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## MATHEMATICAL MODELLING RELIABILITY PARAMETERS OF UNSYMMETRICAL RAMIFIED SYSTEMS

**Main reliability parameters of unrestorable unsymmetrical systems, ramified to level 3, with ageing output elements, are examined in this paper. A method of investigation of reliability parameters of ramified systems by means of generating functions is developed taking account of ageing of the system's output elements. Mathematical models of the probability distribution of count of output working elements, the duration of the system's stay in each of its states and the duration of the system's stay in the prescribed availability state are worked out in case when the lifetime of ageing output elements is circumscribed by the Rayleigh distribution.**

**Key words - reliability parameters, ramified systems, ageing elements, Rayleigh distribution.**

### Introduction

Reliability is a very important concept that is determined in terms of its application to the wide range of activities. Reliability deals with products in service [1].

Reliability analysis of complicated systems is obligatory at their design. Complication of the systems often grows quicker from development of mathematical methods of their researches. Insufficiently high reliability can result in excessive charges on repair and renewal or even to more serious consequences, in particular to the near-accidents or accidents.

The problem of reliability investigation of the systems arose up a long ago together with becoming of the engineering approach to industry. Every engineering object must contain the sign of reliability. At a choice among competitive projects, reliability parameters occupy an important place in the list of requirements. But reliability prognostication is difficult because of multivariate and statistical nature of this phenomenon. In recent years with speed-up development of the computing engineering, the reliability calculations for sufficiently complicated systems became possible taking into account plenty of parameters at cost cutout on such calculations.

Existent traditional methods of systems reliability analysis and estimation are mostly oriented to simple objects and cannot satisfy to a full degree the necessity of analysis of large unsymmetrical ramified systems.

Separate examples of such systems are control systems, measuring systems, some types of computer local networks. On an output level such systems have sensors, printers, keyboards, disk drives that expose to exhaustion and aging. Lifetime of such devices is often described by the Rayleigh distribution or the Weibull distribution. Lifetime of elements on higher levels is described by the exponential distribution [2]. It is necessary to develop the methods of reliability prognostication taking into account features of the systems.

### Mathematical models of reliability parameters

Let us consider a system where 2 elements of level 1 are subordinate to the element of level 0,  $a_2^{(1)}$  elements of level 2 are subordinate to the first element of level 1, every of these  $a_2^{(1)}$  elements subordinates  $a_3^{(1)}$  elements of level 3; the second element of level 1 subordinates  $a_2^{(2)}$  elements of level 2, every of

these  $a_2^{(2)}$  elements subordinates  $a_3^{(2)}$  elements of level 3 (Fig. 1), where  $a_2^{(1)}$ ,  $a_2^{(2)}$  are the ramification coefficients of the first and the second branch correspondingly to level 2,  $a_3^{(1)}$ ,  $a_3^{(2)}$  are the ramification coefficients of the first and the second branch correspondingly to level 3. Without loss of generality assume that  $a_2^{(1)} a_3^{(1)} \leq a_2^{(1)} a_3^{(2)}$ .

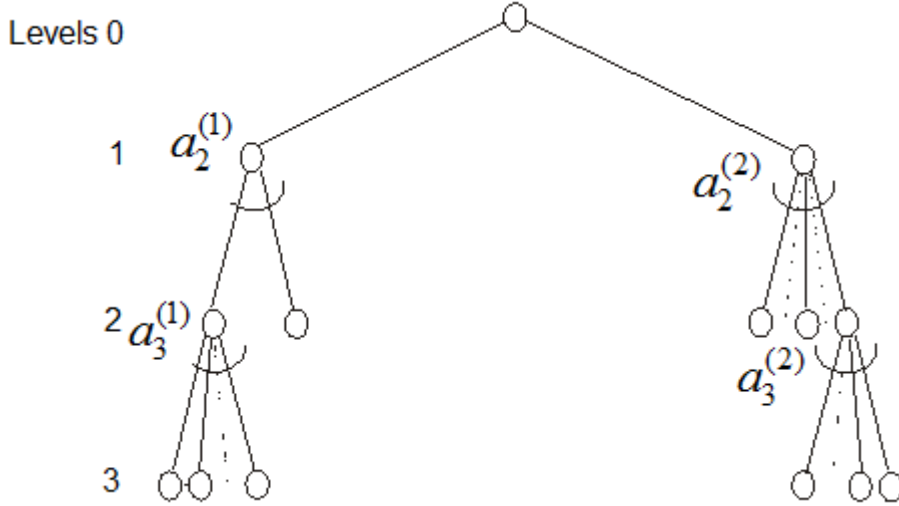


Fig. 1. An unsymmetrical system, ramified to level 3, with 2 branches of unequal value on level 1

The exponential distribution is a special case of the Weibull distribution just as it is of the gamma distribution. The Weibull distribution has different failure rates depending on a shape parameter [3]. The Raileigh distribution is a special case of the Weibull distribution which demarcates two different types of increasing failure rate behaviour, those that are concave upward and those that are concave downward.

We use  $P_{3R}(x_3, t)$  to denote the probability that there are exactly  $x_3$  operating output elements provided the probability of failure-free operation of ageing output elements are circumscribed by the Raileigh distribution. Under conditions  $a_2^{(1)} a_3^{(1)} \leq a_2^{(1)} a_3^{(2)}$ ,  $0 < x_3 \leq a_2^{(1)} a_3^{(1)} + a_2^{(2)} a_3^{(2)}$  we obtain:

$$\begin{aligned}
 P_{3R}(x_3, t) = & e^{-\lambda_0 t} \sum_{x_3^{(1)} = \max\{0, x_3 - a_2^{(2)} a_3^{(2)}\}}^{\min\{x_3, a_2^{(1)} a_3^{(1)}\}} \sum_{x_1^{(1)} = \text{ceil}\left(\frac{x_3^{(1)}}{a_2^{(1)}}\right)}^1 \sum_{x_2^{(1)} = \text{ceil}\left(\frac{x_3^{(1)}}{a_3^{(1)}}\right)}^{a_2^{(1)} x_1^{(1)}} C_{a_2^{(1)} x_1^{(1)}}^{x_2^{(1)}} C_{a_3^{(1)} x_2^{(1)}}^{x_3^{(1)}} \times \\
 & \times \sum_{x_1^{(2)} = \text{ceil}\left(\frac{x_3 - x_3^{(1)}}{a_2^{(2)}}\right)}^1 e^{-\lambda_1 (x_1^{(1)} + x_1^{(2)}) t} (1 - e^{-\lambda_1 t})^{2 - (x_1^{(1)} + x_1^{(2)})} \sum_{x_2^{(2)} = \text{ceil}\left(\frac{x_3 - x_3^{(1)}}{a_3^{(2)}}\right)}^{a_2^{(2)} x_1^{(2)}} C_{a_2^{(2)} x_1^{(2)}}^{x_2^{(2)}} C_{a_3^{(2)} x_2^{(2)}}^{x_3 - x_3^{(1)}} \times \\
 & \times e^{-\lambda_2 (x_2^{(1)} + x_2^{(2)}) t} (1 - e^{-\lambda_2 t})^{a_2^{(1)} x_1^{(1)} + a_2^{(2)} x_1^{(2)} - (x_2^{(1)} + x_2^{(2)})} e^{-\frac{x_3 t^2}{2\sigma_3^2}} \left(1 - e^{-\frac{t^2}{2\sigma_3^2}}\right)^{a_3^{(1)} x_2^{(1)} + a_3^{(2)} x_2^{(2)} - x_3}.
 \end{aligned} \tag{1}$$

In case of  $x_3=0$  the right side of the equation (1) should increase by  $1 - e^{-\lambda_0 t}$ .

The average time to failure of the system  $T_c$  is calculated as an integral from 0 to  $\infty$  of the

probability of trouble-free  $P(t)$  [4]. Analogously, an expression for the probability distribution of count of output working elements having been integrated with respect to  $t$  from 0 to  $\infty$ , under condition  $0 < x_3 \leq a_2^{(1)} a_3^{(1)} + a_2^{(2)} a_3^{(2)}$  we obtain the average duration of the system's stay in each of its working states.

We use  $T_{3R}(x_3)$  to denote the average duration of the system's stay in a state of  $x_3$  operating output elements provided that lifetime of ageing output elements is circumscribed by the Raileigh distribution. In order to calculate  $T_{3R}(x_3)$  we integrate (1) with respect to  $t$  from 0 to  $\infty$ . We obtain the following expression:

$$\begin{aligned}
T_{3R}(x_3) = & \int_0^{\infty} e^{-\lambda_0 t} \sum_{x_3^{(1)} = \max\{0, x_3 - a_2^{(2)} a_3^{(2)}\}}^{\min\{x_3, a_2^{(1)} a_3^{(1)}\}} \sum_{x_1^{(1)} = \text{ceil}\left(\frac{\text{ceil}\left(\frac{x_3^{(1)}}{a_3^{(1)}}\right)}{a_2^{(1)}}\right)}^1 \sum_{x_2^{(1)} = \text{ceil}\left(\frac{x_3^{(1)}}{a_3^{(1)}}\right)}^{a_2^{(1)} x_1^{(1)}} C_{a_2^{(1)} x_1^{(1)}}^{x_2^{(1)}} C_{a_3^{(1)} x_2^{(1)}}^{x_3^{(1)}} \times \\
& \times \sum_{x_1^{(2)} = \text{ceil}\left(\frac{\text{ceil}\left(\frac{x_3 - x_3^{(1)}}{a_3^{(2)}}\right)}{a_2^{(2)}}\right)}^1 e^{-\lambda_1 (x_1^{(1)} + x_1^{(2)}) t} (1 - e^{-\lambda_1 t})^{2 - (x_1^{(1)} + x_1^{(2)})} \sum_{x_2^{(2)} = \text{ceil}\left(\frac{x_3 - x_3^{(1)}}{a_3^{(2)}}\right)}^{a_2^{(2)} x_1^{(2)}} C_{a_2^{(2)} x_1^{(2)}}^{x_2^{(2)}} C_{a_3^{(2)} x_2^{(2)}}^{x_3 - x_3^{(1)}} \times \\
& \times e^{-\lambda_2 (x_2^{(1)} + x_2^{(2)}) t} (1 - e^{-\lambda_2 t})^{a_2^{(1)} x_1^{(1)} + a_2^{(2)} x_1^{(2)} - (x_2^{(1)} + x_2^{(2)})} e^{-\frac{x_3^2 t}{2\sigma_3^2}} \left(1 - e^{-\frac{t^2}{2\sigma_3^2}}\right)^{a_3^{(1)} x_2^{(1)} + a_3^{(2)} x_2^{(2)} - x_3} dt. \tag{2}
\end{aligned}$$

In the equation (2) we transform expressions between brackets raised to some power by the binomial theorem as follows:

$$(1 - e^{-\lambda_2 t})^{a_2^{(1)} x_1^{(1)} + a_2^{(2)} x_1^{(2)} - (x_1^{(1)} + x_1^{(2)})} = \sum_{j_2=0}^{a_2^{(1)} x_1^{(1)} + a_2^{(2)} x_1^{(2)} - (x_1^{(1)} + x_1^{(2)})} C_{a_2^{(1)} x_1^{(1)} + a_2^{(2)} x_1^{(2)} - (x_1^{(1)} + x_1^{(2)})}^{j_2} (-1)^{j_2} e^{-\lambda_2 j_2 t}, \tag{3}$$

$$\left(1 - e^{-\frac{t^2}{2\sigma_3^2}}\right)^{a_3^{(1)} x_2^{(1)} + a_3^{(2)} x_2^{(2)} - x_3} = \sum_{j_3=0}^{a_3^{(1)} x_2^{(1)} + a_3^{(2)} x_2^{(2)} - x_3} C_{a_3^{(1)} x_2^{(1)} + a_3^{(2)} x_2^{(2)} - x_3}^{j_3} (-1)^{j_3} e^{-\frac{j_3 t^2}{2\sigma_3^2}}. \tag{4}$$

Substituting (3), (4) into (2), we obtain:

$$\begin{aligned}
T_{3R}(x_3) = & \sum_{x_3^{(1)} = \max\{0, x_3 - a_2^{(2)} a_3^{(2)}\}}^{\min\{x_3, a_2^{(1)} a_3^{(1)}\}} \sum_{x_1^{(1)} = \text{ceil}\left(\frac{\text{ceil}\left(\frac{x_3^{(1)}}{a_3^{(1)}}\right)}{a_2^{(1)}}\right)}^1 \sum_{x_2^{(1)} = \text{ceil}\left(\frac{x_3^{(1)}}{a_3^{(1)}}\right)}^{a_2^{(1)} x_1^{(1)}} C_{a_2^{(1)} x_1^{(1)}}^{x_2^{(1)}} C_{a_3^{(1)} x_2^{(1)}}^{x_3^{(1)}} \times \\
& \times \sum_{x_1^{(2)} = \text{ceil}\left(\frac{\text{ceil}\left(\frac{x_3 - x_3^{(1)}}{a_3^{(2)}}\right)}{a_2^{(2)}}\right)}^1 e^{-\lambda_1 (x_1^{(1)} + x_1^{(2)}) t} (1 - e^{-\lambda_1 t})^{2 - (x_1^{(1)} + x_1^{(2)})} \sum_{x_2^{(2)} = \text{ceil}\left(\frac{x_3 - x_3^{(1)}}{a_3^{(2)}}\right)}^{a_2^{(2)} x_1^{(2)}} C_{a_2^{(2)} x_1^{(2)}}^{x_2^{(2)}} C_{a_3^{(2)} x_2^{(2)}}^{x_3 - x_3^{(1)}} \times \\
& \times \sum_{j_1=0}^{2 - (x_1^{(1)} + x_1^{(2)})} C_{2 - (x_1^{(1)} + x_1^{(2)})}^{j_1} (-1)^{j_1} \sum_{j_2=0}^{a_2^{(1)} x_1^{(1)} + a_2^{(2)} x_1^{(2)} - (x_1^{(1)} + x_1^{(2)})} C_{a_2^{(1)} x_1^{(1)} + a_2^{(2)} x_1^{(2)} - (x_1^{(1)} + x_1^{(2)})}^{j_2} (-1)^{j_2} \times
\end{aligned} \tag{5}$$

$$\times \sum_{j_3=0}^{a_3^{(1)}x_2^{(1)}+a_3^{(2)}x_2^{(2)}-x_3} C_{a_3^{(1)}x_2^{(1)}+a_3^{(2)}x_2^{(2)}-x}^{j_3} (-1)^{j_3} \int_0^{\infty} e^{-(\lambda_0+\lambda_1(x_1^{(1)}+x_1^{(2)}+j_1)+\lambda_2(x_2^{(1)}+x_2^{(2)}+j_2))t} e^{-\frac{x_3+j_3}{\sigma_3^2}t^2} dt.$$

Under conditions  $a_2^{(1)}a_3^{(1)} \leq a_2^{(1)}a_3^{(2)}$ ,  $0 < x_3 \leq a_2^{(1)}a_3^{(1)} + a_2^{(2)}a_3^{(2)}$  we take down the equation (5) in the form of the next equation:

$$\begin{aligned} T_{3R}(x_3) &= \sum_{x_3^{(1)}=\max\{0, x_3-a_2^{(2)}a_3^{(2)}\}}^{\min\{x_3, a_2^{(1)}a_3^{(1)}\}} \sum_{x_1^{(1)}=\text{ceil}\left(\frac{\text{ceil}\left(\frac{x_3^{(1)}}{a_3^{(1)}}\right)}{a_2^{(1)}}\right)}^1 \sum_{x_2^{(1)}=\text{ceil}\left(\frac{x_3^{(1)}}{a_3^{(1)}}\right)}^{a_2^{(1)}x_1^{(1)}} C_{a_2^{(1)}x_1^{(1)}}^{x_2^{(1)}} C_{a_3^{(1)}x_2^{(1)}}^{x_3^{(1)}} \sum_{x_1^{(2)}=\text{ceil}\left(\frac{\text{ceil}\left(\frac{x_3-x_3^{(1)}}{a_2^{(2)}}\right)}{a_2^{(2)}}\right)}^1 e^{-\lambda_1(x_1^{(1)}+x_1^{(2)})t} (1-e^{-\lambda_1 t})^{2-(x_1^{(1)}+x_1^{(2)})} \times \\ &\times \sum_{x_2^{(2)}=\text{ceil}\left(\frac{x_3-x_3^{(1)}}{a_3^{(2)}}\right)}^{a_2^{(2)}x_1^{(2)}} C_{a_2^{(2)}x_1^{(2)}}^{x_2^{(2)}} C_{a_3^{(2)}x_2^{(2)}}^{x_3-x_3^{(1)}} \sum_{j_1=0}^{2-(x_1^{(1)}+x_1^{(2)})} C_{2-(x_1^{(1)}+x_1^{(2)})}^{j_1} (-1)^{j_1} \sum_{j_2=0}^{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-(x_1^{(1)}+x_1^{(2)})} C_{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-(x_1^{(1)}+x_1^{(2)})}^{j_2} (-1)^{j_2} \times \quad (6) \\ &\times \sum_{j_3=0}^{a_3^{(1)}x_2^{(1)}+a_3^{(2)}x_2^{(2)}-x_3} C_{a_3^{(1)}x_2^{(1)}+a_3^{(2)}x_2^{(2)}-x}^{j_3} (-1)^{j_3} \frac{\sigma_3 \sqrt{\pi}}{\sqrt{2(x_3+j_3)}} e^{\frac{\sigma_3^2(\lambda_0+\lambda_1(x_1^{(1)}+x_1^{(2)}+j_1)+\lambda_2(x_2^{(1)}+x_2^{(2)}+j_2))}{\sqrt{2(x_3+j_3)}}} \text{erfc}\left(\frac{\sigma_3(\lambda_0+\lambda_1(x_1^{(1)}+x_1^{(2)}+j_1)+\lambda_2(x_2^{(1)}+x_2^{(2)}+j_2))}{\sqrt{2(x_3+j_3)}}}\right), \end{aligned}$$

where *erfc* is the complementary error function.

Notice that  $T_{3R}(0)=\infty$ .

The sum of average durations of the system's stay in states over count of output elements from  $k$  to  $a_2^{(1)}a_3^{(1)} + a_2^{(2)}a_3^{(2)}$ , where  $0 < k \leq a_2^{(1)}a_3^{(1)} + a_2^{(2)}a_3^{(2)}$ , equals the average duration of the system's stay in the prescribed availability condition  $k$ .

We use  $T_{\Gamma 3R}(k)$  to denote the average duration of the system's stay in the availability condition  $k$  provided that lifetime of ageing output elements is circumscribed by the Raileigh distribution. Taking (6) into account, under conditions  $a_2^{(1)}a_3^{(1)} \leq a_2^{(1)}a_3^{(2)}$ ,  $0 < k \leq a_2^{(1)}a_3^{(1)} + a_2^{(2)}a_3^{(2)}$  we obtain the following expression:

$$\begin{aligned} T_{\Gamma 3R}(k) &= \sum_{x_3=k}^{a_2^{(1)}a_3^{(1)}+a_2^{(2)}a_3^{(2)}} = \sum_{x_3^{(1)}=\max\{0, x_3-a_2^{(2)}a_3^{(2)}\}}^{\min\{x_3, a_2^{(1)}a_3^{(1)}\}} \sum_{x_1^{(1)}=\text{ceil}\left(\frac{\text{ceil}\left(\frac{x_3^{(1)}}{a_3^{(1)}}\right)}{a_2^{(1)}}\right)}^1 \sum_{x_2^{(1)}=\text{ceil}\left(\frac{x_3^{(1)}}{a_3^{(1)}}\right)}^{a_2^{(1)}x_1^{(1)}} C_{a_2^{(1)}x_1^{(1)}}^{x_2^{(1)}} C_{a_3^{(1)}x_2^{(1)}}^{x_3^{(1)}} \sum_{x_1^{(2)}=\text{ceil}\left(\frac{\text{ceil}\left(\frac{x_3-x_3^{(1)}}{a_2^{(2)}}\right)}{a_2^{(2)}}\right)}^1 e^{-\lambda_1(x_1^{(1)}+x_1^{(2)})t} (1-e^{-\lambda_1 t})^{2-(x_1^{(1)}+x_1^{(2)})} \times \\ &\times \sum_{x_2^{(2)}=\text{ceil}\left(\frac{x_3-x_3^{(1)}}{a_3^{(2)}}\right)}^{a_2^{(2)}x_1^{(2)}} C_{a_2^{(2)}x_1^{(2)}}^{x_2^{(2)}} C_{a_3^{(2)}x_2^{(2)}}^{x_3-x_3^{(1)}} \sum_{j_1=0}^{2-(x_1^{(1)}+x_1^{(2)})} C_{2-(x_1^{(1)}+x_1^{(2)})}^{j_1} (-1)^{j_1} \sum_{j_2=0}^{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-(x_1^{(1)}+x_1^{(2)})} C_{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-(x_1^{(1)}+x_1^{(2)})}^{j_2} (-1)^{j_2} \times \quad (7) \\ &\times \sum_{j_3=0}^{a_3^{(1)}x_2^{(1)}+a_3^{(2)}x_2^{(2)}-x_3} C_{a_3^{(1)}x_2^{(1)}+a_3^{(2)}x_2^{(2)}-x}^{j_3} (-1)^{j_3} \frac{\sigma_3 \sqrt{\pi}}{\sqrt{2(x_3+j_3)}} e^{\frac{\sigma_3^2(\lambda_0+\lambda_1(x_1^{(1)}+x_1^{(2)}+j_1)+\lambda_2(x_2^{(1)}+x_2^{(2)}+j_2))}{\sqrt{2(x_3+j_3)}}} \text{erfc}\left(\frac{\sigma_3(\lambda_0+\lambda_1(x_1^{(1)}+x_1^{(2)}+j_1)+\lambda_2(x_2^{(1)}+x_2^{(2)}+j_2))}{\sqrt{2(x_3+j_3)}}}\right), \end{aligned}$$

where *erfc* is the complementary error function.

As an example we consider a technical equipment of a computer network which includes a server, two hubs, workstations and takes the form of a ramified system shown in Fig. 2 where  $S$  denotes a server,  $H$  denotes a hub,  $W$  denotes a workstation,  $L$  denotes a communication line.

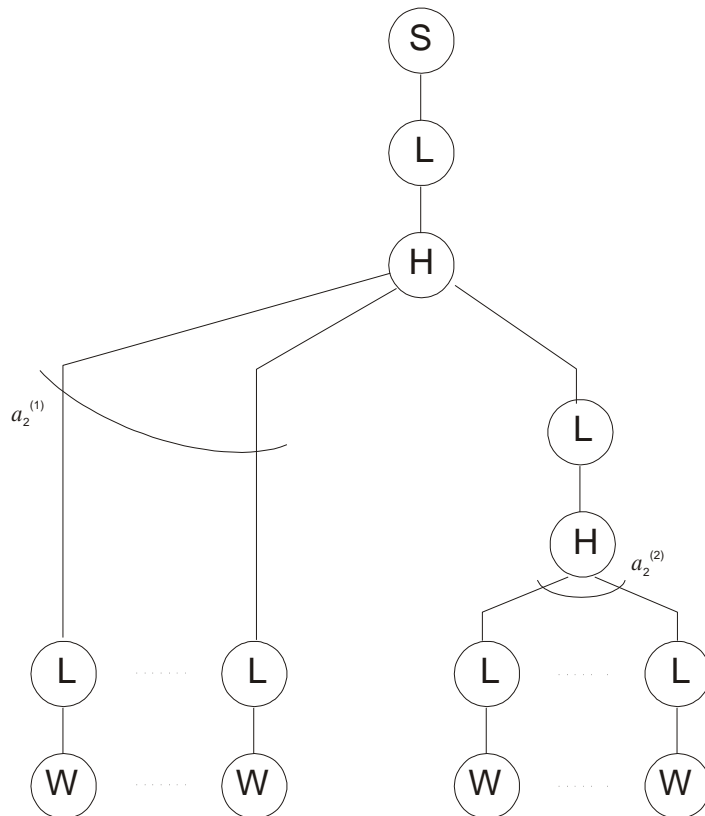


Fig. 2. Representation of a technical equipment of a local area computer network in the form of a ramified system

A municipal enterprise has placed 20 workstations in 2 rooms, moreover 7 workstations are connected to the server by a 8-port hub, and the other 13 workstations are directly connected to a 16-port hub. Only the 8-port hub is directly connected to the server, and both hubs connect each other. Therefore the 8-port hub can be considered as basic.

In Fig. 2  $a_2^{(1)}$ ,  $a_2^{(2)}$  denote counts of workstations directly connected to the basic hub and to the unbasic hub correspondingly. The following inequalities are executed:

$$a_2^{(1)} \leq 7, a_2^{(2)} \leq 16, a_2^{(1)} \leq a_2^{(2)}. \quad (8)$$

Results of calculations by the equation (6) showed that optimal placement of 20 workstations in 2 rooms in relation to reliability is direct connecting as many workstations as possible to the basic hub, that is, a problem of choice of the ramification coefficients  $a_2^{(1)}$ ,  $a_2^{(2)}$  which ensure the highest reliability of the system under conditions (8) and under condition

$$a_2^{(1)} + a_2^{(2)} = 20, \quad (9)$$

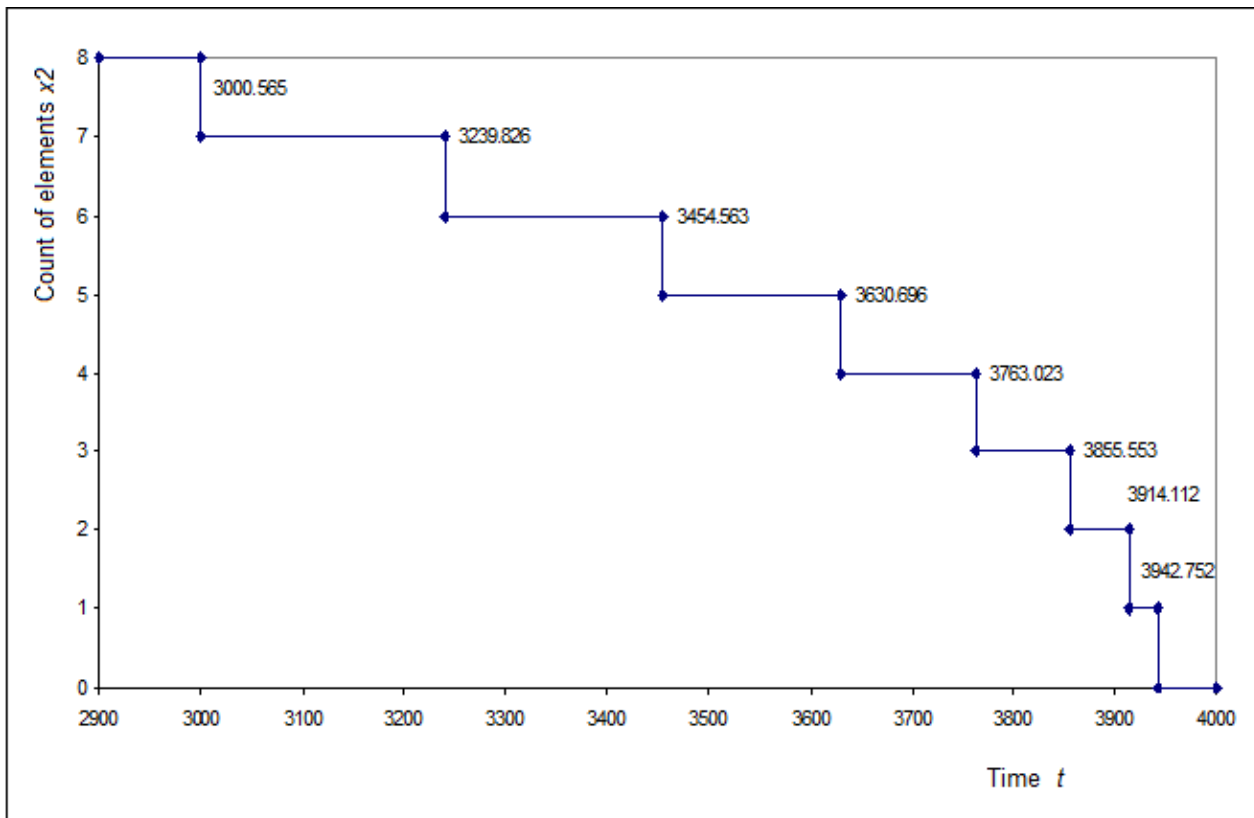


Fig. 3. A fragment of a histogram of operation of an unsymmetrical system in course of time at  $a_2^{(1)} = 7$ ,  $a_2^{(2)} = 13$

results in:  $a_2^{(1)} = 7$ ,  $a_2^{(2)} = 13$ .

The average duration of the system's stay in the state of  $x_2$  working output elements of the system under condition  $8 \leq x_2 \leq 20$  does not depend on the variant of placement responding to conditions (8) and (9). The following results are obtained:

$$\begin{aligned}
 T_2(20) &= 237.077 \text{ hours}, T_2(19) = 222.817 \text{ hours}, T_2(18) = 218.082 \text{ hours}, T_2(17) = 216.856 \text{ hours}, \\
 T_2(16) &= 217.633 \text{ hours}, T_2(15) = 219.854 \text{ hours}, T_2(14) = 223.291 \text{ hours}, T_2(13) = 227.857 \text{ hours}, \\
 T_2(12) &= 233.487 \text{ hours}, T_2(11) = 239.922 \text{ hours}, T_2(10) = 246.257 \text{ hours}, T_2(9) = 250.169 \text{ hours}, \\
 T_2(8) &= 247.263 \text{ hours}.
 \end{aligned}$$

An optimal variant of placement is chosen on the basis of estimating the average durations of the system's stay in the states of  $x_2$  working output elements under  $x_2 < 8$ . No less than 8 output elements of the system will operate on the average over a period of  $\sum_{x_2=8}^{20} T_2(x_2) = 3000.565 \text{ hours}$  from the beginning of the system's operation.

The duration of the system's stay in the state of 7 working output elements has a maximum value which equals  $239.261 \text{ hours}$  at  $a_2^{(1)} = 7$ ,  $a_2^{(2)} = 13$ . At other variants of the system's structure corresponding to conditions (8) and (9) this duration equals  $208.689 \text{ hours}$ . At  $a_2^{(1)} = 7$ ,  $a_2^{(2)} = 13$  no less than 7 output elements will operate until a moment of time  $3239.826 \text{ hours}$ , and at other variants of the system's structure they will operate until a moment of time  $3232.188 \text{ hours}$  from the beginning of the system's operation. In Fig. 3 a fragment of a histogram of the system's operation in the course of time is plotted under conditions  $a_2^{(1)} = 7$ ,  $a_2^{(2)} = 13$ .

We choose the availability condition  $k=18$  which means that no less than workstations operate in the system.

According to the equation (7) the average duration of the system's stay in the prescribed availability state  $k=18$ , which means that no less than 18 output elements operate, equals

$$T_{r_2}(18) = \sum_{x_2=18}^{20} T_2(x_2) = T_2(18) + T_2(19) + T_2(20) = 677.976 \text{ hours.}$$

Generalization of results of modelling reliability parameters for unsymmetrical systems ramified to level 3 makes possible to set up recurrent expressions for calculating the probability distribution of count of output working elements, the duration of the system's stay in each of its states and the failure frequency under the prescribed availability condition for unsymmetrical systems, ramified to level  $n$ , with ageing output elements and with two branches of unequal value on level 1.

### Conclusions

Up to now for ramified systems with complicated structures such as unsymmetrical systems with 2 and more branches of unequal value, ramified to level 3 and more than 3 levels, there were no mathematical models in the form of analytic expressions for such reliability parameters as the duration of the system's stay in each of its states and the duration of the system's stay in the prescribed availability state.

The main thrust of this paper is to reduce the computational time and complexity when evaluating reliability parameters of unsymmetrical ramified systems.

Thus, mathematical models of the probability distribution of count of output working elements, the duration of the system's stay in each of its states and the duration of the system's stay in the prescribed availability state, are worked out for unsymmetrical systems with ageing output elements, ramified to level

3. During designing unsymmetrical ramified systems, decisions about structures of the systems are made on the basis of such results.

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