{\nu}_{n\kappa} \sin \tilde{\nu}_{k\kappa}. \tag{2.32}$$

On the basis of the boundary conditions, the coefficients a_n, b_n, c_n, d_n are the functions of the moving coordinate κ_m and they have the form

$$a_n(\kappa_m) = \frac{\Delta_1(\kappa_m)}{\Delta(\kappa_m)}, \quad b_n(\kappa_m) = \frac{\Delta_2(\kappa_m)}{\Delta(\kappa_m)},$$

$$c_n(\kappa_m) = \frac{\Delta_3(\kappa_m)}{\Delta(\kappa_m)}, \quad d_n(\kappa_m) = \frac{\Delta_4(\kappa_m)}{\Delta(\kappa_m)}, \tag{2.33}$$

where $\Delta_i(\kappa_m$, $(i = 1, 2, 3, 4)$ are determinants of the fourth order. In order to simplify the solving, we denote $Y_m(\tilde{\nu}_{n\kappa}, \tilde{\nu}_{n\kappa}) = Y_m\left(\frac{\nu_n\kappa}{\sqrt{2}}\right)$.

With this, we take into account that

$$Y_1' \left(\frac{\nu_n \kappa}{\sqrt{2}} \right) = \frac{\nu_n}{\sqrt{2}} \left[Y_3 \left(\tilde{\nu}_{n\kappa} \right) - Y_4 \left(\tilde{\nu}_{n\kappa} \right) \right], \quad Y_2' \left(\frac{\nu_n \kappa}{\sqrt{2}} \right) = \frac{\nu_n}{\sqrt{2}} \left[Y_4 \left(\tilde{\nu}_{n\kappa} \right) + Y_3 \left(\tilde{\nu}_{n\kappa} \right) \right],$$

$$Y_3' \left(\frac{\nu_n \kappa}{\sqrt{2}} \right) = \frac{\nu_n}{\sqrt{2}} \left[Y_1 \left(\tilde{\nu}_{n\kappa} \right) - Y_2 \left(\tilde{\nu}_{n\kappa} \right) \right], \quad Y_4' \left(\frac{\nu_n \kappa}{\sqrt{2}} \right) = \frac{\nu_n}{\sqrt{2}} \left[Y_2 \left(\tilde{\nu}_{n\kappa} \right) + Y_1 \left(\tilde{\nu}_{n\kappa} \right) \right]. \tag{2.34}$$

$\Delta(\kappa_m) = |\delta_{ij}|$ are determinants of the system of equations $\Delta(\kappa_m) x_n = f$, where $x_n = (a_n, b_n, c_n, d_n)^T$

$$\delta_{11} = \frac{\nu_n}{\sqrt{2}} \left[Y_3 \left(\frac{\nu_n}{\sqrt{2}} \right) - Y_4 \left(\frac{\nu_n}{\sqrt{2}} \right) \right] + H Y_1 \left(\frac{\nu_n}{\sqrt{2}} \right),$$

$$\delta_{12} = - \left\{ \frac{\nu_n}{\sqrt{2}} \left[Y_3 \left(\frac{\nu_n}{\sqrt{2}} \right) + Y_4 \left(\frac{\nu_n}{\sqrt{2}} \right) \right] + H Y_2 \left(\frac{\nu_n}{\sqrt{2}} \right) \right\},$$

$$\delta_{13} = \frac{\nu_n}{\sqrt{2}} \left[Y_1 \left(\frac{\nu_n}{\sqrt{2}} \right) - Y_2 \left(\frac{\nu_n}{\sqrt{2}} \right) \right] + H Y_3 \left(\frac{\nu_n}{\sqrt{2}} \right),$$

$$\delta_{14} = - \left\{ \frac{\nu_n}{\sqrt{2}} \left[Y_1 \left(\frac{\nu_n}{\sqrt{2}} \right) + Y_2 \left(\frac{\nu_n}{\sqrt{2}} \right) \right] + H Y_4 \left(\frac{\nu_n}{\sqrt{2}} \right) \right\},$$

$$\delta_{21} = \delta_{12}, \quad \delta_{22} = -\delta_{11}, \quad \delta_{23} = \delta_{14}, \quad \delta_{24} = -\delta_{13},$$

$$\delta_{31} = Y_1 \left(\frac{\nu_n}{\sqrt{2}} \kappa_m \right), \quad \delta_{32} = -Y_2 \left(\frac{\nu_n}{\sqrt{2}} \kappa_m \right), \ \delta_{33} = Y_3 \left(\frac{\nu_n}{\sqrt{2}} \kappa_m \right), \quad \delta_{34} = -Y_4 \left(\frac{\nu_n}{\sqrt{2}} \kappa_m \right),$$

$$\delta_{41} = -Y_2 \left(\frac{\nu_n}{\sqrt{2}} \kappa_m \right), \ \delta_{42} = -Y_1 \left(\frac{\nu_n}{\sqrt{2}} \kappa_m \right), \ \delta_{43} = -Y_4 \left(\frac{\nu_n}{\sqrt{2}} \kappa_m \right), \ \delta_{44} = -Y_3 \left(\frac{\nu_n}{\sqrt{2}} \kappa_m \right). \tag{2.35}$$

The column of constant terms has the form

$$f = \begin{pmatrix} H\alpha_n \\ H\beta_n \\ 0 \\ 0 \end{pmatrix}. \tag{2.36}$$

A solution of the boundary value problem (2.23), (2.24) we write as follows

$$T^*(\kappa, \tau) = \int_{\kappa_m}^{1} G(\kappa, \kappa_m, \xi, \tau) f_1(\xi) \, d\xi = \sum_{n=1}^{\infty} A_n \frac{\sin \mu_n (\kappa - \kappa_m)}{\mu_n} \exp \left(-\mu_n^2 \tau \right), \tag{2.37}$$

$$A_n = \frac{1}{\Delta_n(\mu_n, \kappa_m)} \int_{\kappa_m}^{1} f_1(\xi) \left[\cos \mu_n (1 - \xi) + \frac{H}{\mu_n} \sin \mu_n (1 - \xi) \right] d\xi, \tag{2.38}$$

$$\Delta_n \left(\mu_n, \kappa_m \right) = \cos \mu_n \left(1 - \kappa_m \right) \left[\left(\mu_n^2 + H^2 \right) \left(1 - \kappa_m \right) + H \right], \tag{2.39}$$

$$f_1(\kappa) = f(\kappa) - \bar{T}_0, \tag{2.40}$$

where

$$\bar{T}_0 = T_1(\kappa, 0) = \sum_{n=0}^{P} \varphi_n(\kappa).$$

If the initial temperature distribution is uniform, i.e. $T(\kappa, 0) = T_0$, with taking into account (2.17), the contribution of the initial temperature into $T^*(\kappa, \kappa_m, \tau)$ has the form

$$T_0^*(\kappa, \kappa_m) = \sum_{n=1}^{\infty} \frac{\sin \mu_n (\kappa - \kappa_m)}{\mu_n} A_n^*(\kappa_m) e^{-\mu_n^2 \tau}, \tag{2.41}$$

$$A_n^* = \frac{2 T_0 H}{\Delta_n} \left\{ \frac{\sin \mu_n (1 - \kappa_m)}{\mu_n} - \frac{H}{\mu_n^2} \left[\cos \mu_n (1 - \kappa_m) - 1 \right] \right\}. \tag{2.42}$$

The contribution of the sum $\sum_{n=1}^{p} \varphi_n(\kappa)$ into $T^*(\kappa, \kappa_m)$ we represent as

$$T_\varphi^* = - \int_{\kappa_m}^{1} \sum_{k=1}^{p} \varphi_n(\xi) \times G(\kappa, \kappa_m, \xi, \tau) d\xi,$$

$$T^* = \sum_{n=1}^{\infty} \frac{\sin \mu_n (\kappa - \kappa_m)}{\mu_n} A_n(\kappa_m) e^{-\mu_n \tau}, \tag{2.43}$$

where

$$A_n(\kappa_m) = \sum_{k=1}^{p} \left[a_k \tilde{Z}_{nk1}^* - b_k \tilde{Z}_{nk2}^* + c_k \tilde{Z}_{nk3}^* - d_k \tilde{Z}_{nk4}^* \right]; \tag{2.44}$$

$$\tilde{Z}_{nki}^* = \frac{2H}{\Delta_n} Z_{nki}^*; \tag{2.45}$$

$$Z_{nk1}^* (\mu_n, \nu_k, 1, \kappa_m) = (\Delta_{1n}) \left\langle g_{1kn} \, \Phi_{11kn} [\alpha_k \xi, \beta_{1kn} \xi] + g_{2kn} \, \Phi_{12kn} [\alpha_k \xi, \beta_{2kn} \xi] \right\rangle_{\kappa_m}^1 +$$
$$+ (\Delta_{2n}) \left\langle g_{1kn} \, \Phi_{21kn} [\alpha_k \xi, \beta_{1kn} \xi] + g_{2kn} \, \Phi_{22kn} [\alpha_k \xi, \beta_{2kn} \xi] \right\rangle_{\kappa_m}^1 ;$$
$$Z_{nk2}^* (\mu_n, \nu_k, 1, \kappa_m) = (\Delta_{1n}) \left\langle g_{2kn} \, \Phi_{32kn} [\alpha_k \xi, \beta_{2kn} \xi] - g_{1kn} \, \Phi_{31kn} [\alpha_k \xi, \beta_{1kn} \xi] \right\rangle_{\kappa_m}^1 +$$
$$+ (\Delta_{2n}) \left\langle g_{2kn} \, \Phi_{42kn} [\alpha_k \xi, \beta_{1kn} \xi] - g_{1kn} \, \Phi_{41kn} [\alpha_k \xi, \beta_{1kn} \xi] \right\rangle_{\kappa_m}^1 ;$$
$$Z_{nk3}^* (\mu_n, \nu_k, 1, \kappa_m) = (\Delta_{1n}) \left\langle g_{1kn} \, \Phi_{41kn} [\alpha_k \xi, \beta_{1kn} \xi] + g_{2kn} \, \Phi_{42kn} [\alpha_k \xi, \beta_{2kn} \xi] \right\rangle_{\kappa_m}^1 -$$
$$- (\Delta_{2n}) \left\langle g_{2kn} \, \Phi_{32kn} [\alpha_k \xi, \beta_{2kn} \xi] + g_{1kn} \, \Phi_{31kn} [\alpha_k \xi, \beta_{1kn} \xi] \right\rangle_{\kappa_m}^1 ;$$
$$Z_{nk4}^* (\mu_n, \nu_k, 1, \kappa_m) = (\Delta_{1n}) \left\langle g_{1kn} \, \Phi_{21kn} [\alpha_k \xi, \beta_{1kn} \xi] - g_{2kn} \, \Phi_{22kn} [\alpha_k \xi, \beta_{2kn} \xi] \right\rangle_{\kappa_m}^1 +$$
$$+ (\Delta_{2n}) \left\langle g_{2kn} \, \Phi_{12kn} [\alpha_k \xi, \beta_{2kn} \xi] - g_{1kn} \, \Phi_{11kn} [\alpha_k \xi, \beta_{1kn} \xi] \right\rangle_{\kappa_m}^1 . \tag{2.46}$$

$$\Delta_{1n} = \cos \mu_n + \frac{H}{\mu_n} \sin \mu_n, \quad \Delta_{2n} = \sin \mu_n - \frac{H}{\mu_n} \cos \mu_n,$$

$$\{ \alpha_k \, Y_3 [\alpha_k \xi, \beta_{jkn} \xi] + \beta_{jkn} Y_4 [\alpha_k \xi, \beta_{jkn} \xi] \} = \Phi_{1jkn}(\xi);$$
$$\{ \alpha_k \, Y_2 [\alpha_k \xi, \beta_{jkn} \xi] - \beta_{jkn} Y_1 [\alpha_k \xi, \beta_{jkn} \xi] \} = \Phi_{2jkn}(\xi);$$
$$\{ -\alpha_k \, Y_4 [\alpha_k \xi, \beta_{jkn} \xi] + \beta_{jkn} Y_3 [\alpha_k \xi, \beta_{jkn} \xi] \} = \Phi_{3jkn}(\xi);$$
$$\{ \alpha_k \, Y_1 [\alpha_k \xi, \beta_{jkn} \xi] + \beta_{jkn} Y_2 [\alpha_k \xi, \beta_{jkn} \xi] \} = \Phi_{4jkn}(\xi); \tag{2.47}$$

$$\alpha_k = \frac{\nu_k}{\sqrt{2}}, \quad \beta_{1kn} = \frac{\nu_k}{\sqrt{2}} + \mu_n, \quad \beta_{2kn} = -\frac{\nu_k}{\sqrt{2}} + \mu_n,$$

$$g_{1kn} = \frac{1}{2 \left[\frac{\nu_k^2}{2} + \left(\frac{\nu_k}{\sqrt{2}} + \mu_n \right)^2 \right]}, \quad g_{2kn} = \frac{1}{2 \left[\frac{\nu_k^2}{2} + \left(-\frac{\nu_k}{\sqrt{2}} + \mu_n \right)^2 \right]}, \tag{2.48}$$

$$T^* + T_0^* = \sum_{n=1}^{\infty} \frac{\sin \mu_n (\kappa - \kappa_m)}{\mu_n} B_n(\kappa_m) e^{-\mu_n \tau}, \tag{2.49}$$

$$B_m(\kappa_m) = A_m(\kappa_m) + A_m^*(\kappa_m). \tag{2.50}$$

This sum contributes to the temperature change just within the interval $[\kappa_m, 1]$ of variable thickness.

Thereby, the temperature under random convective heating, depending on the time, can be written as

$$T(\kappa, \tau) = \varphi_0(\kappa) + \sum_{n=1}^{p} \left[\varphi_n(\kappa) \cos \nu_n^2 \tau + \chi_n(\kappa) \sin \nu_n^2 \tau \right] + \sum_{n=1}^{\infty} \frac{\sin \mu_n (\kappa - \kappa_m)}{\mu_n} B_n(\kappa_m) e^{-\mu_n^2 \tau}, \tag{2.51}$$

where

$$B_n(\kappa_m) = \frac{2T_0 H}{\Delta_n} \left\{ \frac{\sin \mu_n (1 - \kappa_m)}{\mu_n} - \frac{H}{\mu_n^2} \left[\cos \mu_n (1 - \kappa_m) - 1 \right] \right\} +$$

$$+ \sum_{k=1}^{p} \left\{ a_k(\kappa_m) \left[\tilde{Z}_{nk1}^* \left(\frac{\nu_k}{\sqrt{2}} \right) - \tilde{Z}_{nk1}^* \left(\frac{\nu_k \kappa_m}{\sqrt{2}} \right) \right] - b_k(\kappa_m) \left[\tilde{Z}_{nk2}^* \left(\frac{\nu_k}{\sqrt{2}} \right) - \tilde{Z}_{nk2}^* \left(\frac{\nu_k \kappa_m}{\sqrt{2}} \right) \right] +$$

$$+ c_k(\kappa_m) \left[\tilde{Z}_{nk3}^* \left(\frac{\nu_k}{\sqrt{2}} \right) - \tilde{Z}_{nk3}^* \left(\frac{\nu_k \kappa_m}{\sqrt{2}} \right) \right] - d_k(\kappa_m) \left[\tilde{Z}_{nk4}^* \left(\frac{\nu_k}{\sqrt{2}} \right) - \tilde{Z}_{nk4}^* \left(\frac{\nu_k \kappa_m}{\sqrt{2}} \right) \right] \right\}.$$

This formula shows that

$$B_n(\kappa_m = 1) = 0.$$

It should be noted that in the formula (2.51), the temperature of phase transition is still unknown.

3. Solving the problem of moisture conduction during convective drying

Transfer of vapor-air mixture in a dried area occurs mainly by means of the mechanism of molecular diffusion, effusion, and filtration. For diffusion transfer, the driving force is the partial pressure gradient, for filtration – the total pressure gradient in the gas phase. The structure of the medium occupied by the gas phase does not significantly affect the value of the effective diffusion coefficient. In contrast to diffusion, gas filtration in wetted porous matrices depends to a large extend on the structure. The gas filtration coefficient increases with increasing porosity, the size of particles of input material, and the reduction of liquid content in pores. The dependence of the filtration coefficient on the porosity is described by the formula $K = c\Pi^3 (1 - \Pi)^{-2} d \exp \left(-5\kappa_m^2 \right)$, where Π is the porosity, d is the average particle size of the skeleton, κ_m is the saturation of the pore space with liquid, c is the empirical constant $c \approx 1.6 \cdot 10^{-7} \div 3.4 \cdot 10^{-7}$.

If into the equations (2.2) we introduce the dimensionless variables $\gamma_a = \gamma_{a0} \xi$, $\gamma_v = \gamma_n \eta$, $y = L_0 \kappa$, then we calculate the values of the dimensionless variables

$$\tilde{a} = \frac{D_1 M_a \mu_g}{K \gamma_{a0} R T_0}, \quad \tilde{b} = \frac{\gamma_n}{\gamma_{a0}} \frac{M_a}{M_v}, \tag{3.1}$$

and assume D_1 to be $D_1 = \left(D + 1.064 \varepsilon \sqrt{\frac{RT_0}{M_v}} \right)$ for a given change in temperature of the drying agent, where T_0 is the initial temperature; after that we can linearize the original system of equations. Here γ_{ao} is the air density on the outer walls of the plate. If we integrate the equation of the system (2.2) under the condition that the air density is not significantly changed along the length of the capillary, the density of vapour $\eta(\kappa, \kappa_m)$ will be determined as follows $\eta = \left[-A + \sqrt{(A + 1)^2 - 2B(\eta_1 - \eta_0)(\kappa - \kappa_m)} \right],$

where $A = \frac{(1+\tilde{a})}{\tilde{b}}$, $B = -\beta'A$, $\beta' = \frac{L_0\tilde{\beta}}{D_1}$. On the surface $\kappa = 1$, the equation is satisfied

$$\frac{\tilde{b}}{(1+\tilde{a})}\frac{(\eta_1^2 - 1)}{2} + \eta_1 - 1 + \beta'(\eta_1 - \eta_0)(1 - \kappa_m) = 0, \tag{3.2}$$

the solution of which determines the value of the dimensionless density of vapour on the outer surface of the plate. Here η_1, η_0 are the relative saturations of vapour on the surface of the plate and in the drying agent. Denote $z_m = 1 - \kappa_m$, where z_m is the relative moisture loss in the drying process, and denote the change of the width of the dried area. A physically reasonable solution has the form

$$\eta_1 = -(A + Bz_m) + \sqrt{U + Sz_m + B^2 z_m^2}, \tag{3.3}$$

where

$$U = (A+1)^2, \quad S = 2B(A + \eta_0).$$

After determining the vapour density value on a wall, according to the condition (3.3) the value of the flux j we present in the form

$$j = H_1\left\{-(a_1 + Bz_m) + \sqrt{U + Sz_m + B^2 z_m^2}\right\}, \tag{3.4}$$

where $a_1 = A + \eta_0$, $H_1 = \tilde{\beta}\gamma_{n0}/\Pi L\gamma_L$, and the pressure at arbitrary point is equal to

$$P(\kappa, \kappa_m) = \frac{\gamma_n \eta}{M_v}RT(\kappa) = \left[-A + \sqrt{(A+1)^2 - 2B(\eta_1 - \eta_0)(\kappa - \kappa_m)}\right]\frac{\gamma_n}{M_v}RT(\kappa, \kappa_m), \tag{3.5}$$

where γ_n is the unknown value.

4. Determination of the phase transition temperature with taking into account temperature gradient with the thickness of the plate

Let us determine phase transition temperature T_m from the condition (2.3)

$$-\lambda\frac{\partial T}{\partial \kappa}\Big|_{\kappa=\kappa_m} = r_k\frac{K_g}{\mu_g}\frac{\partial P}{\partial \kappa}\Big|_{\kappa=\kappa_m}. \tag{4.1}$$

where

$$T'_\kappa(\kappa_m, \tau) = (\alpha_0 - T_m)\frac{H}{1+H} + \sum_{n=1}^{p}\left[\varphi'_n(\kappa_m)\cos\nu_n^2\tau + \chi'_n(\kappa_m)\sin\nu_n^2\tau\right] + \sum_{n=1}^{\infty}B_n(\kappa_m)e^{-\mu_n^2\tau}.$$

Note, that $\varphi'_n(\kappa_m)$, $\chi'_n(\kappa_m)$, $B_n(\kappa_m)$ are related with with expansion coefficients of the drying agent temperature control function. Therefore, we present

$$T'_\kappa(\kappa, \kappa_m, \tau)|_{\kappa_m} = \Phi(\kappa_m, \tau, \alpha_n, \mu_n, H) - T_m\frac{H}{1+H} = \Phi - \alpha_{12}T_m, \quad \alpha_{12} = \frac{H}{1+H},$$

$$\Phi(\kappa_m, \tau, \alpha_n, \mu_n, \nu_n, H) = \alpha_0\frac{H}{1+H} + \sum_{n=1}^{p}\left[\varphi'_{n\kappa}(\kappa_m)\cos\nu_n^2\tau + \chi'_{n\kappa}(\kappa_m)\sin\nu_n^2\tau\right] + \sum_{n=1}^{\infty}B_n(\kappa_m)e^{-\mu_n^2\tau}$$

is a part of the temperature gradient, which is dependent on the expansion coefficients and the frequency associated with a change in temperature of the drying agent, moisture saturation of the porous medium, the rate drying agent.

Considering the transport coefficients and specific heat of phase transition r_k to be constant value (when with the change in temperature of the phase transition from 100 to 150ºC, the change of r_k represents less than 20%) [7], and taking into account the linearized dependence between the temperature and the pressure of phase transition $T_m = T_{mk} + \alpha_{mk} P_n$, where the coefficient $\alpha_{mk} \approx 16 \cdot 10^{-5}$ has the dimension $\left[\frac{\text{ºK}}{\text{Па}} \right]$, and $T_{mk} \approx (83 + 273.15)$ºK, we obtain the quadratic equation to determine T_m

$$\sum_{i=0}^{2} C_i T_m^i = 0, \tag{4.2}$$

with the following coefficients

$$C_2 = \frac{B(\eta_1 - \eta_0)}{A+1} + \alpha_{12}(1 + c_{11}), \quad C_1 = -\left(\frac{B(\eta_1 - \eta_0)}{A+1} + \Phi(1 + c_{11}) + T_{mk}\alpha_{12} \right), \quad C_0 = T_{mk}\Phi,$$

$$c_{11} = \frac{\alpha_{mk}\lambda\mu_g}{r_k K_g} = \frac{16\lambda\mu_g}{10^5 r_k K_g}.$$

The solution of this equation

$$T_{m1} \approx T_{mk} \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12} \right] \bigg/ \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12}(1 + c_{11}) \right] +$$

$$+ T_{mk} \left[\frac{B}{A+1}(\eta_1 - \eta_0)c_{11}\Phi \right] \bigg/ \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12}(1 + c_{11}) \right] \times$$

$$\times \left\{ T_{mk} \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12} \right] - (1 + c_{11})\Phi \right\}, \tag{4.3}$$

$$T_{m2} = \Phi(1 + c_{11}) \bigg/ \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12}(1 + c_{11}) \right] -$$

$$- T_{mk} \left[\frac{B}{A+1}(\eta_1 - \eta_0)c_{11}\Phi \right] \bigg/ \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12}(1 + c_{11}) \right] \times$$

$$\times \left\{ T_{mk} \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12} \right] - (1 + c_{11})\Phi \right\}$$

where

$$\alpha_{12} = \frac{H}{1+H}.$$

Here and further, the argument next to the function Φ we do not write for simplification. The second root of the equation depends mainly on the temperature gradient and is crucial for hard drying.

By linearizing of the expression (4.3), and neglecting the temperature gradient squares, we obtain

$$T_m \approx T_{mk} \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12} \right] \bigg/ \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12}(1 + c_{11}) \right] + \tag{4.4}$$

$$+ T_{mk} \left[\frac{B}{A+1}(\eta_1 - \eta_0)c_{11}\Phi \right] \bigg/ \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12}(1 + c_{11}) \right] \left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12} \right].$$

5. Determining the influence of changes in time in the temperature of the drying agent on the drying process

After determining the flux of vapour on the surface, from the equations (3.2), (3.3), and (3.4) and taking into account that $1 - \kappa_m = z_m$ is the loss of relative moisture in the drying process and the change of the width of dried area, we obtain the equation for determining the relative moisture changes

in time and the equations of the liquid-gas interface movement

$$\frac{dz_m}{d\tau} = \frac{j(z_m, T(\tau))}{\Pi \gamma_L L} = \frac{\tilde{\beta} \gamma_{n0} T_0}{\Pi \gamma_L L \left(T_0 + T(\tau)\right)\big|_{\kappa_m}} \left[-(a_1 + Bz_m) + \sqrt{U + Sz_m + B^2 z_m^2}\right] \tag{5.1}$$

under the initial condition $\tau = 0$:

$$z_m = 0. \tag{5.2}$$

For the solving the Cauchy problem (5.1), (5.2) we use the change of variables:

$$\sqrt{U + Sz_m + B^2 z_m^2} = \tilde{\chi} + z_m B.$$

Considering

$$z_m = \frac{\tilde{\chi}^2 - U}{S - 2\tilde{\chi}B}, \quad \tilde{\chi} - a_1 = \frac{2B\tilde{\chi} - S}{2B} \quad \text{and} \quad \frac{dz_m}{d\chi} = 2\frac{\left(-B\tilde{\chi}^2 + S\tilde{\chi} - BU\right)}{(S - 2B\tilde{\chi})^2}, \tag{5.3}$$

we obtain the expression

$$4B\frac{\left(-B\tilde{\chi}^2 + S\tilde{\chi} - BU\right)}{\left((2B\tilde{\chi} - S)^3\right)} \left(1 + \frac{T_m}{T_0}\right) d\tilde{\chi} = H d(t_{iz} + t_{niz}), \tag{5.4}$$

where $H = \tilde{\beta} \gamma_{n0} / \Pi \gamma_L L$.

The first members of the left and right sides of the equation correspond to isothermal drying

$$4B\frac{\left(-B\tilde{\chi}^2 + S\tilde{\chi} - BU\right)}{\left((2B\tilde{\chi} - S)^3\right)} d\tilde{\chi} = H dt_{iz}. \tag{5.5}$$

The effect of change in temperature of the drying agent in time is determined from the equation

$$4B\frac{\left(-B\tilde{\chi}^2 + S\tilde{\chi} - BU\right)}{\left((2B\tilde{\chi} - S)^3\right)} \frac{T_m}{T_0} d\tilde{\chi} = H \, dt_{niz} \tag{5.6}$$

Substituting the expression of the phase transition temperature from the equation (4.4) into (5.6), we obtain

$$\frac{d\tilde{\chi}}{dt} = \frac{HT_0}{4BT_{mk}} \frac{(2B\tilde{\chi} - S)^3}{[-B\tilde{\chi}^2 + S\tilde{\chi} - BU]} \left\{\frac{(\chi - a_3)}{(\chi - a_2)} \left[1 - \frac{(\chi - a_1)\frac{A+1}{B}c_{11}\Phi}{(\chi - a_2)^2}\right]\right\}, \tag{5.7}$$

where

$$a_1 = A + \eta_0, \quad a_2 = a_1 - \frac{A+1}{B}\alpha_{12}, \quad a_3 = a_1 - \frac{A+1}{B}\alpha_{12}(c_{11} + 1), \tag{5.8}$$

$\kappa_m = 1 - \frac{\tilde{\chi}^2 - U}{S - 2\tilde{\chi}B}$, $\tau = \frac{at}{L^2}$, $\varphi'_{n\kappa}(\kappa_m)$, $\chi'_{n\kappa}(\kappa_m)$ are the derivatives of the functions $\varphi_n(\kappa)$, $\chi_n(\kappa)$ for $\kappa = \kappa_m$. The solution of the equation (5.7) must satisfy the initial condition

$$\chi\big|_{t=0} = \sqrt{U}. \tag{5.9}$$

The solution of the equation we try approximately by method of iterations. Integration the equation (5.7) from zero to t with taking into account (5.9) yields

$$\chi = \int_0^t F(\chi, t)dt + \chi_0. \tag{5.10}$$

In this equation the unknown function χ is under the integral sign. Thus, we have come to the integral equation for the function χ that satisfies the equation (5.10) and the initial condition (5.9). We assume $\chi_0 = \chi|_{t=0} = \sqrt{U}$ to be zero approximation of the solution.

The first approximation we obtain from (5.10) as

$$\tilde{\chi}_1(t) = \frac{HT_0}{4BT_{mk}} \frac{\left(2B\sqrt{U}-S\right)^3}{[-2BU+S\tilde{\chi}]} \left\{ \frac{\left(\sqrt{U}-a_3\right)}{\left(\sqrt{U}-a_2\right)} \left[t - \frac{\left(\sqrt{U}-a_1\right)\frac{A+1}{B}c_{11}\Phi_1(1,t)}{\left(\sqrt{U}-a_2\right)^2} \right] \right\} + \sqrt{U},$$

where

$$\Phi(1,t) = \alpha_0 \frac{H}{1+H} t + \frac{L^2}{a} \sum_{n=1}^{p} \frac{1}{\nu^2} \left[\varphi'_{n\kappa}(1) \sin \nu_n^2 \frac{at}{L^2} + \chi'_{n\kappa}(1) \left(\cos \nu_n^2 \frac{at}{L^2} - 1 \right) \right]. \qquad (5.11)$$

Substituting the first approximation into the integrand in the equation (5.10), we obtain the second approximation

$$\chi_2(t) = \int_0^t F(\chi_1(t), t) dt + \chi_0,$$

or

$$\chi_2(t) = \frac{HT_0}{4BT_{mk}} \frac{\left(2B\tilde{\chi}_1(t) - S\right)^3}{\left[-B\tilde{\chi}_1(t)^2 + S\tilde{\chi}_1(t) - BU\right]} \times$$

$$\times \left\{ \left[1 - \frac{\frac{A+1}{B}c_{11}\alpha_{12}}{(\chi_1(t) - a_2)} \right] \left[t - \frac{[\chi_1(t) - a_1]\frac{A+1}{B}c_{11}\Phi_2(\chi_1(t), t)}{[\chi_1(t) - a_2]^2} \right] \right\} + \sqrt{U},$$

$$\Phi_2(\tilde{\chi}_1(t), t) = \alpha_0 \frac{H}{1+H} t +$$

$$+ \frac{L^2}{a} \sum_{n=1}^{p} \frac{1}{\nu_n^2} \left[\varphi'_{n\kappa}\left(1 - \frac{\tilde{\chi}_1^2(t) - U}{S - 2B\tilde{\chi}_1(t)} \right) \cos \nu_n^2 \frac{at}{L^2} + \chi'_{n\kappa}\left(1 - \frac{\tilde{\chi}_1^2(t) - U}{S - 2B\tilde{\chi}_1(t)} \right) \sin \nu_n^2 \frac{at}{L^2} \right] +$$

$$+ B_0 \left(1 - \frac{\tilde{\chi}_1^2(t) - U}{S - 2B\tilde{\chi}_1(t)} \right) + \sum_{n=1}^{\infty} B_n \left(1 - \frac{\tilde{\chi}_1^2(t) - U}{S - 2B\tilde{\chi}_1(t)} \right) e^{-\mu_n^2 \frac{at}{L^2}},$$

where $\kappa_{m1}(t) = 1 - \frac{\tilde{\chi}_1^2(t) - U}{S - 2B\tilde{\chi}_1(t)}$ are the arguments to functions $\varphi'_{n\kappa}(\kappa_{m1})$, $\chi'_{n\kappa}(\kappa_{m1})$, $B_0(\kappa_{m1})$, $B_n(\kappa_{m1})$. Continuing this process, we obtain formulae for the sth approximation

$$\chi_s(t) = \frac{HT_0}{4BT_{mk}} \frac{\left(2B\tilde{\chi}_{s-1}(t) - S\right)^3}{\left[-B\tilde{\chi}_{s-1}(t)^2 + S\tilde{\chi}_{s-1}(t) - BU\right]} \times$$

$$\times \left\{ \left[1 - \frac{\frac{A+1}{B}c_{11}\alpha_{12}}{(\chi_{s-1}(t) - a_2)} \right] \left[t - \frac{[\chi_{s-1}(t) - a_1]\frac{A+1}{B}c_{11}\Phi_2(\chi_{s-1}(t), t)}{[\chi_{s-1}(t) - a_2]^2} \right] \right\} + \sqrt{U}$$

$$\Phi_2(\tilde{\chi}_{s-1}(t), t) = \alpha_0 \frac{H}{1+H} t + \frac{L^2}{a} \sum_{n=1}^{p} \frac{1}{\nu_n^2} \left[\varphi'_{n\kappa}\left(1 - \frac{\tilde{\chi}_{s-1}^2(t) - U}{S - 2B\tilde{\chi}_{s-1}(t)} \right) \cos \nu_n^2 \frac{at}{L^2} + \right.$$

$$+ \chi'_{n\kappa}\left(1 - \frac{\tilde{\chi}_{s-1}^2(t) - U}{S - 2B\tilde{\chi}_{s-1}(t)} \right) \sin \nu_n^2 \frac{at}{L^2} \right] +$$

$$+ B_0 \left(1 - \frac{\tilde{\chi}_{s-1}^2(t) - U}{S - 2B\tilde{\chi}_{s-1}(t)} \right) + \sum_{n=1}^{\infty} B_n \left(1 - \frac{\tilde{\chi}_{s-1}^2(t) - U}{S - 2B\tilde{\chi}_{s-1}(t)} \right) e^{-\mu_n^2 \frac{at}{L^2}}. \qquad (5.12)$$

5.1. Particular cases

In the case when in the solution of equation for determining the phase transition temperature (4.3) the member $T_{mk}\left[\frac{B}{A+1}(\eta_1 - \eta_0)c_{11}\Phi\right]$ can be neglected, we obtain two solutions of the equation (4.2)

$$T_m = T_{mk}\left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12}\right]\bigg/\left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12}(1 + c_{11})\right], \tag{5.13}$$

$$T_m = \Phi(1 + c_{11})\bigg/\left[\frac{B}{A+1}(\eta_1 - \eta_0) + \alpha_{12}(1 + c_{11})\right]. \tag{5.14}$$

The first expression (5.13) can be treated as an approximate linearized solution to the phase transition temperature of soft modes, the second expression (5.14) — of hard modes, in which the temperature gradient in the body is the main factor represented by $\Phi(\kappa_m, \tau, \alpha_n, \mu_n, H)$. Thus, we have defined the non-stationary fields of potentials of energy and matter transference under the boundary conditions of the third kind. The obtained approximate solutions establish the relations between heat-mass transfer dependence and kinetic and geometric coefficients.

Define the relationship between time and dried area width for soft modes, when we can neglect the second member for expression T_m.

From the equations (5.1) we obtain the equation for determining the relative moisture change in time and the equation of movement of the liquid-gas interface

$$\frac{dz_m}{d\tau} = \frac{j(z_m)}{\Pi\gamma_L L} = \frac{\tilde{\beta}\gamma_{n0}T_0}{\Pi\gamma_L L\,(T_0 + T_m(\tau))}\left[-(a_1 + Bz_m) + \sqrt{U + Sz_m + B^2 z_m^2}\right] \tag{5.15}$$

with the initial condition

$$\text{if} \quad \tau = 0, \quad z_m = 0. \tag{5.16}$$

Substituting the expressions for the phase transition temperature from (5.13) into (5.15) and using replacement (5.3), we obtain

$$\frac{4B\left[-B\tilde{\chi}^2 + S\tilde{\chi} - BU\right]}{(2B\tilde{\chi} - S)^3}d\tilde{\chi} + \frac{4BT_{mk}}{T_0}\frac{\left[-B\tilde{\chi}^2 + S\tilde{\chi} - BU\right](\chi - a_2)}{(2B\tilde{\chi} - S)^3(\chi - a_3)}d\tilde{\chi} = Hd\tau. \tag{5.17}$$

After factorizing (to simple multipliers) the left side of the equation (5.17), we obtain

$$4B\left[\frac{A_0}{(\tilde{\chi} - a_3)} + \frac{B_0}{(2B\tilde{\chi} - S)} + \frac{C_0}{(2B\tilde{\chi} - S)^2}\right] +$$
$$+ \frac{4BT_{mk}}{T_0}\left[\frac{A_1}{(\tilde{\chi} - a_3)} + \frac{B_1}{(2B\tilde{\chi} - S)} + \frac{C_1}{(2B\tilde{\chi} - S)^2} + \frac{D_1}{(2B\tilde{\chi} - S)^3}\right]d\tilde{\chi} = d\tau, \tag{5.18}$$

where

$$A_0 = -\frac{1}{4B}, \quad C_0 = B\left(a_1^2 - U\right), \quad B_0 = 0, \quad A_1 = -\frac{1}{4B}\left[1 + \frac{\left(-a_3^2 + 2a_1 a_3 - U\right)(a_3 - a_2)}{(a_3 - a_1)^3}\right],$$

$$D_1 = \frac{\left(-a_3^2 + 2a_1 a_3 - U\right)(a_3 - a_2)}{8B^2(a_3 - a_1)^3}, \quad B_1 = \frac{(a_2 - a_3)}{2}\left[1 + \frac{\left(-a_3^2 + 2a_1 a_3 - U\right)}{(a_3 - a_1)^2}\right],$$

$$C_1 = \frac{(a_1 - a_2)}{(a_1 - a_3)}B\left(a_1^2 - U\right).$$

The solution of the equation (5.18) has the following form

$$\frac{4B}{H}\left\{\frac{A_0}{2B}\ln\left|\frac{2\tilde{\chi}B-S}{2\sqrt{U}B-S}\right|-\frac{C_0}{2B}\left[\frac{1}{(2\tilde{\chi}B-S)^2}-\frac{1}{(2\sqrt{U}B-S)^2}\right]\right\}+\frac{4B}{H}A_1\ln\left|\frac{\tilde{\chi}-a_3}{\sqrt{U}-a_3}\right|+$$

$$+\frac{4B}{H}\left\{\frac{B_1}{2B}\ln\left|\frac{2\tilde{\chi}B-S}{2\sqrt{U}B-S}\right|-\frac{C_1}{2B}\left[\frac{1}{2\tilde{\chi}B-S}-\frac{1}{2\sqrt{U}B-S}\right]-$$

$$-\frac{D_1}{2B}\left[\frac{1}{(2\tilde{\chi}B-S)^2}-\frac{1}{(2\sqrt{U}B-S)^2}\right]\right\}=\tau,$$

where

$$\tilde{\chi}=\sqrt{U+Sz_m+B^2z_m^2}-z_mB\qquad(z_m=1-\kappa_m). \tag{5.19}$$

6. Conclusions

The expression (5.19) defines the relations between the time of drying and the dried area width z_m as the function of the heat transfer coefficient (by means of the parameters a_2, a_3), the mass transfer coefficient (by means of the parameters B, S, C_{55}), permeability, diffusion, and average temperature (by means of the parameters A, U). The term in the first braces gives us the expression of the relative moisture change in time for the stationary temperature of soft regime when drying conditions are such that the temperature gradient can be neglected. Comparing the expression (5.19) with the expression of isothermal drying, we see that if in the stationary case the time of drying depends on the relative moisture of the drying agent and the mass transfer coefficient; in the non-stationary mode all the coefficients depend on the state equation at the interface as well as on the heat transfer coefficient, which is not observable in isothermal convective drying. If $z_m=1$, we obtain the complete time of drying. As can be seen in the soft drying mode, we can control the process by increasing or decreasing the blow rate by means of the heat H or mass B transfer coefficients, moisture of the drying agent η_0 by means of the coefficients a_1, S, and varying the time of the drying agent temperature change by means of the coefficients α_n, β_n of the expansion of the control function $u(\tau)$. All expressions we have explicitly, so we can choose and minimize appropriate desired criteria of optimal mode of drying.

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Побудова розв'язку задачі про конвективно-теплове осушення пористих тіл в сушильних установках

Чапля Є. Я.[1], Гайвась Б. І.[2], Торський А. Р.[2]

[1]*Інститут механіки і прикладної інформатики Університету Казиміра Великого в Бидгощі*
вул. Коперніка, 1, 85-064 м. Бидгощ, Польща
[2]*Центр математичного моделювання ІППММ ім. Я. С. Підстригача НАН України*
вул. Дж. Дудаєва, 15, 79005, Львів, Україна

Отримані в роботі наближені розв'язки одномірної нелінійної математичної моделі процесу сушіння пористих тіл дають можливість встановити температуру фазового переходу при довільних змінах температурних режимів сушильного агента, зміну відносної вологості в тілі в процесі осушення, як функцію геометричних та фізичних параметрів, вплив швидкості та відносної вологості сушильного агента на процес сушіння з метою оптимізації процесу.

Ключові слова: *тепловий режим, сушильний агент, рухома координата границі області фазового переходу*

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