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IDENTIFYING THE ELASTIC MODULI OF COMPOSITE PLATES BY USING HIGH-ORDER THEORIES

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Abstract. The study aims to predict elastic and damping properties of composite laminated plates from the measured mechanical characteristics. The elastic constants and damping properties of a laminated element are determined by using experimental data and the results of a multi level theoretical approach. Solution examples for particular problems are given. On the basis of static three-point bending tests, measured eigenfrequencies, and refined calculation schemes, the elastic properties of layered composite beams were identified. For determining Young's and shear moduli, the method of genetic minimization of error function was used. It is shown that, by employing combined criteria, the transverse elastic moduli can be determined uniquely. It is shown that, by employing combined criteria, the transverse elastic moduli can be determined. The elastic modules were also determined from measured vibration eigen-frequencies of the beams. New combined criteria of identification – schemes averaged over the calculation results for a homogeneous beam and for a sandwich with a core identical to the homogeneous beam and rigid outer layers are considered. The error function is chosen as the sum of error functions for the homogeneous beam, and for the sandwich. In the present study, combined identification schemes making it possible to unequivocally determine the transverse modules and Poisson ratio are suggested.

Introduction

Structures made of layered materials are most frequently used in modern machine building and, especially, in aero -space industry. Due to their light weight and high strength, such structures are also increasingly utilized in civil engineering, transport, and machine building. The rapid growth of the industrial use of these structures requires the development of new analytical and computation tools for analyzing and investigating their mechanical behavior. The determination of stiffness parameters for composite materials, particularly fibrous composites, is much more complicated than for isotropic ones, since composites are anisotropic and heterogeneous. At the same time, for identification of the physical parameters directly describing the structural behavior of composites, a great variety of approaches is suggested. Recently, the elaboration of new numerical and experimental techniques for identifying the mechanical characteristics of materials has been started.

Problem statement

The problems on mechanical vibrations of three-layer structures were investigated in many studies [1-14]. Review and assessment of various theories for modelling sandwich composites are presented in [15, 16]. Most the considered models are based on the following assumptions: (i) the viscoelastic layer is subjected only to shear deformations, and hence the volume energy of the core is neglected; (ii) the face layers are elastic and isotropic, and their contribution to the shear deformation is neglected; (iii) the plane cross sections of face layers remain plane and normal to the deformed median lines of face layers. How ever, with increasing vibration frequency, the results of calculations on the basis of these models do not coincide well enough with the results of measurements.

Thus, in modeling composite layered plates, it is important to have an efficient general theory for an exact estimation of the influence of transverse shear stresses on the behavior of the plates. It is known that the high-order theories of layered plates allow us to predict the stress-strain state of composite thick plates subjected to bending loads quite accurately. We should note that the use of such theories, which take into account the transverse shear and compression, is a reason able compromise between the accuracy and simplicity, al though they are usually connected with boundary conditions whose interpretation presents difficulties in practical applications.

Simple theories of thin layered plates usually can not determine the three-dimensional stress field in a plate. There fore, an analysis of layered composites requires an independent layerwise theory of plates or the three-dimensional theory of elasticity. The exact three-dimensional solutions of elasticity theory [12, 13] have shown that the continuity conditions for the displacements and transverse stresses on interfaces considerably affect the results of an exact analysis of thick multilayer composite plates, in investigation of which the prevailing role is played by the transverse normal stresses. How ever, it is not easy to derive these exact solutions. There fore, approximate theories of high-order accuracy based on expansion of the displacement field into power series [17-23] are used.

The identification of mechanical characteristics of thin-walled layered elements is a rather complicated problem. Many studies are dedicated to this problem, both theoretical, with a mathematical substantiation of algorithms, and experimental, where one or several parameters are determined [24-27]. An adequate description of the mechanical properties of layered thin-walled elements is based on the construction of a refined theoretical model and on the choice of a testing scheme making it possible to uniquely determine some modulus or a group of moduli.

The method of static experiments used in [24] allows one to separately find the shear modulus of an orthotropic composite from tension tests at a certain angle to its symmetry axes. In [25], a more general approach with a specific choice of the strain field is suggested, which undoes the contribution of all components of the strain tensor except that needed for determination of a certain modulus. This enables one to theoretically determine each elastic modulus separately. But there still remains the question of practical realization of such strain fields in actual experiments. Experimental schemes for determining the elastic moduli by using plates eigen-frequencies are known ([26, 27]). However, they are unsuitable for determining the elastic modulus along the normal to a plate.

Statement of purpose and problems of research

The purpose of our study is the elaboration of a stable identification algorithm allowing one to uniquely determine the elastic moduli, including the transverse ones. The problem of identification necessarily includes the planning of experiments, the construction of a calculation model, and the identification schemes themselves. In this study, the construction of the model is considered in two plans: first, adaptation of the model to the kind of specimens and experimental equipment employed and, second, adaptation of the model to the identification scheme. The reliability of the results obtained was estimated by analyzing the robustness of the calculation schemes suggested. The verification of the model was considered in [17-23]:

verification of the model $\Rightarrow$ verification of the identification scheme $\Rightarrow$ identification $\Rightarrow$ analysis of reliability (postanalysis)

In this study, a simple calculation procedure is suggested, which allows one to determine the level of a refined high-order theory necessary for an accurate and efficient analysis of vibrations in layered composite plates and to obtain sufficiently accurate results compared with the analytical solutions available. A new adaptive procedure for identifying the stiffness parameters of layered composites is developed proceeding from the minimization of discrepancies between the experimentally found static characteristics and eigen-frequencies and those calculated based on the method suggested.

Certain aspects of the laminated elements parametric discretization

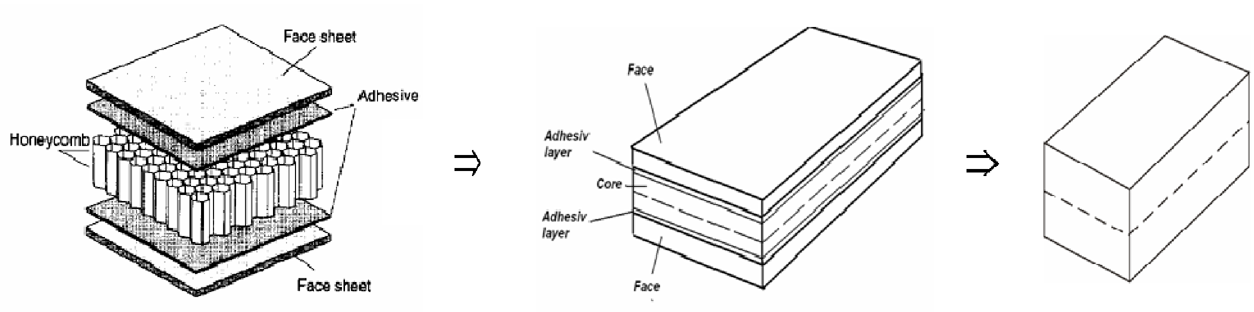
We will consider the discretization of the laminated thin-walled element A in some detail. The parameters of $l_1, l_2,..., l_{N_R}$ describe the geometrical form of A, the number and thickness of the layers, and the mechanical properties of the materials of the layers. Their amount is limited. For composite materials, it is often necessary to determine the mechanical properties from the structural properties of the materials. In such cases, some of the parameters of $l_1, l_2,..., l_{N_R}$ will be derived from a great number of primary parameters of $x_1, x_2, ..., x_{N_P}$, which characterize the mechanical properties of the fibres, the structure of composition of the material and some features of the process of polymerization. It should be noted that the thin-walled element A properties not directly dependent on the aggregate of the parameters $l_1, l_2, ..., l_{N_R}$, but only on a certain combination of these parameters. It is actually possible to consider only the optimization of the parameters $s_1, s_2, ..., s_{N_C}$ which depend on $l_1, l_2, ..., l_{N_R}$. If we consider the set accordingly:

$$x_i \in \Xi, \quad l_i \in \Lambda, \quad s_i \in Z,$$

then the surjective reflections (when one element l_i of Λ corresponds a few elements $x_1, x_2, ..., x_{N_P}$ of Ξ and every element l_i has a prototype) of W_A, W_B will take place

$$\Xi \xrightarrow[\Omega_A]{} \Lambda \xrightarrow[\Omega_B]{} Z.$$

The actual calculation and consequently the optimization will be conducted in the set of Z, which is far narrower than Λ, and even narrower than Ξ. Numerical schemes (*NS*) using hypotheses for the entire package of the laminated elements will especially demonstrate this narrowing. The scheme for the condensed modelling of the sandwich element wit the honeycomb core is presented in Fig. 1.



a) *b)* *c)*

Fig. 1. Heterogeneous 3-D model (a), (b) heterogeneous 2-D model; (c) homogeneous 2-D model

Detailed description of various types of sandwich panel theories may be found in [1-16].

Analytical modelling of the cylindrical bending of laminated plates

Exact solutions may be obtained in only a few cases for the deformations of laminated plates. These exact solutions exist in the case of static cylindrical bending of composite laminas.

Cylindrical bending of laminated plates subjected to loading by a moment. The governing equations for the stresses are (the axis *x* is oriented along the middle line of the plate and the axis *z* is oriented normal to it).

$$\frac{\partial s_{xx}}{\partial x} + \frac{\partial t_{xz}}{\partial z} = 0, \qquad \frac{\partial t_{xz}}{\partial x} + \frac{\partial s_{zz}}{\partial z} = 0. \tag{1}$$

The solutions which express Hooke's law with respect to the stress components have the form

$$s_{xx} = C_{xx} e_{xx} + C_{xz} e_{zz}, \ s_{zz} = C_{zx} e_{xx} + C_{zz} e_{zz}, \ t_{xz} = G g_{xz}. \tag{2}$$

For pure bending, the stress-strain state is uniform, thus

$$\frac{\partial t_{xz}}{\partial z} = -\frac{\partial s_{xx}}{\partial x} = 0 \quad and \quad t_{xz} = 0, \tag{3}$$

$$\frac{\partial s_{zz}}{\partial z} = -\frac{\partial t_{xz}}{\partial x} = 0 \quad and \quad s_{zz} = 0. \tag{4}$$

If the stress s_{xx} is not equal to zero, the following assumption must be made

$$s_{xx} = S(z). \tag{5}$$

The expressions for the displacements and the stress are obtained from Eqs. (1)–(5), The result is as follows.

$$u = xz\,,\ w = -0.5(a\,z^2 + x^2)\,,$$
$$s_{xx} = z\left(C_{xx} - a\,C_{xz}\right), \tag{6}$$

where $a = C_{xz} / C_{zz}$.

For the bending moment we obtain

$$M = \int_{-H_p}^{H_p} z^2 \left(C_{xx} - a\,C_{xz}\right) dz\,. \tag{7}$$

where H_p is one-half of the thickness of a lamina.

Since the materials may be non-homogeneous, $C_{xx}(z), \alpha(z), C_{xz}(z)$ may all be functions of z. For a uniform Timoshenko beam of the same thickness, an equation for the bending rigidity of a uniform equivalent beam may be written with regard to the condition $\frac{dg}{dx} = 1$.

$$E_T I = M\left(\frac{dg}{dx}\right)^{-1} = \int_{-H_p}^{H_p} z^2 \left(C_{xx} - a\,C_{xz}\right) dz\,. \tag{8}$$

Cylindrical bending of laminated plates subjected to loading by a force. In this case the primary assumptions are

$$s_{xx} = x\,S(z), \qquad t_{xz} = T(z)\,. \tag{9}$$

The following expressions are obtained from Hooke's law

$$e_{xx} = \frac{\partial u}{\partial x} = xS^*\,,\ e_{zz} = \frac{\partial w}{\partial z} = -a_1 x S^*\,,$$
$$S^* = \frac{S(z)}{1 - a_1 a_2}\,,\quad a_1 = \frac{C_{xz}(z)}{C_{xx}(z)}\,,\quad a_2 = \frac{C_{xz}(z)}{C_{zz}(z)}\,. \tag{10}$$

Next, by integrating Eq. (10) for the displacement we obtain

$$u = \frac{x^2}{2} S^* + j(z), \quad w = -x\int_0^z a_1 S^* dz + y(x)\,. \tag{11}$$

By substituting Eq. (11) into Eq. (1), we can derive the following equation for a symmetrically laminated plate. It also holds for plates with arbitrary laminations

$$u = \frac{c_1 x^2 z}{2C_{xx}} + c_1 \int_0^z \left(\int_0^z \frac{a_2 z}{C_{xx}} dz + \frac{t_{xz}}{G} \right) dz - c_2\,z\,,\quad w = -\frac{c_1 x^3}{6C_{xx}} - c_1 x \int_0^z \frac{a_2 z}{C_{xx}} dz + c_2\,x\,, \tag{12}$$

where for the tangential stress t_{xz} and the constant c_2 we have

$$t_{xz} = -c_1 \int_z^H (1 - a_1 a_2) z dz\,,\quad c_2 = \frac{c_1}{H_p} \int_0^{H_p} dz \left(\int_0^z \frac{a_2 z}{C_{xx}} dz + \frac{t_{xz}}{G} \right) dz\,. \tag{13}$$

In the above equations, c_1 is an arbitrary constant. If the tangential force Q is equated to unity, then the following equation may be obtained from Eqs. (12) and (13)

$$Q = \int_{-H_p}^{H_p} t_{xz}\,dz = 1 \ \Rightarrow\ c_1 = \left(\int_{-H_p}^{H_p} \left(\int_z^{H_p} (1 - a_1 a_2) z dz \right) dz \right)^{-1}\,. \tag{14}$$

Thus, by comparison with a uniform Timoshenko beam of the same cross section, we may write an equation for the transverse rigidity of a uniform equivalent beam

$$\frac{1}{2H_p G_T} = -c_1 \int_0^z \frac{a_2 z}{C_{xx}} dz + c_2 = -c_1 \int_0^z \frac{a_2 z}{C_{xx}} dz + \frac{c_1}{H_p} \int_0^{H_p} dz \left(\int_0^z \frac{a_2 z}{C_{xx}} dz + \frac{t_{xz}}{G} \right) dz . \quad (15)$$

Here the value t_{xz} is given by Eq. (13); G_T and E_T denote the Timoshenko beam moduli. It may be seen that the same bending rigidity is obtained from Eq. (12) as in Eq. (8).

Higher order asymptotic approach

Various high-order displacement models have been developed in the literature by considering combinations of displacement fields for in-plane and transverse displacements inside a mathematical sub-layer. In order to obtain more accurate results for the local responses, another class of laminate theories, commonly named as the layer-wise theories, approximate the kinematics of individual layers rather than a total laminate using the 2-D theories. These models have been used to investigate the phenomena of wave propagation as well as vibrations in laminated composite plates. Numerical evaluations obtained for wave propagation and vibrations in isotropic, orthotropic and composite laminated plates have been used to determine the efficient displacement field for economic analysis of wave propagation and vibration. The numerical method developed in this paper follows a semi-analytical approach with an analytical field applied in the longitudinal direction and a layer-wise displacement field employed in the transverse direction. The goal of the present paper is to develop a simple numerical technique, which can produce very accurate results compared with the available analytical solution. The goal is also to provide one with the ability to decide upon the level of refinement in higher order theory that is needed for accurate and efficient analysis. Let us consider now such kinematic assumptions ($U=U_e+U_d$) for a symmetrical three-layered plate of thickness $2H_p$ (only cylindrical bending is considered):

$$U_e - \begin{cases} u = \sum_{i,k} u_{ik}{}^e z^{2i-1} j_k(x), & 0 < z < H, \\ w = \sum_{i,k} w_{ik}{}^e z^{2i-2} g_k(x), & 0 < x < L, \end{cases} \quad (16)$$

$$U_d - \begin{cases} u = \sum_{i,k} u_{ik}{}^d (z-H)^i j_k(x), & H < z < H_p, \\ w = \sum_{i,k} w_{ik}{}^d (z-H)^i g_k(x) & 0 < x < L. \end{cases} \quad (17)$$

Here $j_k(x)$, $g_k(x)$ – are apriory known coordinate functions (for every beam clamp conditions), $u_{ik}{}^e$, $w_{ik}{}^e$, $u_{ik}{}^e$, $w_{ik}{}^e$ – unknown set of parameters.

By substituting Eqs. (16) by (2) into the following variation equation

$$\int_{t_1}^{t_2} \int_V (s_{xx} de_{xx} + s_{zz} de_{zz} + t_{xz} de_{xz} - r \frac{\partial u}{\partial t} d \frac{\partial u}{\partial t} - r \frac{\partial w}{\partial t} d \frac{\partial w}{\partial t}) dV dt = \int_S P dU , \quad (18)$$

and also assuming single frequency vibration ($u_{ik}^e = \bar{u}_{ik}^e e^{iwt}$, $w_{ik}^e = \bar{w}_{ik}^e e^{iwt}$, $u_{ik}^d = \bar{u}_{ik}^d e^{iwt}$, $w_{ik}^d = \bar{w}_{ik}^d e^{iwt}$) we obtain the set of linear algebraic equations for the amplitudes

$$[A]\bar{U} = \begin{bmatrix} A_1 & A_d \\ A_d^T & A_2 \end{bmatrix} \begin{bmatrix} \bar{U}_e \\ \bar{U}_d \end{bmatrix} = f . \quad (19)$$

For a greater number of lamina this equation has the following form for each additional layer

$$U_d^n - \begin{cases} u = \sum_{i,k} u_{ik}\left(z - H^{(n)}\right)^i j_k(x), & H^{(n)} < z < H^{(n+1)}, \\ & 0 < x < L, \\ w = \sum_{i,k} w_{ik}{}^n\left(z - H^{(n)}\right)^i g_k(x), & n = 1, ..., N, \end{cases} \quad (20)$$

Here $H_p^{(n+1)} - H_p^{(n)} = H_n$, $H_p^{(1)} = H$; H and H_n are one-half of the thicknesses of the core and the outer n-*th* layer, respectively. Matrix $[A]$ are found by double integration through the thickness and along the length of the beam. Note that, $N = 1$ and $N = 2$ represent the cases of symmetrical three- and five-layered plates, respectively. The corresponding frequency equation for the material with the viscous damping should be written such

$$-w^2[M]\overline{U} + iw[C]\overline{U} + [K]\overline{U} = [A]\overline{U} = \overline{f}. \quad (21)$$

This is the traditional frequency domain method which is normally used in linear elastic system investigations [29, 30].

Symmetric Three-Layer Beam (Sandwich)

Let us consider vibrations of a simply supported symmetric three-layer beam (see Fig. 2) with the following physicomechanical characteristics of layer materials: compression modulus 1.076 GPa, tension modulus 3.96 GPa, bending modulus 1.020 GPa, and shear modulus 0.638 GPa for the core (a honeycomb polymeric filler); tension modulus 26 GPa, bending modulus ≈ 6 GPa, and shear modulus ≈ 0.6 GPa for the face layer (a fibrous composite material).

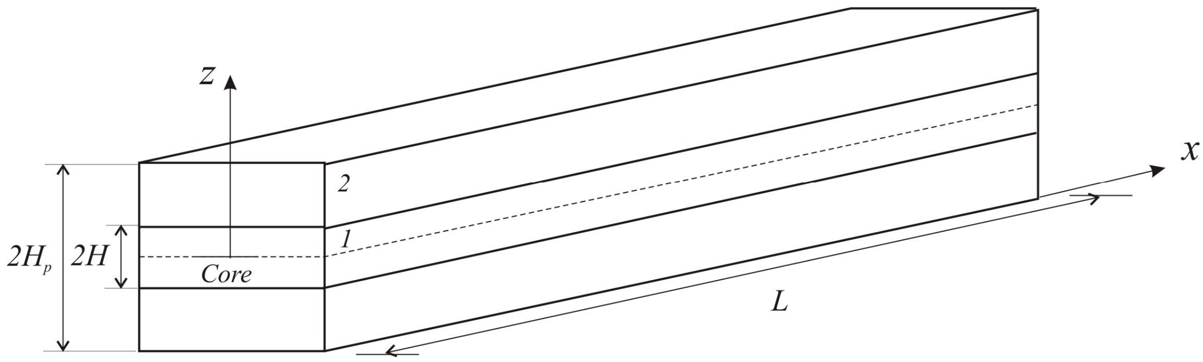
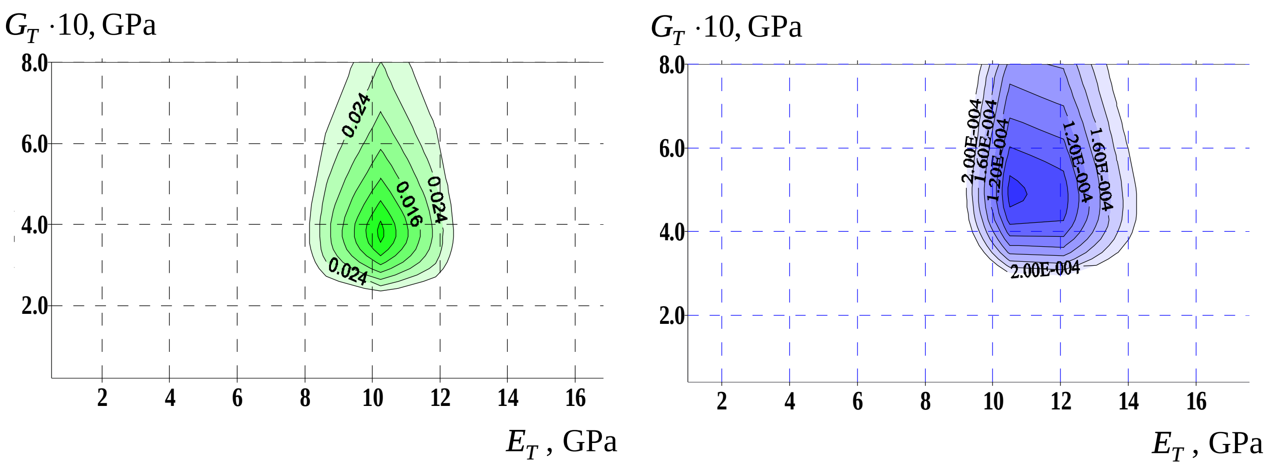


Fig. 2. Three-layer beam with internal viscoelastic (core, 1) and face (2) layers. The designations are also used in Figs. 7-9

Figure 3 shows the elastic constants E_T and G_T of an equivalent Timoshenko beam as an analogue of the beam specimen (see Fig. 2). The identification of elastic moduli was carried out by using the procedure of minimization of discrepancies between the elastic energies of homogeneous and inhomogeneous beams. The isolines in Fig. 3 correspond to constant values of the difference between the elastic energy of the homogeneous Timoshenko beam and the elastic energy $\int_V (s_{xx}de_{xx} + s_{zz}de_{zz} + t_{xz}de_{xz})dV$ of the inhomogeneous beam modeled by using the analytical (1)-(15) and approximate (16)-(22) approaches. The small difference between these values is caused by the fact that, in both the cases, different components of the elastic energy are taken into account. In the approximate method, the contribution of the normal transverse stress ($\int_V (s_{zz}de_{zz})dV$) is a small quantity) to the elastic energy is neglected. The equiv a lent rigidity of the Timoshenko beam was found from Eqs. (8) and (15).



a) *b)*

Fig. 3. Equivalent modules E_T and G_T : (a) – analytical approach; (b) – approximation approach

Properties of a layered beam in the frequency domain

A range of numerical experiments must be made to ensure that this theoretical approach is correct. Let us compare the eigen-frequencies of a clamped-free uniform isotropic beam for the following geometrical parameters: length L was chosen to be 0.3 m, thickness was chosen to be $H = 0.0130$ m. The elastic moduli were assumed to be as follows: $C_{xx} = C_{zz} = 240$ MPa, $G = 120$ MPa, and $C_{xz} = 103$ MPa (n =0.3). The eigen-frequencies were obtained by means of Eqs. (21.) by $j_k(x) = g_k(x) = \sin(kpx/2L)$ in (20) and FEM. (See Tables 1-4).

Table 1

Eigen-frequencies of a clamped-free uniform isotropic beam obtained by means of FEM

N_f	(N_x x N_z)60x9	150x9	300 x 9	600 x 9	600x9(2)
1	20.5	19.9	19.7	19.6	19.2
2	128.3	124.3	123.2	122.9	120.4
3	356.7	345.4	342.3	341.4	335.4
4	690.0	667.7	661.6	659.9	649.8
5	1121.1	1083.7	1073.6	1070.9	1056.4
6	1641.0	1584.0	1568.8	1564.7	1546.1
7	2240.8	2159.3	2137.7	2131.9	2109.5
8	2954.5	2800.6	2771.2	2763.4	2737.5

Table 2

Eigen-frequencies of a clamped-free uniform isotropic beam obtained by means of Eqs. (22), $N_z = 1$

N_f	$N_x = 17$, $N_z = 1$	$N_x = 23$, $N_z = 1$	$N_x = 27$, $N_z = 1$	$N_x = 29$, $N_z = 1$	$N_x = 31$, $N_z = 1$
1	21.8	21.5	21.5	21.4	21.4
2	135.6	134.2	133.8	133.4	133.4
3	375.7	371.5	370.3	369.7	369.7
4	725.7	717.4	715.8	714.1	714.1
5	1178.7	1165.0	1161.8	1159.7	1158.7
6	1728.4	1702.9	1699.0	1697.8	1696.5
7	2363.6	2327.7	2318.8	2315.8	2312.8
8	3084.4	3024.6	3011.0	3006.0	3002.6

In the Table 1, in the first row, the mesh of the FEM grid in the X, Z plane is presented. In the last column, the mesh in the Y direction was doubled (N_y =20). In Tables 2-4, the first row presents the number of approximations in the X and Z directions. (See Eq. (16)). The modes of vibration are presented in Fig. 4. In Tables 1- 4 may be seen the rate of convergence of two methods and the agreement between the results. (See the last column in Tables 1 and 4).

Table 3

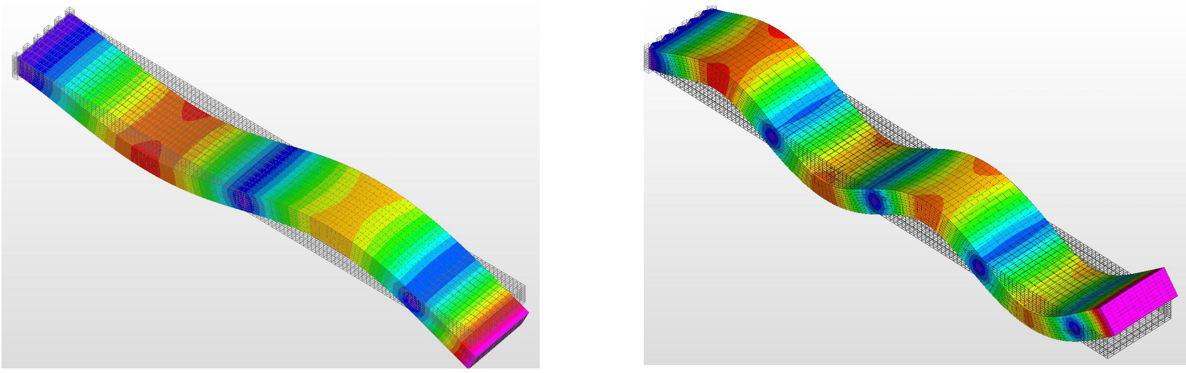
Eigen-frequencies of a clamped-free beam obtained by means of Eqs.(22), N_z =2

N_f	$N_x = 17, N_z = 2$	$N_x = 23, N_z = 2$	$N_x = 27, N_z = 2$	$N_x = 29, N_z = 2$	$N_x = 31, N_z = 2$
1	19.8	19.5	19.5	19.4	19.4
2	122.7	121.7	121.3	121.0	121.0
3	340.7	337.3	336.1	335.6	335.6
4	659.1	652.0	650.5	648.9	648.9
5	1073.1	1061.0	1057.0	1055.0	1053.9
6	1574.1	1554.6	1548.5	1546.1	1543.6
7	2158.1	2126.6	2115.2	2112.4	2109.5
8	2819.2	2760.5	2750.8	2745.9	2742.7

Table 4

Eigen-frequencies of a clamped-free beam obtained by means of Eqs.(22), N_z = 3

N_f	$N_x = 17, N_z = 3$	$N_x = 23, N_z = 3$	$N_x = 27, N_z = 3$	$N_x = 29, N_z = 3$	$N_x = 31, N_z = 3$
1	19.9	19.5	19.5	19.5	19.4
2	124.0	121.7	121.3	121.0	121.0
3	343.5	337.3	336.1	336.1	335.6
4	671.1	652.8	650.5	649.7	648.1
5	1066.0	1061.0	1057.0	1056.0	1053.9
6	1563.1	1554.6	1547.3	1544.8	1543.6
7	2153.8	2123.8	2115.2	2112.4	2109.5
8	2816.0	2763.8	2750.8	2745.9	2742.7

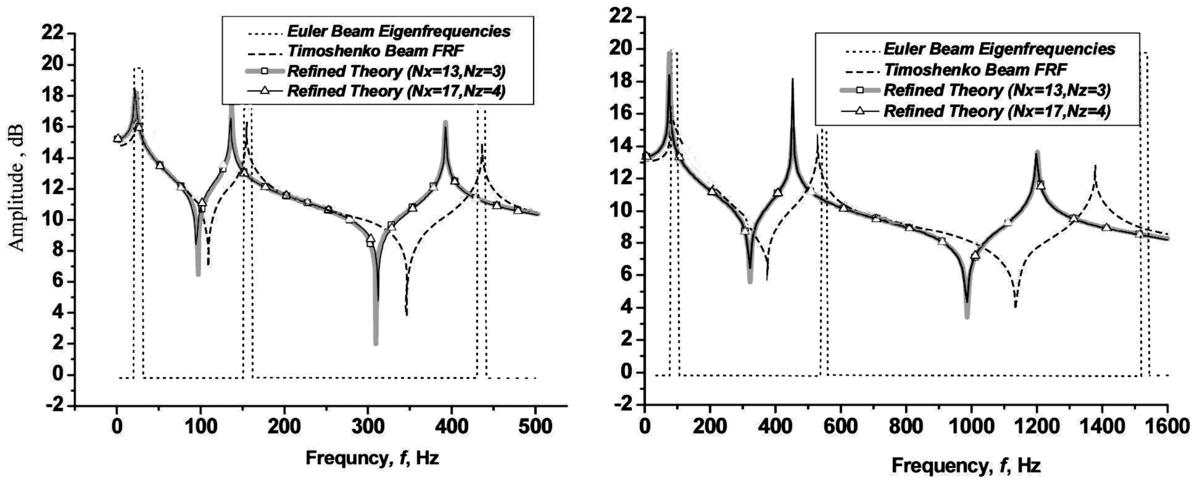


a) *b)*

Fig. 4. Modes of beam vibration: (a) – the third mode, (b) – the sixth mode

Consider now vibration testing of an anisotropic beam having the following geometrical parameters: length L varied from 0.6 m to 0.06 m. and thickness $H = 0.0127$ m. The elastic moduli are assumed to be as follows: $C_{xx} = C_{zz} = 250$ MPa, $G = 58$ MPa, and $C_{xz} = 40$ MPa (foam material). The frequency response functions (FRF) for these beams are presented in Fig. 5 (Eq's.(16-21) were used). These were obtained for various ranges of approximations for various frequency domains (various beam lengths).

From the results obtained, it follows that the Euler beam theory overestimates the natural frequency values. It can be observed that the Timoshenko beam theory also overestimates the natural frequency values but less so than the Euler theory. Note that in practice the refined theory produces asymptotic estimated values for the natural frequencies when the degree of approximation in the longitudinal direction $N_x > 13$ and in the thickness direction $N_z \geq 2$.



a) *b)*

Fig. 5. Modes of beam vibration: (a) – the third mode, (b) – the sixth mode

Damping properties in the frequency domain

Analogue theories and order-dependent results may be obtained also for the prediction of damping. This result may be achieved by direct computation by use of the stiffness matrix if the damping matrix is congruent to the stiffness matrix

$$h = \frac{h_1[q_1]^T[K_1][q_1]+h_2[q_2]^T[K_2][q_2]+...+h_N[q_N]^T[K_N][q_N]}{[q]^T[K][q]}. \quad (22)$$

Here: $[K]$ is the stiffness matrix, $|q|$ is a vector of displacement components, $[K_i]$is a stiffness matrix component corresponding to (i^{th}) layer ($[K]=\sum_i [K_i]$). Components of damping matrix C are, us usually, taken proportional to components of the rigity matrix: $C_i = h_i[K_i]$.

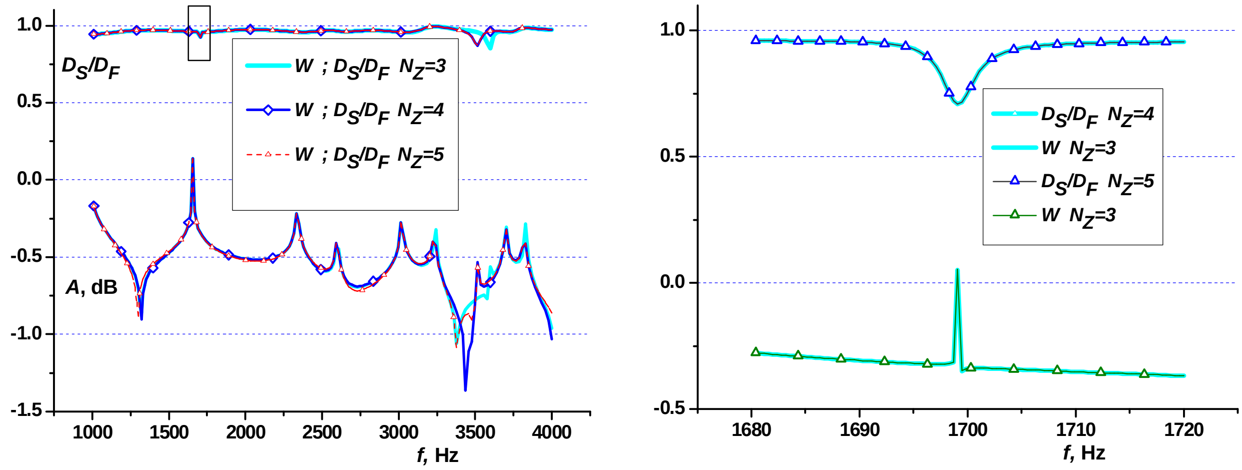
In the previous section of this paper, the frequency influence on the properties of sandwich panels may be seen. For this purpose, fewer investigations were made for damping properties investigation in the frequency domain.

In Fig.6, influence of the theoretical order on the damping prediction accuracy is presented for the sandwich with the damping core ($h_i = 0, i \neq 1$) for different orders of approximations in (16,17). for simply supported centrally loaded beam ($j_k(x) = \sin\left((2k-1)px/2L\right)$, $g_k(x) = \cos\left((2k-1)px/2L\right)$) for the following geometrical parameters: length L was chosen to be 0.1 m, core thickness was chosen to be $H = 0.030$ m. The core elastic moduli were assumed to be as follows: $C_{xx} = 180, C_{zz} = 150$ MPa, $G = 40$ MPa, and $C_{xz} = 75$ MPa (n =0.3); density $r = 2400\, kg/m^3$. Face layers thickness was chosen to be $H = 0.002$ m. The elastic moduli were assumed to be as follows: $C_{xx} = 5400, C_{zz} = 750$ MPa, $G = 200$ MPa, and $C_{xz} = 375$ MPa; $r = 240\, kg/m^3$.

Here D_S / D_F is the ratio of the whole damping of sandwich to the damping in face layers. The behaviour of these curves is not similar to the ones in the lower frequency range. For the sandwich beam with thick damping core, the damping is seen to decrease just above 1,700 Hz. Now consider this region in more detail. Damping fluctuations may be seen. These fluctuations are correlated with the appropriate FRF fluctuations (Fig. 7b).

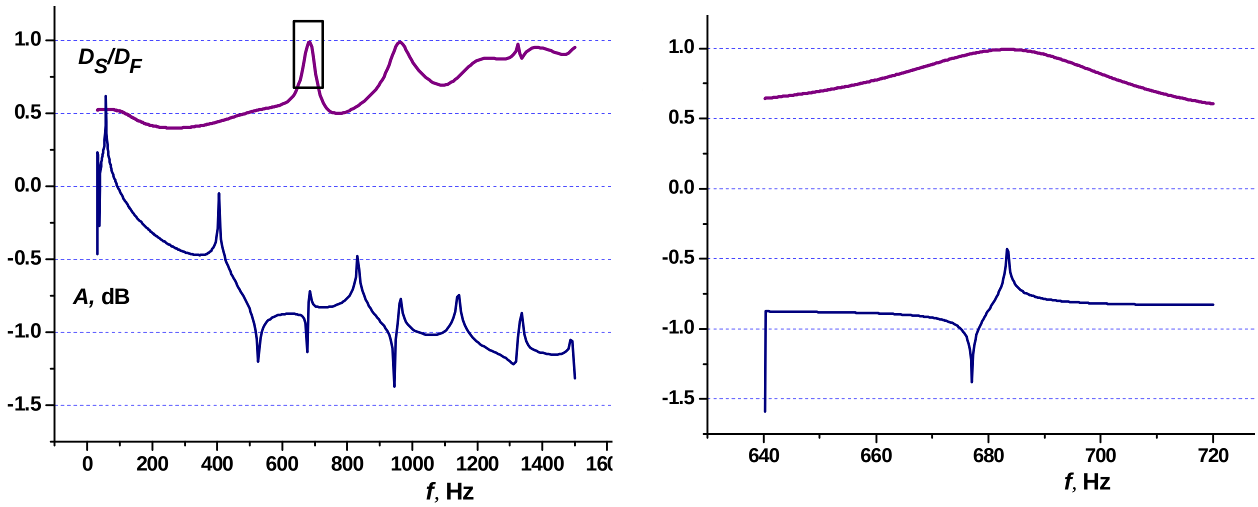
In Fig.7, the values of the amplitude and the damping is presented for the symmetrical three-layered beam with the damping face sheets ($h_i = 0, i = 1$) (us above the influence of the orders of approximations in

(16,17) is practically equivalent for $N_Z \geq 2$). Here core thickness was chosen to be $H = 0.008$ m, face layers thickness was $H = 0.016$ m. The core and face layers mechanical characteristics were assumed to be identical to the face layers and core characteristics us above for sandwich.



a) *b)*

Fig. 6. Frequency dependence for: (a) – amplitude, damping (*Ds/D1*) fluctuations; (b) – in the fluctuation region



a) *b)*

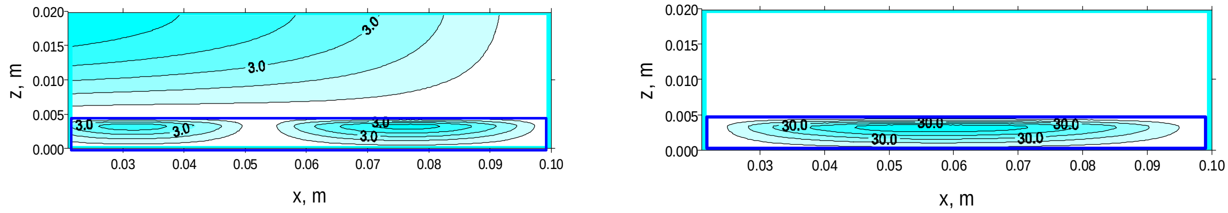
Fig. 7. Frequency dependence for: (a) – amplitude, damping (*Ds/D1*) fluctuations; (b) – in the fluctuation region

Damping fluctuations (increasing) also may be seen.

In Fig. 8,9 stress distributions through one-half of the beam thickness are presented for this symmetrical beams in the region of the damping fluctuations (f = 680 Hz) and in the region of small damping (f = 250 Hz).

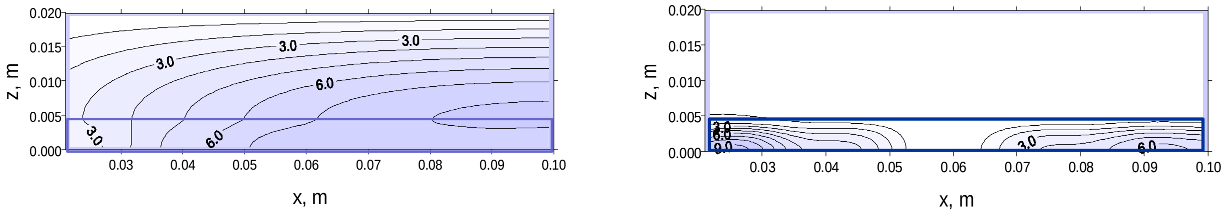
It can be seen in Fig. 9, 10, b that the stresses in the damping layer (damping is nonzero only in layer: 4 mm < H < 20 mm) are small compared to the stresses presented in Fig. 8, 9, a.

In Fig. 10, stress distributions through one-half of the beam thickness are presented for symmetrical beams for $x=0.1L$ in the region of the damping fluctuations (f = 1,700 Hz, sandwich) and also near f = 680 Hz (thick damping face sheets).



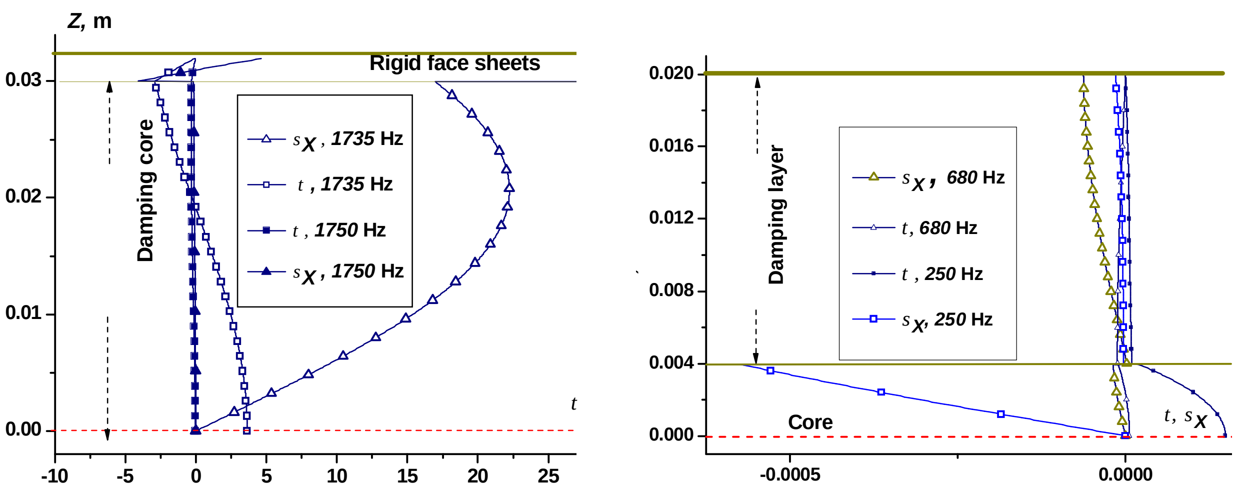
a) *b)*

Fig. 8. Normal stress s_X distributions through one-half of the beam thickness: (a) – in the region of the damping fluctuations (f = 685 Hz); (b) – near f = 250 Hz



a) *b)*

Fig. 9. Shear stress t distributions through one-half of the beam thickness: (a) – in the region of the damping fluctuations (f = 685 Hz); (b) – near f = 250 Hz



a) *b)*

Fig. 10. Stress distributions near fluctuations: (a) – sandwich, f = 1,700 Hz; (b) damping face sheets, f = 680 Hz

Experimental investigations

Beams made of plastic foams of trademarks 3715 and 6718 (General Plastic, USA) and sandwich beams with CFRP load-carrying layers and a core made of the plastic foam of trademark 3715 were tested in bending (see Fig. 1). The thickness of CFRP layers, made of a plain-weave fabric, was 0.5 mm, and the fibers were oriented along and across the layers. The three-layer beam had a rectangular cross section, length 0.6 m, width 0.028 m, and height 0.0264 m. The apparent density of the plastic foams of trademarks 3715 and 6718 was 240 and 277 kg/m3, respectively, and the weight per unit length of the sandwich was 0.207 kg/m. The beams were loaded stepwise. The maximum deflections (Tables 5 and 6) were measured by dial gages with a scale-division value of 0.01 mm. The radii of cylindrical supports and the width of the loading indenter were 5 mm.

The vibration eigen frequencies of the beam, measured by the method suggested in [11], are presented in Table 7.

Table 5

Deflections of Beams $\delta \cdot 10^3$ m Made of Plastic Foams (*b* . 2*H* = 6.4 . 25.4 mm2; *F* = 2*bH*) as Functions of Load *P* and Span Length *L*

L, m	*P*, N				
	4.41	14.22	24.03	38.84	43.65
0.06	0.020* / 0.015	0.060 / 0.045	0.090 / 0.075	0.135 / 0.100	0.170 / 0.135
0.08	0.035 / 0.025	0.140 / 0.090	0.230 / 0.145	0.315 / 0.205	0.390 / 0.365
0.10	0.065 / 0.045	0.220 / 0.165	0.360 / 0.275	0.515 / 0.385	0.660 / 0.505
0.12	0.105 / 0.080	0.350 / 0.280	0.600 / 0.465	0.855 / 0.645	1.100 / 0.825
0.14	0.165 / 0.125	0.580 / 0.420	0.970 / 0.725	1.345 / 1.025	1.755 / 1.335

In the numerator and denominator, the deflections of beams made of plastic foams of trademarks 3715 and 6718, respectively, are given.

Table 6

Deflections $\delta \cdot 10^3$ m of a Sandwich as Functions of Load *P* and Span Length *L*

P, N	19,62	39,24	58,86	78,48	98,10	L, m
$\delta \cdot 10^3$, m	0,15	0,300	0,440	0,590	0,730	0,38
	0,09	0,185	0,275	0,360	0,445	0,30
		0,100	0,140	0,180	0,220	0,20

Table 7

Experimental Eigenfrequencies of a Beam (Trademark 6718)

Mode	Frequency $f_i^{\exp}$, Hz	Mode	Frequency $f_i^{\exp}$, Hz
1	60	5	1154
2	144	6	1666
3	376	7	2279
4	713	8	2953

Identification criteria and an analysis of reliability

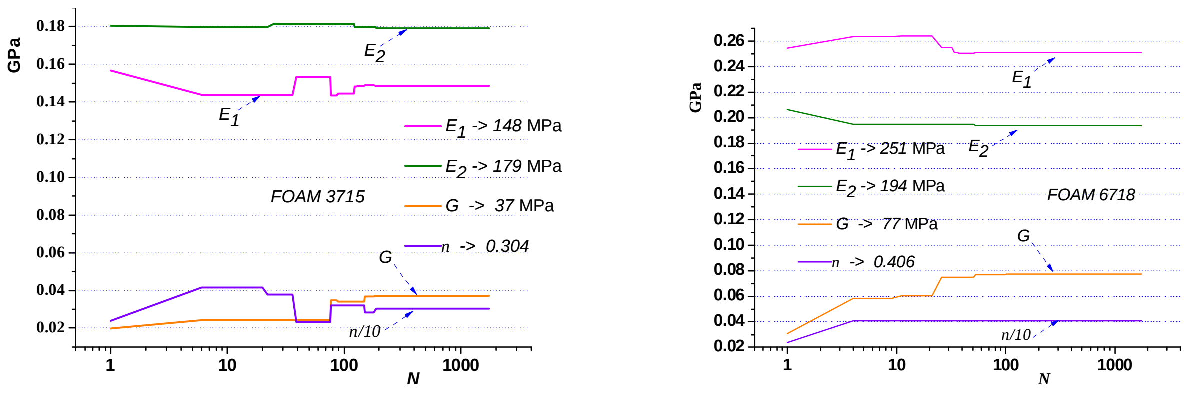
The identification of elastic moduli by the results of static tests is realized by the method of minimization of the error function

$$\Delta = \sum_{i}^{N_P} \frac{\left| w_i^{\exp} - w_i \right|}{w_i^{\exp}}, \qquad (23)$$

which is the relative difference, averaged over all loads, between the deflections $w_i^{\exp}$ and w_i determined experimentally and theoretically on the basis of refined model (16-20), respectively.

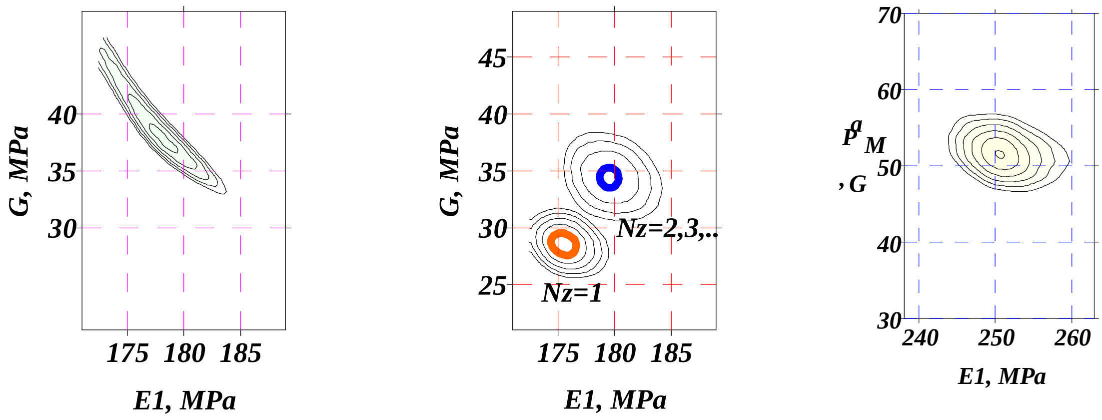
The process of identification of the elastic moduli E_1, E_2 and G and the Poisson ratio n of homogeneous beams by using the genetic algorithm and the results obtained are shown in Fig. 11, where *N* is the number of steps of minimization of error function (23).

Fig. 12 shows the $E_1 - G$ maps — the level lines of error function (5) upon variation of the moduli E_1 and G of homogeneous beams.



a) *b)*

Fig. 11. Stepwise identification of the elastic moduli of plastic foams of trademarks 3715 (a) and 6718 (b)



a) *b)* *c)*

Fig. 12. $E_1 - G$ maps: a — Timoshenko beam, trademark 3715, b — the refined model at different orders of approximation, and c — trademark 6718

The map shown in Fig. 14a was obtained by using the known solution for deflection of a beam in three-point bending [27]

$$d = w(0) = P\left(\frac{L}{4GF} + \frac{L^3}{48EI}\right). \tag{24}$$

The values of E_1 and G were varied in some specified intervals D_E and D_G with steps depending on the number of division points N_E and N_G:

$$E_1 = E_{10} + (p_E - 0.5)D_E, \quad G = G_0 + (p_G - 0.5)D_G,$$
$$p_E = \frac{i_E - 1}{N_E}, \quad p_G = \frac{i_G - 1}{N_G}, \quad i_E = 1, \ldots, N_E + 1, \quad i_G = 1, \ldots, N_G + 1. \tag{25}$$

To the required values of moduli there corresponds a point at the centre of the region bounded by the line with the least value of error function.

As seen from Fig. 12, the Timoshenko beam model gives somewhat underestimated values of Young's E_1 and shear G, moduli, which are close to those obtained from the refined model at $N_z = 1$. For an exact approximation based on Eqs. (16-20), it suffices to use two terms of expansion (1) along the normal $N_z = 2$ (see Fig. 12b). We should note that, in the case of sandwich, we need only one term of approximation for each layer. The convexity of the parameter maps indicates that the solution of the identification problem is unique and robust. Maps for the transverse modulus E_1 and Poisson ratio n are not shown in Fig. 12, because they are determined ambiguously.

In the case of dynamic tests, the error function is chosen in the form of quadratic deviation of experimental f_i^{exp} from calculated f_i^c values of vibration eigen-frequencies:

$$F_c = \sum_{i}^{N_f} \left(\frac{\left| f_i^{\exp} - f_i^c \right|}{f_i^{\exp}} \right)^2 . \tag{26}$$

The numerical calculation of vibration eigenfrequencies was performed on the basis of relations (16-20), with account of elastic and inertial properties of the beam and elements of the vibrator.

Figure 12 illustrates the process of identification of the moduli of a homogeneous beam with the use of criterion (25).

Figure 13 shows the maps of error function for different vibration frequencies according to formula (26). We should note that, if more than six frequencies are allowed for, the results obtained are practically identical. The first eigen-frequency was neglected because it greatly depended on fastening conditions of the beam and on the characteristics of vibrator.

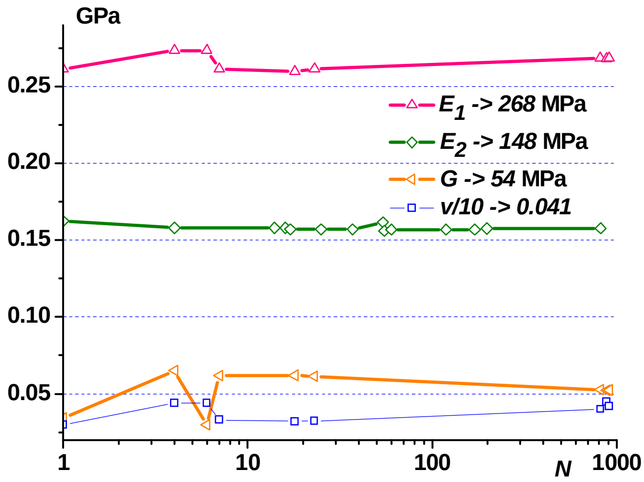


Fig. 12. Stepwise identification of the elastic moduli (GPa) and Poisson ratio of a homogeneous beam (trademark 6718)

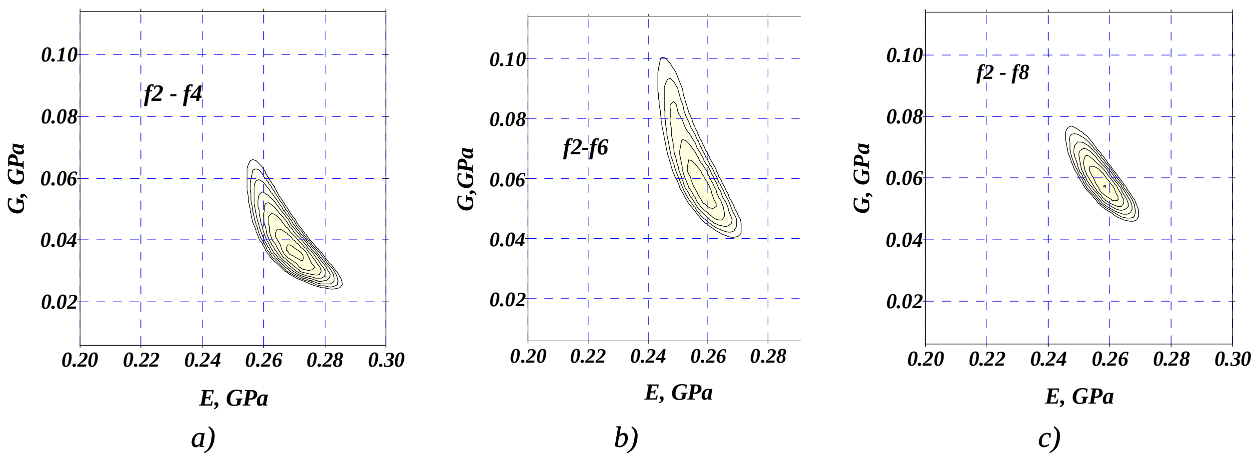


Fig. 13. E_1 –G maps (trademark 6718) for the frequencies f_2-f_4 (a), f_2-f_6 (b), and f_2-f_8 (c)

The results of a thorough analysis of sensitivity [25-27] show that the solution of the identification problem is very sensitive to changes in one group of elastic moduli and less sensitive to those in other moduli. The moduli E_1 and G are determined rather unequivocally both from static and dynamic tests, but the transverse modulus E_2 and the transverse Poisson ratio $,n$ are determined ambiguously. In [26], it is suggested to overcome this difficulty by employing two-step identification schemes. Note that, if an identification scheme is a priori inadequate for determining these moduli, then even splitting the identification process into several stages can turn out to be inefficient.

Verification of the identification scheme. To verify the identification scheme used, we will consider additionally several maps of error function which include the transverse modulus and Poisson ratio. The error function D0 is defined as the average of differences between the calculated static deflections w_i of a homogeneous beam in specified ranges of E_2 and ν and the deflections $w_i^{(0)}$ calculated with average values $E_{10}, E_{20}, G_0, \nu_0$ and ν_0 (6).

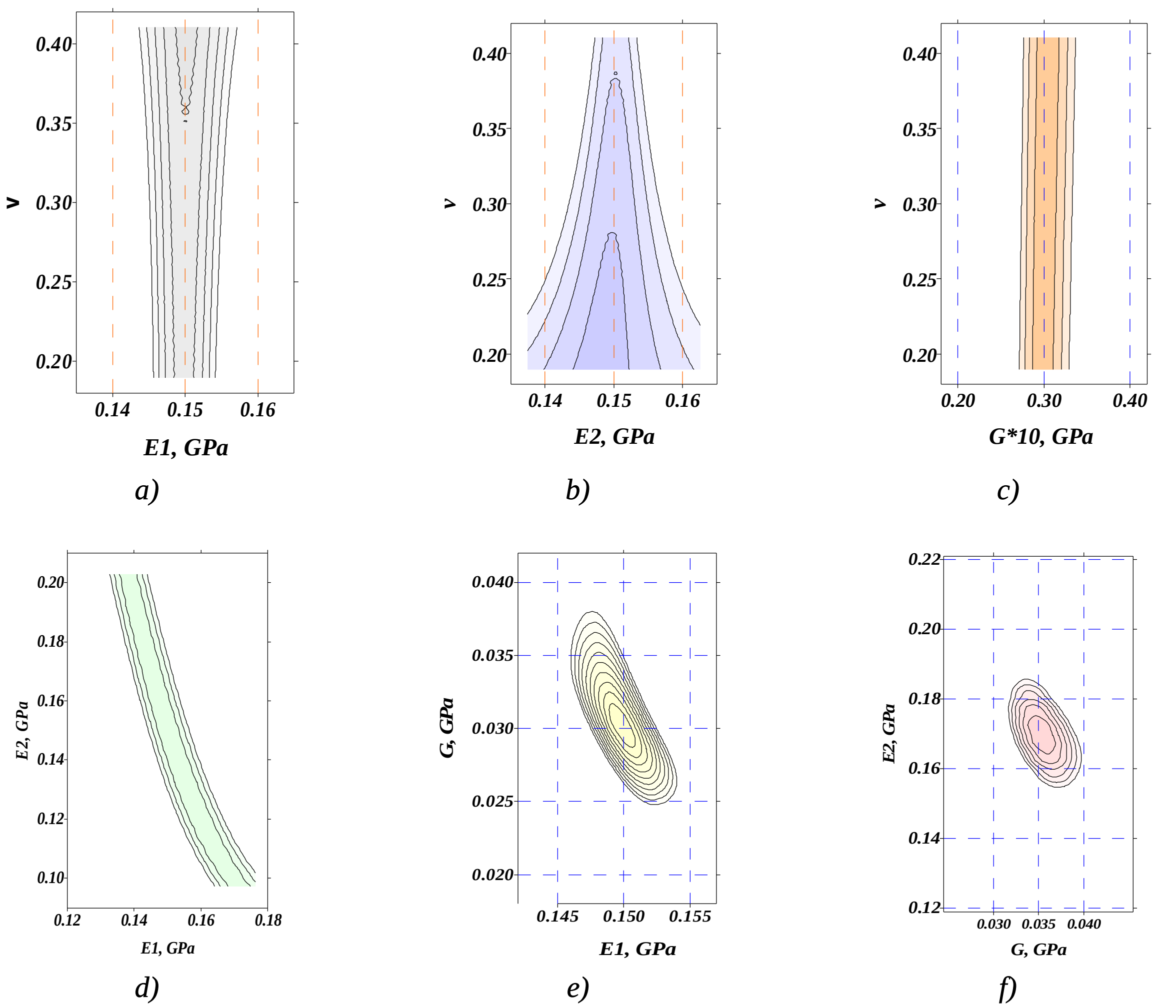


Fig. 14. $E_1-\nu$ (a), $E_2-\nu$ (b), $G-\nu$ (c), E_1-E_2 (d), E_1-G (e) and $G-E_2$ (f) maps (in statics)

Figure 14 depicts some maps of the error function Δ_0 in pairs with the transverse moduli. Similar results were also obtained in dynamic tests. As seen from data of the figure, in the identification scheme used, the transverse modulus E_2 and coefficient n cannot be determined unequivocally. Theoretically, they can be found from static tests by reducing the minimum length of beam span to two or three thicknesses of the beam.

Let us consider the three-point test by simultaneously use of beams of various lengths *L, 2L/3, L/3* (Fig. 15 – beam thickness *H=0.0254* m). The difference function is

$$\Delta = \left(w_0(0) \quad - w(0)\right)\Big|_{L=L_0/3} + \left(w_0(0) \quad - w(0)\right)\Big|_{L=2L_0/3} \Big/ 8 + \left(w_0(0) \quad - w(0)\right)\Big|_{L=L_0} \Big/ 27 \,. \tag{27}$$

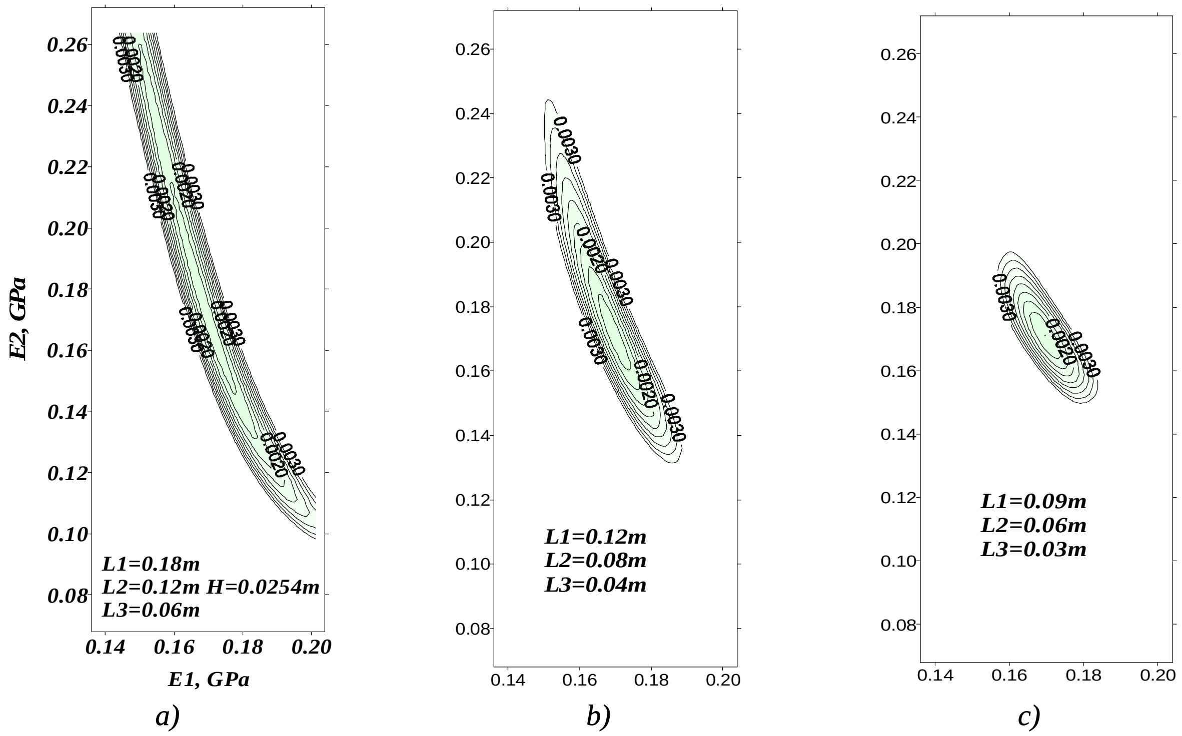


Fig. 15. Map for parameters E_1, E_2 test , beam thickness *H=0,0254* m: a – length *L=0.18, 0.12, 0.06* m; b – length *L=0.12, 0.08, 0.04* m; c – length *L=0.09, 0.06, 0.03* m

But, at such spans, which are practically comparable with beam thickness, the local deflections in supports must be taken into account. In dynamics, a considerable improvement in identification of the moduli is possible only in a range of sufficiently high frequencies. Such high-frequency measurements are rather complicated too. The difficulties in determining the transverse moduli can be explained if the vibration forms of the beam are examined (Fig. 16). One can notice that only at a frequency of about 10-15 kHz does the vibration wavelength approach the minimum length of a beam in static tests at which it is possible to separate the moduli E_1 and E_2

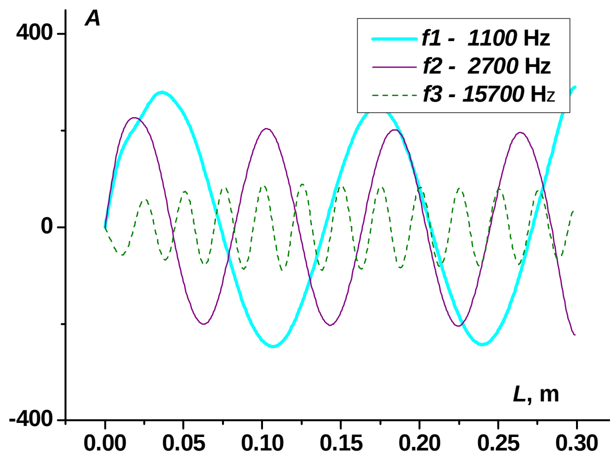


Fig. 16. Vibration modes of a homogeneous beam at f_1 = 1.1 (1), f_2 = 2.7 (2), and f_3 = 15.7 (3) kHz

In Fig. 15 consideration can be noticed, that as for dynamic so for static tests some modules little influence on mechanical behave of beam. For dynamic tests only shorter beams or high frequency numerical experiments are informing (Fig. 16, b, c). For static tests only shorter and deeper beams give the variance in the difference function (Fig. 16, b, 17, b, 18, b). It serves as basis for the following algorithm of identification of the modules: identification of the longitudinal module E_1 on the basis of testing enough long and thin beams and determination of transverse modules on the basis of testing of short beams or use of information about eigen-frequencies of higher range.

Combined criteria of identification. Let us consider new combined criteria of identification — schemes averaged over the calculation results for a homogeneous beam and for a sandwich with a core identical to the homogeneous beam and rigid outer layers. The error function is chosen as the sum of error functions for the homogeneous beam, Δ_C, and for the sandwich, Δ_S, in static tests

$$\Delta = \Delta_c + \Delta_s = \alpha \sum_i^{N_f} \left(\frac{\left| w_i^{\exp,c} - w_i^c \right|}{w_i^{\exp}} \right) + \beta \sum_i^{N_f} \left(\frac{\left| w_i^{\exp,s} - w_i^s \right|}{w_i^{\exp,s}} \right) \tag{28}$$

or as the sum of dynamic discrepancies for the homogeneous beam, Δ_{DC}, and the sandwich, Δ_{Df}, namely

$$\Delta_D = \Delta_{Dc} + \Delta_{Df} = \alpha \sum_i^{N_f} \left(\frac{\left| f_i^{\exp} - f_i^c \right|}{f_i^{\exp}} \right)^2 + \beta \sum_i^{N_f} \left(\frac{\left| f_i^{\exp,s} - f_i^{c,s} \right|}{f_i^{\exp,s}} \right)^2 . \tag{29}$$

Here, a and b are the weight factors. They were determined by numerical experiments during the minimization process and roughly corresponded to an equal contribution of both summands. Combined static-dynamic criteria, for example, $\Delta_S = \Delta + \Delta_D$ can also be employed.

In [26], it is proved that function (26) is convex. However, we should note that, for some parameters, its level lines are excessively extended (see Fig. 14, 15). In the case of application of the combined scheme of identification, they become more convex (see Fig. 17).

Let us consider the same maps of parameters for a sandwich in which the mechanical characteristics of core are the same as those of a homogeneous beam. Here, the $E_2 - n$ and $G - n$ maps for the inner layer are still nonconvex, but the $E_1 - E_2$ maps for the core and the $v - G_f$, $v - E_{xz}^f$, and $v - E_{zz}^f$ maps become convex (Fig. 17).

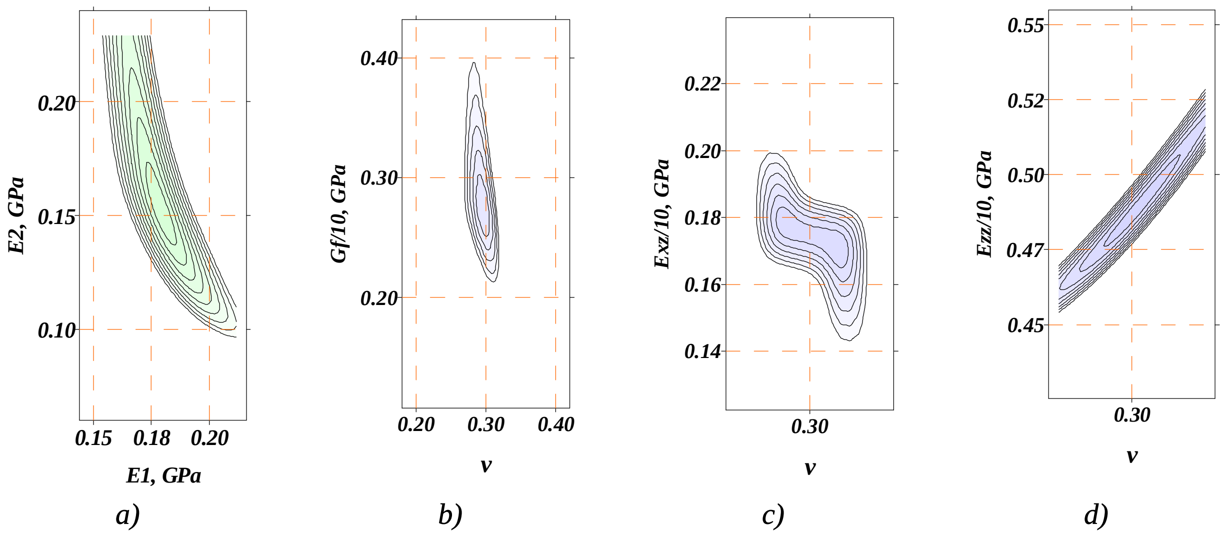
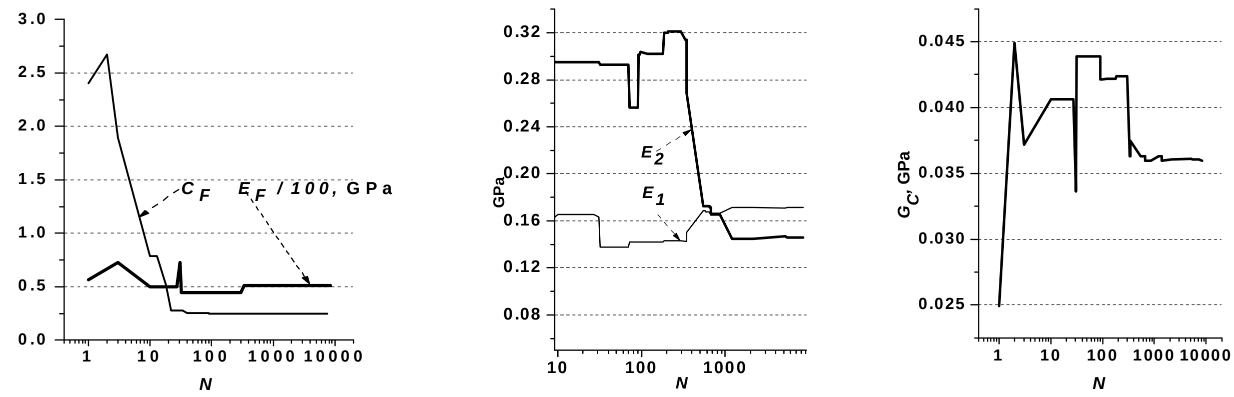


Fig. 17. $E_1 - E_2$ (a), $v - G_f$ (b), $v - E_{xz}^f$ (c), and $v - E_{zz}^f$ (d) maps (sandwich in statics)

As seen from the data of Figs. 17, additional information can be used for an unequivocal determination of the transverse modulus E_2 and Poisson ratio ν of the core and, simultaneously, of the moduli of outer layers. For this, it is sufficient that the equal-value level lines of the error function be convex and contract to a point as the error decreases. If, by some criteria, the maps (level lines) are striplines (see Fig. 14), it is necessary to search for variants where, with other schemes of experiments, they become convex. In this connection, it is expedient to consider combined criteria of identification that take into account this information.

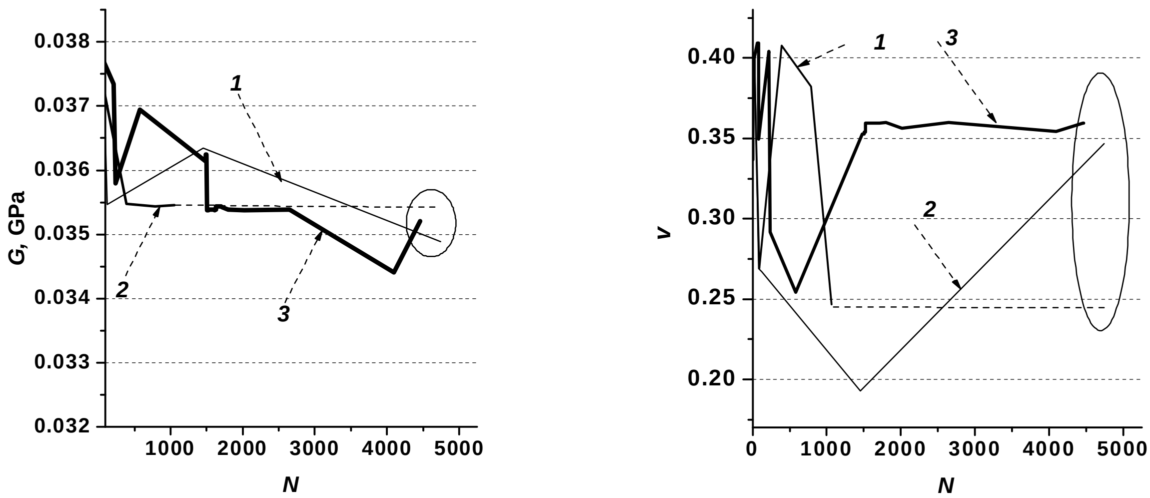
Let us analyze the identification of the moduli in the case of simultaneous use of data on a homogeneous beam and a beam with a core (material 3715). The error function is taken in form (27), with the moduli E_f, G_c, E_{1c} and E_{2c} as the parameters to be optimized. The other moduli of the sandwich are assigned approximately because of their weak influence on the dynamic and static properties of layered plates. Fig. 18 illustrates the identification processes.



a) *b)* *c)*

Fig. 18. Stepwise identification of the moduli E_f (a), E_{1c}, E_{2c} (b), and G_c (c) in the combined scheme. C_F – error function

Let us consider five variable parameters in the combined identification scheme, E_f, G_c, E_{1c}, E_{2c}, and ν. Here, we will additionally consider the Poisson ratio ν of the homogeneous beam. The processes of identification of the moduli according to different schemes are illustrated in Fig. 19.



a) *b)*

Fig. 19. Stepwise identification of the parameters G_c (a) and ν (b) in the combined scheme: 1 – homogeneous beam, 2 – sandwich beam, and 3 – combined scheme

We should note that, in the combined identification scheme, the Young's modulus E_2 and Poisson ratio n are also determined. The final results of the process of identification of the moduli in the different schemes are enclosed in ovals. The results of identification of the elastic moduli and Poisson ratio are presented in Table 4.

As seen from data of the table, the identified Young's E_1 and shear *G* moduli for materials 3715 and 6718 coincide, accurate to 1.5%, with data of the General Plastic company (USA), while the Young's modulus E_2 differs by 6-11.5%. The accuracy of agreement with the modulus *E2* determined with the help of combined criterion (27) in bending of a three-layer beam makes 1%.

Table 8

Comparison of the Elastic Characteristics of Materials

Trademark of plastic foam	Identification *	E_1	E_2	G	n
		GPa			
3715	1	179,2	139	$34 \div 35{,}4$	≈ 0.3
	2	176,5	141	35,2	0.355
6718	1	251,7		$57{,}5 \div 58{,}0$	
	2	251,5		51,15	
	3	258		54	

*1 – according to the General Plastics Company; 2 and 3 – from static and dynamic tests; *K* – with the use of combined criterion (8).

Conclusions

In the present study, theoretical models for investigations of the vibrations and damping of layered composite plates are developed. A rational approximation of the field of displacements is established, which allows one, at a small number of parameters, to predict the dynamic behavior of a beam. Based on this model, for a three-layer composite beam, not only the damping from the shear deformation in the core, but also the damping associated with the normal and bending deformations of layers, which is of importance for analyzing their damping properties under vibrations with moderate and high frequencies, was investigated. A new procedure for determining the parameters of the dynamic rigidity of three-layer plates is suggested, which was used to find the equivalent values of elastic moduli for a Timoshenko beam. We should note that the method presented does not require rigorous assumptions concerning the plate model. In the second part of the study, based on the multilevel theoretical approach described and a two-step procedure of identification for some composite plates, their elastic moduli will be determined.

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