UDS 517.958:536.12

L. Zhuravchak*, O. Kruk
* The Carpathian Branch of S.I.Subbotin Institute of Geophysics
of the National Academy of Sciences of Ukraine,
Lviv Polytechnic National University,
Software Department

MATHEMATICAL MODELING OF DISTRIBUTION OF THERMAL FIELD IN PARALLELEPIPED WITH CONSIDERING COMPLEX HEAT TRANSFER ON ITS BOUNDARY AND INNER SOURCES

We compare the efficiency using indirect methods of boundary and near-boundary elements for building numerical-analytical solution of three-dimensional stationary heat conduction problems considering the difficult conditions heat and intensity of inner sources. We built discrete-continual model for problems with boundary conditions of the first, second and third kind using integral representations for the temperature. The computing experiments are presented to estimate errors of discretization and mathematical model approximation. Influence of the thermal flat and three-dimensional internal sources on the distribution of temperature field in the object was investigated.

Keywords: heat field, complex heat transfer, indirect near-boundary elements method, indirect boundary elements method, internal sources.

Introduction

Simulation and optimization of thermal processes are essential in a variety of industries and technology particularly in instrument-making and mechanical engineering in the design of microelectronic devices, a cover construction and equipment by fireproof materials.[7,8]. Mathematical modeling of temperature fields in such objects remove the need for test experimental research for predict their durability and reliability requires the development of well-known and development of new computational methods for finding the stationary thermal fields in not canonical form area and conduction on this basis of detailed fundamental research. The basis of a mathematical model of a heat stationary process, as filtering incompressible fluids, electrostatics, serves as a differential equation in partial derivatives of elliptic type (the Laplace and the Poisson), supplemented by the boundary conditions of the first kind, the second kind, the third kind and mixed (at their combination).

Since exact analytical solutions of these problems can be obtained for a small number of objects, numerical-analytical and numerical methods are used, which include finite difference and finite elements, integral and boundary integral equations, boundary and near-boundary elements, which have advantages and a disadvantages [1, 2, 4, 5, 9-12]. In the indirect method of boundary and near-boundary elements integrated image output differential equation written to a fold of its fundamental singular solution of intensities "fictitious" sources distributed on the edge of an object or external to it near-boundary region. By themselves, the intensity functions have no physical meaning, but when they are found, the value of the desired temperature inside the body can be obtained by using integration.

In this paper, by modeling of stationary heat processes in the parallelepiped we compared indirect boundary and near-boundary elements methods. Discrete-continuous model for the intensities of the unknown source are introduced onto the boundary or near-boundary elements and approximated by constant, is reduced to a system of linear algebraic equations (SLAE), formed as a result collocation of satisfaction of boundary condition in collocation forms.

Mathematical model for finding the thermal field

We consider a homogeneous isotropic parallelepiped in Cartesian coordinate system x_1, x_2, x_3, in region

$$\Omega = \{(x_1, x_2, x_3) : a_1 < x_1 < a_2, b_1 < x_2 < b_2, c_1 < x_3 < c_2\}. \quad (1)$$

with a boundary $\Gamma = \cup_{j=1}^{6} \Gamma^{(j)}$, where $\Gamma^{(1)} = \{(x_1, x_2, x_3) : x_1 = a_1, b_1 < x_2 < b_2, c_1 < x_3 < c_2\}$,
$\Gamma^{(2)} = \{(x_1, x_2, x_3) : x_1 = a_2, b_1 < x_2 < b_2, c_1 < x_3 < c_2\}$,
$\Gamma^{(3)} = \{(x_1, x_2, x_3) : a_1 < x_1 < a_2, x_2 = b_1, c_1 < x_3 < c_2\}$,
$\Gamma^{(4)} = \{(x_1, x_2, x_3) : a_1 < x_1 < a_2, x_2 = b_2, c_1 < x_3 < c_2\}$,
$\Gamma^{(5)} = \{(x_1, x_2, x_3) : a_1 < x_1 < a_2, b_1 < x_2 < b_2, x_3 = c_1\}$,
$\Gamma^{(6)} = \{(x_1, x_2, x_3) : a_1 < x_1 < a_2, b_1 < x_2 < b_2, x_3 = c_2\}$.

To find the unknown temperature $\theta(x)$ we have an original equation

$$\sum_{i=1}^{n} \frac{\partial^2 \theta(x)}{\partial x_i^2} = -\psi(x)\chi_\psi, \; x \in \Omega \subset \mathbf{R}^3, \quad (2)$$

and boundary conditions of the first kind, second kind and third kind:

$$\theta(x) = f_\Gamma^{(1)}(x), x \in \partial\Omega^{(1)}, \qquad -\lambda_0 \frac{\partial\theta(x)}{\partial \mathbf{n}(x)} = f_\Gamma^{(2)}(x), x \in \partial\Omega^{(2)}, \quad (3)$$

$$-\lambda_0 \frac{\partial\theta(x)}{\partial \mathbf{n}(x)} + \upsilon(x)\theta(x) = \upsilon(x) f_\Gamma^{(3)}(x), x \in \partial\Omega^{(3)}. \quad (4)$$

Where $\psi(x) = \tilde{\psi}(x)/\lambda_0$; $\tilde{\psi}(x)$ – the intensity sources in $\Omega_{\psi n} \subset \Omega$; $\Omega_{\psi n}$ – rectangle (n=2) or parallelepiped (n=3); χ_ψ – characteristic function in area Ω_ψ, for example $\chi_\psi = 1$ at $x \in \chi_\psi$, $\chi_\psi = 0$ at $x \notin \chi_\psi$; $\partial\Omega^{(1)} \cup \partial\Omega^{(2)} \cup \partial\Omega^{(3)} = \Gamma$ – boundary of Ω; $x = (x_1, x_2, x_3)$; $f_\Gamma^{(1)}(x)$, $f_\Gamma^{(2)}(x)$ – known functions describing the temperature and heat flow at the boundary of the parallelepiped; $f_\Gamma^{(3)}(x)$ – ambient temperature; λ_0, $\upsilon(x)$ – coefficient of the thermal conductivity of the material and coefficient of heat transfer from the surface of the object.

Integral representation of solution

To construct an algorithm for solving the problem (1)-(4) we use an indirect boundary (IBEM) and indirect near-boundary elements methods (INBEM). According to the main provisions of these methods on the boundary of the object Γ or to the external near-boundary Ω area G we introduce unknown functions $\varphi^\gamma(x)$, $\gamma \in \{\Gamma, G\}$, describing the distribution of fictitious heat sources.

After the expansion of the function $\theta(x)$ domain on the whole *=0.95; (e) on the front and (f) on the top faces of parallelepiped*

Conclusions

We realized the approbation of the proposed approaches that are based on the combined use of the advantages of analytical and numerical methods. They include the fundamental solution of the Laplace equation, the basic idea of indirect boundary and near-boundary elements methods and collocation method. The error of satisfying the boundary conditions decreases when the number of boundary elements or near-boundary elements increases. However, complications of the procedure of numerical integration (using, for example, the distribution of unknown functions within the boundary or near-boundary elements as continuous functions, and not constant) would significantly reduce the computational error, even with a smaller number of elements. Note also that the benefits of both approaches include the fact that they do not require differentiation of numerical values.

For the calculation of research instruments C# was used. To visualize the results research, we use Gnuplot.

The approach can be extended for consideration of three-dimensional solid of arbitrary shape, then we must calculate integrals over 8 nodes of boundary element instead of 4 and 24 nodes of near-boundary element instead of 8.

1. Benerdzhy P., Batterfyld R. The boundary element method in engineering science. – M.: Myr, 1984. – 494 p. 2. Brebbia K., Telles Zh., Vroubel L. Boundary element methods.– M.: Mir, 1987.– 524 p. 3. Vladimirov V.S. The mathematical physics equations.– M.: Nauka, 1971.– 512 p. 4. Zhuravchak L.M., Hrytsko E.G. Near-boundary element method applied to problems of mathematical physics. – Lviv: The Carpathian Branch of S.I.Subbotin Institute of Geophysics of the NAS of Ukraine, 1996.– 220 p. 5. Zhuravchak L.M., Zabrodska N. V. Unsteady thermal field in inhomogeneous materials with nonlinear behavior of the component // Phys.-Chem. Mechanics. – 2010. – № 1. – C. 33-41. 6. G. Karslou, D. Eger. Conductivity of solid bodies: translation with English. – M.: Nauka, 1964. – 488 p. 7. Krukovskiy P.G. Determination of thermal characteristics spuchuvalnoho fireproof coating method of inverse heat conduction problems // journal "Fire safety: theory and practice ".– 2008. – №1. – C. 49–53. 8. Fedasyuk D.V. Methods and tools for thermal design of microelectronic devices. – L.: Publish DU " Lviv Polytechnic ", 1999. – 228 p. 9. Shaw R. P. An outer boundary applied to transient wave scattering in an inhomogeneous medium. - Appl. Mech., 42,1975. – C. 147–152 . 10. Ozisik, M.N. Heat Conduction, 2nd edition.- John Wiley and Sons, N.Y., 2002. – 512 c. 11. Beer G., Smith I., Duenser C. The Boundary Element Method with Programming: for Engineers and Scientists. - Wienn: Springer-Verlag, 2008. –- 496 p. 12. Marczak R.J. An Object-Oriented Programming Framework for Boundary Integral Equation Methods / R.J. Marczak // Computers and Structures, 2004. – Vol. 82. – P. 1237-1257.