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RANK-ORDER FILTERING BASED ON ANALOGUE K-WINNERS-TAKE-ALL NEURAL CIRCUIT

The problem of rank-order filtering is solved on the base of analogue neural circuit which determines maximal value signals among signal set. The filter is described by system of algebra-differential equations and combines such properties as high accuracy and speed, low computational and hardware implementation complexity, and independency on initial conditions. The filter can be used for processing of constant signals, variable signals, and also equal signals. The filter simulation examples confirming theoretical statements are provided.

Keywords: rank-order filtering, analogue neural circuit, a system of algebra-differential equations, computational complexity, hardware implementation, computer simulation.

1. Introduction

As known, rank-order filters are nonlinear filters which have many applications including digital image processing, speech processing, coding and digital TV, etc.[1] – [5]. A rank-order filter functions by working by selecting its input with a certain rank as its output. Rank-order filters entails substantial processing power to implement, which limits their real-time signal processing applications. Nevertheless, rank-order filters can benefit from their parallelist realizations.

Numerous approaches have been developed to design rank-order filters using hardware [1] – [4]. In particular, a rank order filter design based on two KWTA models are proposed in [3]. A KWTA model with K winners is used in parallel to another KWTA model with K–1 winners to select the input with its rank-order being K in [5].

2. Rank-order filtering

Let us derive the filtered output signal as

$$c = a^T\left(S^K - S^{K-1}\right), \tag{1}$$

where S^K is a step vector function which can be defined or encoded as the following binary functions:

$$S_k^K(x) = \begin{cases} 1, & \text{if} \quad a_{n_k} - x > 0; \\ 0, & \text{if} \quad a_{n_k} - x \le 0, \end{cases} \tag{2}$$

where $k = 1,2,\ldots,N$, $a = \left(a_{n_1}, a_{n_2}, \ldots, a_{n_N}\right)^T$ is an input vector with the elements distinct and arranged in a descending order of magnitude satisfying the inequalities

$$\infty > a_{n_1} > a_{n_2} > \cdots > a_{n_N} > -\infty, \tag{3}$$

$n_1, n_2, ..., n_N$ are numbers of the first largest input, the second largest input and so on up to Nth largest input inclusive, $c = \left(c_{n_1}, c_{n_2}, \ldots, c_{n_N}\right)^T$ is an output vector of the rank-order filter which can be realized by using the state equation of continuous-time model of analogue KWTA neural circuit given by

$$\dot{x} = -\alpha\begin{cases} x, & \text{if} \quad E(x) > 0; \\ 0, & \text{if} \quad E(x) = 0; \\ x - A, & \text{if} \quad E(x) < 0, \end{cases} \tag{4}$$

with a state variable $x \in \Re$ and an initial condition $0 \le x_0 \le A$,

$$E^K(x) = K - \sum_{k=1}^{N} S_k^K(x) \tag{5}$$

is a residual function, α is a constant parameter or decaying coefficient that is used for controlling a convergence speed of the state variable trajectories to the KWTA operation [6]. As it was pointed out above, in view that (2) and (6) are discontinuous functions of x, the state equation (4) is an ordinary differential equation with a discontinuous right-hand side.

The results presented above are valid not only for sets of time-constant inputs (3). The model (1) can be also used in the case of time-varying input signals $a_{n_k}(t)$, $k = 1,2,\ldots,N$ if the speed module of such signals is much less than that of the state variable x during transients. In other words, in this case, condition

$$\left|da_{n_k} / dt\right|_{max} << \left|dx / dt\right|_{min}. \tag{6}$$

should be satisfied for each $t < t^*$, $k = 1,2,\ldots,N$. In order to match the condition (7), the value of the decaying coefficient α should be large enough.

Let us consider the case where two or more inputs of the model (4) are equal to one another. If such inputs belong to K winners or to $N - K$ losers, then the outputs of the model converge to KWTA operation [6]. However, if the model should distinguish equal largest inputs and split them into positive and negative planes when there exists a number of maximal inputs less or greater than K only, then the outputs of the model which do not have KWTA operation will be obtained. The model outputs will demonstrates a phenomenon of chattering in the time points in which the condition (2) is violated, in other words, where input signals are equal one to another. Specifically, a dynamic shift x of inputs will take place in a sliding mode around a certain point e_e of equal inputs of vector **a**. In this case, the conditions of the existing sliding modes $\dot{s}_{s\to -0} > 0$, $\dot{s}_{s\to +0} < 0$ are satisfied and the sliding mode equation is given by

$$s = x - a_e = 0. \tag{7}$$

A derivative $\dot{x}$ in this case is not defined on the surface of discontinuity Δ since K largest inputs do not exist and, therefore, $E(x) \ne 0$. Outputs $b_{n_k}, k = 1,2,..., N$ will slide around the point $a_{n_k} - x$ [7] and therefore a residual function (5) will accept sequentially the following two values: $E^K(x) = -1$ and $E^K(x) = 1$.

In order to remove indicated oscillations let us generalize the model (4) on the case of processing such time-varying input signals, some of them can be equal one to another in some time moments. In such time moments inequalities (3) are not satisfied and therefore K maximal input signals do not exist. Let us extend for this case the model (4) to the following form:

$$c = a^T\left(S^K - S^{K-1}\right), \text{ if } E^K(x) = 0 \text{ and } E^{K-1} = 0;$$
$$\frac{dc}{dt} = 0,\ c_0 = 0, \text{ otherwise}, \tag{8}$$

where c_0 is an initial condition. It is not hard to see, that in the steady state KWTA mode the system of algebra-differential equations (8) is reduced to the expression (1) which is a particular case of the system (8). In the transients output signals of the rank-order filter are described by the degenerative differential equation of the system (8).

3. Computer simulation results

In order to illustrate the theoretical results presented above, let us consider concrete example with corresponding computer simulations which demonstrate the rank-order filtering by the filter model (8) based on the continuous-time model of an analogue KWTA neural circuit (4). Let us design for this purpose corresponding program in the codes of high-performance language for the technical computing Matlab. To run such programs we use a 1.81 GHz desktop PC.

Example. Let us set input sinusoidal signals defined as $a_i = \sin(\omega t + i\varphi) + d$ (i=1,2,…,m-1), where ω is the angular frequency, φ is the phase shift, and d is the bias. Let us also use as an additional input signal a_m uniformly distributed on the interval (-1,1) random noise. In order to improve a performance of differential equations (4) and (8) solving we use corresponding finite difference equations with the time step $\Delta t = 1 \times 10^{-3}$. The first subplot in Fig. 1 depicts the eight input sinusoidal signals, the second subplot shows the random noise n uniformly distributed on the interval [-1, 1], and the last subplot presents the filtered output signal c obtained by the expression (1) and model (4) for m=9, $\alpha = 1000$, $x_0 = 0$, K=5, d=2, ω = 2π, and φ = π/4. As one can see, this signal presents a phenomenon of chattering in the time points in which the condition (2) is violated in accordance with the predictions. For the same data, the filtered output signal c obtained using the system of algebra-differential equations (8) is shown in Fig. 2. According to prognosis the oscillations in the time points where input signals are equal one to another have been completely removed. It is not hard to see that the system presents a median filter because of K=5. Since $\left|da_{n_k}/dt\right|_{max} = 1$ and $\left|dx/dt\right|_{min} = 100$ therefore inequality (6) is satisfied. As one can see from the results shown in Fig. 2, the analogue rank-order filter built on the base of the KWTA neural circuit described by the model (8) presents good performance including the time points of equal input signals.

Note that if the problem considered in this example is solved for the same data using the rank-order filter presented in [5] which is described by expression (1) and one of the most simple models of discrete-time KWTA neural circuits, then output signal of the filter contains distortions similar to ones presented in Fig. 1 in the time points in which the condition (3) is not satisfied.

4. Conclusions

As one can see from the results obtained by computer simulations, the output signals of the model (8) of the analogue rank-order filter, are correct ones including the time points of equal input signals. Such filter can have various applications, especially for real time signal processing. In particular, in the special case if N=3, such analogue rank-order filter can be used in analog fault tolerant systems. As known, analogue fault tolerant systems can be designed based on the analog Tripple-Modular Redundancy (TMR). The TMR approach is a frequently used design technique in fault tolerant systems, especially for critical-computation applications [8]. The basic concept of TMR is to triplicate the hardware and perform a majority vote to determine the output of the system. If one of the module becomes faulty, two-remaining fault-free modules mask the result of the faulty module. One major problem arises if the result of each module is an analog signal. In such cases, the three signals may not completely agree in value even if the system functions with no-fault. One technique that alleviates this problem is the mid-value select technique [9], which selects the middle value of each triplet. Obviously, a 3-input 2^{nd} WTA network perfectly matches this selection method and thus can be directly used for this application.

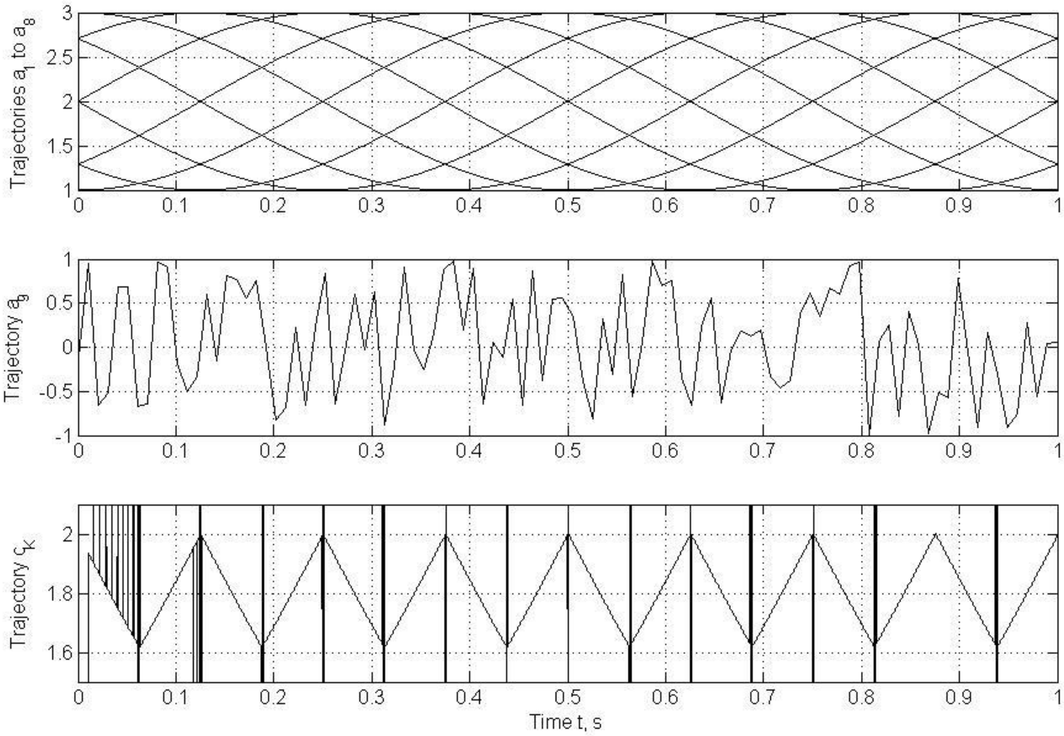

Fig. 1. *Trajectory of eight sinusoidal input signals, one noised input signal, and filtered output signal of the rank-order filter described by the expression (1) and KWTA model (4).*

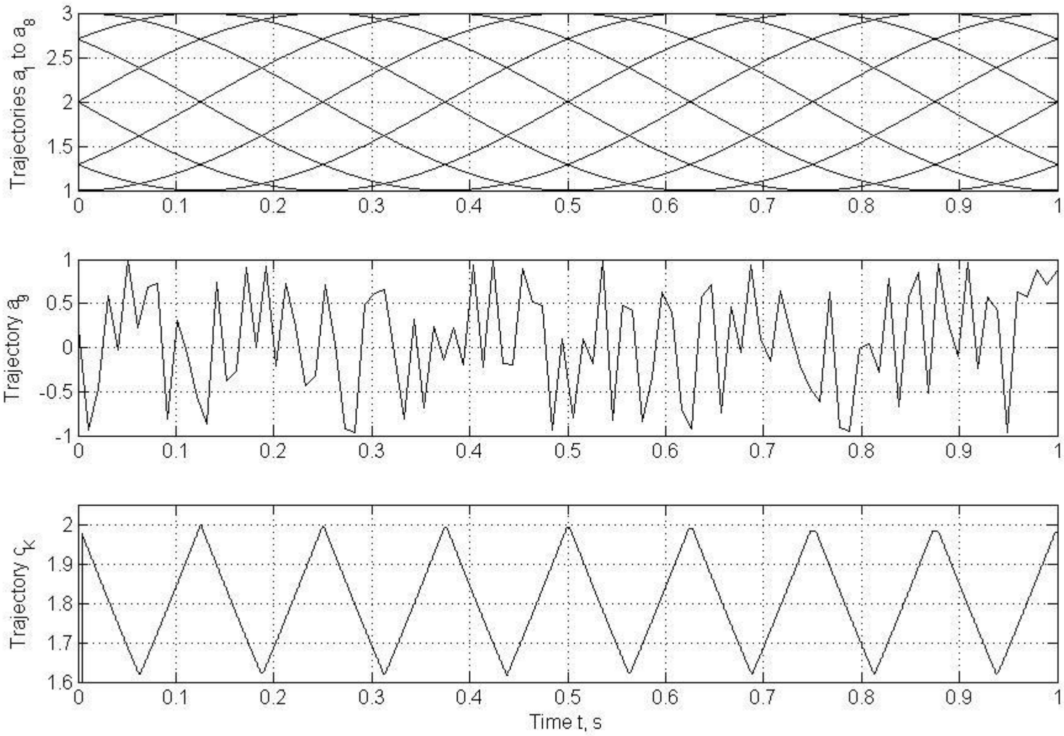

Fig. 2. *Dynamics of eight sinusoidal input signals, one noised input signal, and filtered output signal of the rank-order filter described by the system of algebra-differential equations (8).*

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