

METHOD OF SURFACE RECONSTRUCTION FROM WHITE LIGHT INTERFEROGRAM BASED ON HILBERT TRANSFORM METHOD

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In this paper theoretical fundamentals of white light interferometry and mathematical model of interferogram were analyzed. The method of topography reconstruction from phase of white-light interferogram based on amplitude demodulation using Hilbert transform was developed. The verification of introduced algorithm on tilted and sphere surface is presented and the main sources of reconstruction errors were analyzed.

Keywords – surface topology reconstruction, white light interferometry, Michelson interferometer, analytic signal, Hilbert transform.

Introduction

White light interferometry is a non-contact measurement method that is widely used for determination of mechanical quantities such as geometric dimensions, surface topography and position of the measured object. The measurements can be performed in the range from a few centimeters to a few micrometers. The advantages of interferometry are a high speed and large area of measurement.

White light interferometry combined with atomic force microscopy and Raman spectroscopy is widely used to measure a number of parameters with nanometer resolution such as geometrical size, shape, thickness, roughness, waviness, curvature. The phenomenon of interference is the basis of the work of many measuring instruments, such as microscopes, profilers, vibrometers, pressure and displacement sensors. The main areas of use of white light interferometers are micro-and nanotechnology, biomechanics, polymer chemistry, semiconductor equipment, and others. In nanotechnology the white light interferometry is applied for nanopositioning in 3D- coordinate system, standardization of metrological characteristics of material standards, which is later used for calibration of atomic force microscope. An interesting technology for increasing the hardness and mechanical stability of plastics is coverage with nanodiamond films for quality control of which can be used white light interferometry [1].

At present time, the mechanical and electronic assemblies of interferometers provide interferograms' registration with high accuracy and reproducibility. But the authenticity of surface topography reconstruction depends not only on the accuracy of interference patterns measurement, but also on algorithms that helps carrying out its analysis and reconstruction. Thus, it is important to ensure the stability of the algorithms to the effects of noises, optical nonlinearities and other undesirable factors. Another important requirement for these algorithms is the possibility of surface reconstruction without a priori information about it. The relevance of the tasks of developing the algorithms is evidenced by numerous publications [2,3,4].

Within the literature overview it was found that most modern methods for surfaces topology determining from white-light interferogram are based on so-called phase shift algorithms. This approach requires precision mechanical movement of some interferometers assemblies (usually the reference mirror), which in turn leads to a complication of the interferometers structure and increasing of measuring time. In many cases when it comes to design of a low-cost miniature systems or data processing in real time, these disadvantages are critical.

The aim of work is to develop an efficient and accurate algorithm to determine the surface topography from its interferogram based on a minimal number of interference patterns.

The basis of white light interferometry

The interference is a superposition phenomenon of two or more electromagnetic waves, which leads to the weakening and strengthening of the resultant (total) wave in each point of space. The main condition for appearance of the interference pattern is the coherence of the waves that overlap. Since the coherent wave must have a constant in time phase difference, in practice they can be obtained by separating the light beam generated by the same source of radiation. The result of coherence is a steady-in-time interference pattern that provides an unambiguous dependence of its nature from parameters of the object under test. It should also be noted that interference can occur for both monochromatic (laser light) and white (halogen lamp) light.

The measuring device, the principle of which is based on the interference phenomenon, is called an interferometer. One of the most common types of interferometer is Michelson interferometer. The structure of this device is shown in Fig. 1. The basic structural elements of the interferometer are light source Q , beam splitter ST , two mirrors $S1$ and $S2$ and screen. Also, to improve the quality of interference pattern a number of additional elements, such as polarizers, lenses, collimators, light pipes and so on are used. Because these elements are not principle for the interferogram formation, then to facilitate the further analysis their impact will be ignored.

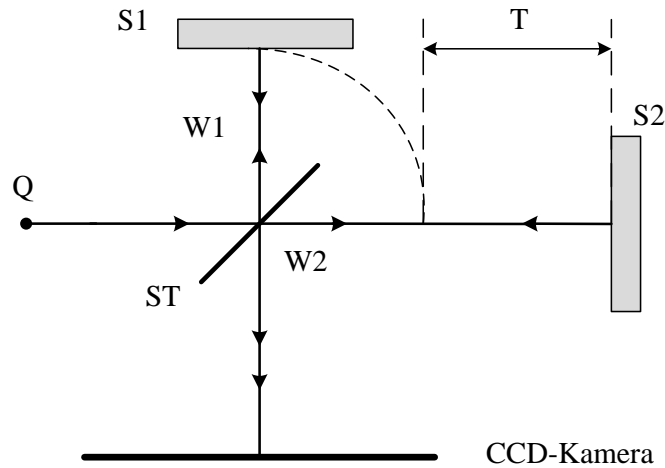


Fig. 1. The optical scheme of the Michelson interferometer [5]

Michelson interferometer works as follows: the wave of the light source is divided by beam splitter into two waves $W1$ and $W2$, which are directed corresponding to mirrors $S1$ and $S2$. The waves are reflected from the mirrors and return to the beam splitter ST , where they overlap. The result of waves' superposition is observed on the screen and recorded CCD- camera. The phenomenon of strengthening and weakening of the resultant wave is observed due to the optical phase difference T . It should be noted that the maximal intensity of interferogram (light areas) is observed at the points where the optical phase difference T is zero and, conversely, dark areas correspond to points of interferogram where the waves are in antiphase and compensate each other.

Fig. 2 shows the white light interferograms for a spherical surface obtained for light sources with different parameters. It should be noted that the form of interferogram depends on the spectral bandwidth and central wavelength. The interferogram on the right contains colored rings of varying thicknesses, which damped quicker with distance from the center as obtained by using a light source with a wider spectral bandwidth located in the center of the visible range. The interferogram on the left is obtained by using a light source with a narrower spectral bandwidth located on the edge of the visible range, so interference ring width varies slightly, their contrast is better preserved with increasing distance from the center, and the interference pattern is more monochromatic.

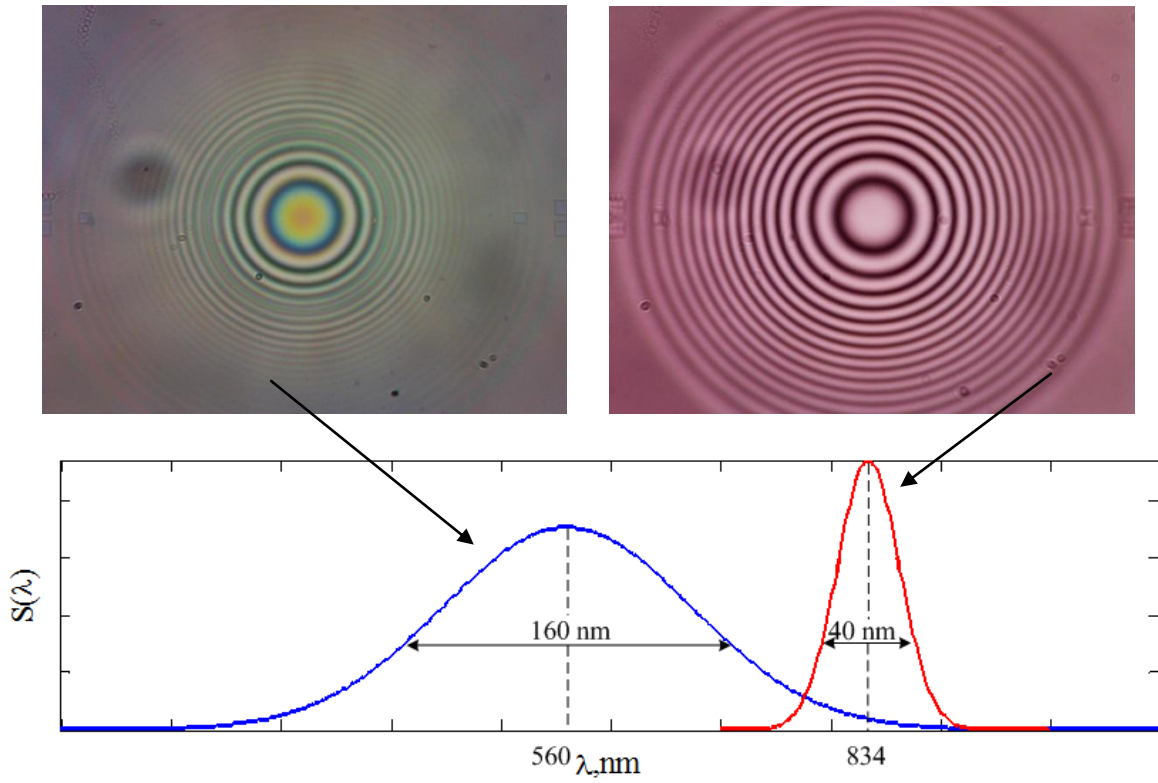


Fig. 2. Real interferograms of spherical surface for different light sources

Mathematical model of white light interferogram is described by the following formulas [5,6]:

$$I(T) = I_0 + E(T) \cdot C(T), \quad (1, a)$$

$$E(T) = I_M \cdot \exp\left(-\frac{4 \cdot \Delta\lambda^2 \cdot T^2}{\lambda_0^4}\right) \text{ (envelope)}, \quad (1, b)$$

$$C(T) = \cos\left(\frac{4 \cdot \pi}{\lambda_0} \cdot T\right) \text{ (carrier)}, \quad (1, b)$$

where I_0 i I_M – corresponding constant component and the envelope amplitude of the interferogram signal intensity;

T – the optical phase difference;

λ_0 i $\Delta\lambda$ – the central wavelength and spectral bandwidth of the light source.

As follows from the presented equations, the form of carrier of interferogram signal is determined by two parameters: the optical phase difference T and the central wavelength λ_0 . On the nature of the envelope, in addition to these two parameters greatly affects also the spectral bandwidth $\Delta\lambda$. The confirmation of this effect can serve fig. 2. Such a mathematical model is valid also for interferogram of monochromatic light, as in this case, the spectrum width $\Delta\lambda$ tends to zero, and consequently there is no amplitude modulation.

The concept development of the method for surface topology reconstruction

Analyzing equation (1) we can make a conclusion that the desired value T is both a function of the envelope $E(t)$ and the carrier $C(T)$. However, in the phase analysis of the carrier the higher sensitivity and accuracy can be achieved than in the amplitude analysis of the envelope. In addition, frequency or phase as informative parameter versus amplitude is resistant to the effects of noise. Thus, it is reasonable to develop methods to determine the surface topology from phase of the interferogram carrier.

In terms of theory of signal analysis the interferogram can be interpreted as a signal of amplitude-phase modulation in which the informative parameter is the optical phase difference T , and the modulation coefficients are the parameters of the light source. Obviously, to solve this task (determining the phase of the carrier) first the influence of amplitude modulation, which is described by the carrier, should be eliminated. For this, an analytical representation of the signal can be applied, which allows you to extend the concept of amplitude and phase in non-harmonical signals [7].

Analytical signal is a complex signal, the real part of which is analyzed signal and the imaginary - its orthogonal complement:

$$X(T) = E(T)\cos(\omega \cdot T) + Q(T)\sin(\omega \cdot T), \quad (2)$$

where $E(T)$ and $Q(\omega T)$ – the amplitude of the in-phase and quadrature components;
 $\omega = 4\pi / \lambda_0$.

If during the processing of interference signal first separate the in-phase $s(T) = E(T)\cos(\omega \cdot T)$ and quadrature $s_{\perp}(T) = Q(T)\sin(\omega \cdot T)$ components, then divide them into one another, in the result we obtain a signal proportional to the tangent of the carrier phase:

$$\frac{s_{\perp}(T)}{s(T)} = \frac{Q(T)\sin(\omega \cdot T)}{E(T)\cos(\omega \cdot T)} = \frac{Q(T)}{E(T)} \cdot \operatorname{tg}(\omega \cdot T). \quad (3)$$

If $E(T) = Q(T)$ the equation becomes:

$$\frac{s_{\perp}(T)}{s(T)} = \operatorname{tg}(\omega \cdot T), \quad (4)$$

and phase can be determined using the arctangent function

$$\phi = \operatorname{arctg} \frac{s_{\perp}(T)}{s(T)}. \quad (5)$$

When comparing the expressions (1) and (2) it becomes apparent that interferogram signal $I(T)$ can be considered as in-phase component of $s(T)$, but it's necessary to remove the additive offset I_0 . Therefore, to get the expression (5) to form a suitable quadrature signal (with the same amplitude and phase shifted by 90°), and then the measuring value T can be calculated as follows:

$$T = \frac{\lambda_0}{4 \cdot \pi} \cdot \operatorname{arctg} \frac{I_{\perp}(T)}{I(T) - I_0}, \quad (6)$$

where $I_{\perp}(T)$ – quadrature signal, which is received from the interferogram signal after removing the constant component.

In fact, the practical implementation of this method is reduced to the problem of forming quadrature signal of interferogram intensity. Graphical interpretation of the proposed method is shown in Fig. 3.

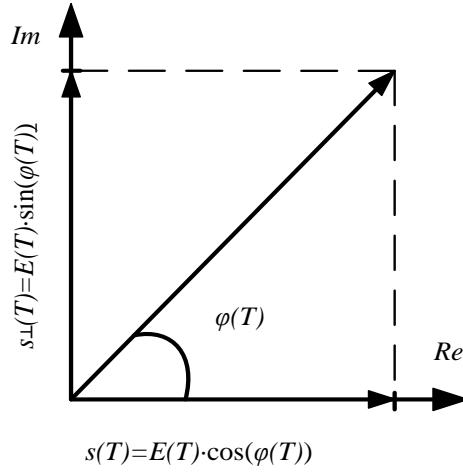


Fig. 3. Graphical interpretation of phase demodulation using quadrature signal

Theoretically, the most accurate method for determining the quadrature signal of interferogram without a priori information about the surface type is based on Hilbert transform [8]. Hilbert transform for some real signal $s(T)$ ensures the formation of its nonzero "quadrature equivalent" $s_{\perp}(T)$, which satisfies the following condition:

$$\int_{-\infty}^{\infty} s(T) \cdot s_{\perp}(T) dT = 0. \quad (7, a)$$

Analytical expression of Hilbert transform has the following form [7]:

$$s_{\perp}(T) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{s(\tau)}{T - \tau} d\tau, \quad (7, b)$$

where $s(\tau)$ and $s_{\perp}(T)$ – analyzed input signal and its quadrature component (output signal); τ and T – argument (independent variable) for input and output signals.

The given expression can be interpreted as a convolution of the signal $s(T)$ with the function $h(T) = 1/\pi T$, which, in essence, is the core of the Hilbert transform. So Hilbert transform generates an orthogonal (quadrature) complement of the input signal, making the phase shift in 90° of each harmonic that is available in a spectrum of the analyzed real signal $s(T)$.

In a digital form the Hilbert transform is described by the expression:

$$s_{\perp}(m) = \frac{1}{\pi} \sum_{n=1}^{N-1} \frac{s(n)}{n - m}. \quad (8)$$

where $s(n)$ and $s_{\perp}(m)$ – input and output samples of quadrature signals; n and m – the arguments of the input and output signals.

Thus, the surface reconstruction algorithm with a help of phase demodulation method consists of the following parts:

- the download or simulation of interferogram and parameters of the interferometer;
- removal of constant component I_0 ;
- calculation of quadrature signal of interferogram using the Hilbert transform;
- interferogram demodulation with a following phase calculation;
- visualization of the reconstructed surface.

The expected benefits of such algorithm are its independence of the surface form of the object under test, light source, as well as the actual numerical values of the mathematical model parameters. In addition, the proposed algorithm has the potential high speed due to the lack of recursive or iterative computation.

The next step after the development of the concept is its practical implementation on a particular platform with a following verification and analysis of results.

The verification of developed algorithm and results analysis

Implementation and verification of algorithm will be carried out in a software environment MATLAB, which has convenient user interface, flexible scripting language and multiple libraries for data analysis and visualizing. In order to eliminate the influence of different potential destabilizing factors (noise, noise, optical distortion, etc.) in the analysis of algorithm work the simulated data were used. For research two types of surfaces were selected: tilted (Fig. 4, a) and spherical (Fig. 4, b).

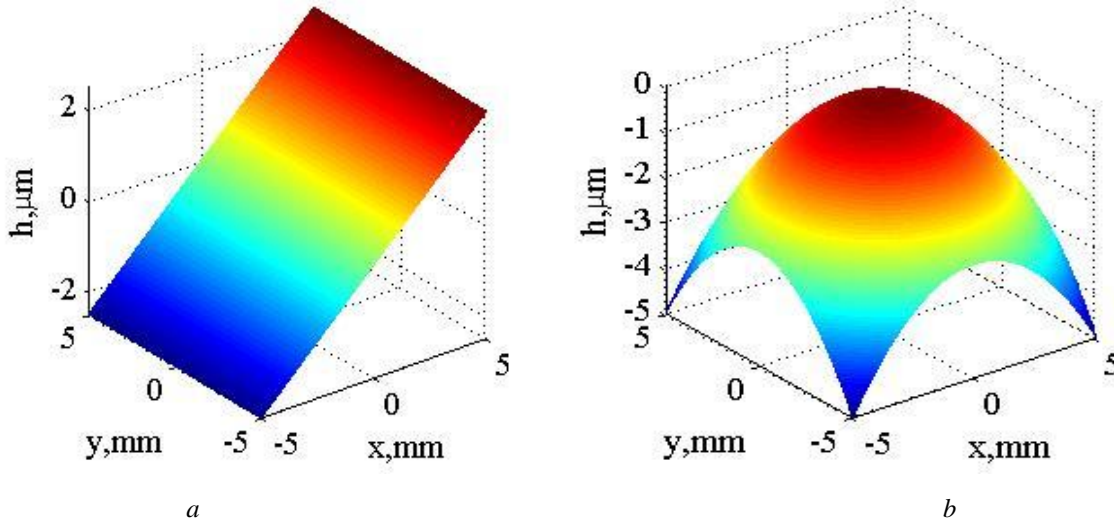


Fig.4. The form of simulated surfaces: a – tilted, b – spherical

The corresponding interferograms (Fig. 5) were obtained from the formulas (1), with the following parameters of light source and CCD-camera:

- constant component of signal- $I_0 = 2$;
- modulation amplitude - $I_M = 2$;
- central wavelength - $\lambda_0 = 620$ nm;
- spectral bandwidth - $\Delta\lambda = 62$ nm;
- number of data in x and y axis - 800×800 points;
- the optical phase difference in center of interferogram is zero.

In order to simplify, at first development of algorithms relate only to one-dimensional data, that is the one line of interferogram. As noted above, for the correct determination of the quadrature signal first the constant component must be removed. For this the MATLAB function *mean (*)* was used [9]. It should be noted that the Hilbert transform automatically eliminates the constant component, so this operation can be performed both before the formation of the quadrature signal or after it.

In MATLAB discrete Hilbert transform is implemented by inbuilt-function *hilbert (*)*. The feature of this function is that it returns not only the quadrature $s_{\perp}(T)$, but also the real $s(T)$ component:

$$s(m) + js_{\perp}(m) = \text{hilbert}[s(n)]. \quad (9)$$



Fig.5. The interferograms of simulated surfaces: *a* – tilted, *b* – spherical

Therefore, to determine the phase of the signal by the formula (5) first the real and imaginary parts must be identified. This task can be implemented using the appropriate function *real(*)* and *imag(*)*.

The signal that is obtained after calculating of interferogram phase, is characterized by a number of discontinuous transitions due to neglect of the periodicity of the arctangent function. Therefore, the next step is to remove these gaps with a help of special procedure - the so-called phase unwrapping. In practice, the phase unwrapping can realize by using the in-built function *unwrap(*)* in the package MATLAB [9]. This function works as follows: if the difference between two adjacent values is greater than $\pm \pi$, then the value is complemented at $\pm 2\pi$.

After phase unwrapping step the calculation of surface topology based on the expression (6) can be performed. However it should be noted, that between the original and reconstructed surface the systematic drift is observed. In case of original surface the *y* axis has an absolute character as zero point ($T=0$) correspond to the position of the surface where the optical path difference of the rays is equal to zero. For the reconstructed surface coordinate system is relative because the arctangent function always returns the value within the first period $[-\pi; +\pi]$ without member $2\pi \times k$, that is, information about the absolute value of the period is lost. Since smoothing using the function *unwrap(*)* going from left to right, the leftmost point will always lie within the first period. This effect is not critical while in detection of the surface topology is important relative rather than absolute position of the surfaces points. However, this phenomenon can be eliminated by analyzing the function of the envelope. The point where the envelope reaches a maximum (symmetry point of interferogram) coincides with the true origin (optical path difference is equal to zero), and the value of the reconstructed surface at this point is essentially a constant displacement. To eliminate systematic drift is necessary to subtract from each point of the reconstructed surface value at the point where the envelope is maximal:

$$h_{corrected}(x, y) = h_{reconstructed}(x, y) - h(x_{Ienv, max}, y_{Ienv, max}). \quad (10)$$

The results of using the method of interferogram phase demodulation for reconstruction of surface topology with a help of the described algorithm is shown in Fig. 6. As seen from the graphs, in the case of tilted surfaces, the reconstructed and original surfaces are identical, and for spherical one the significant deviations are observed, especially at the edges. A possible reason is that in the case of spherical surface the interferogram has a distinctly non-sinusoidal character. Therefore, it is necessary to carry out additional research on the legality of the formula (5) for such signals. It is also important to note that while using the phase demodulation method for surface reconstruction is desirable to use an integer periods of interferogram because under such conditions the results of the algorithm are the best.

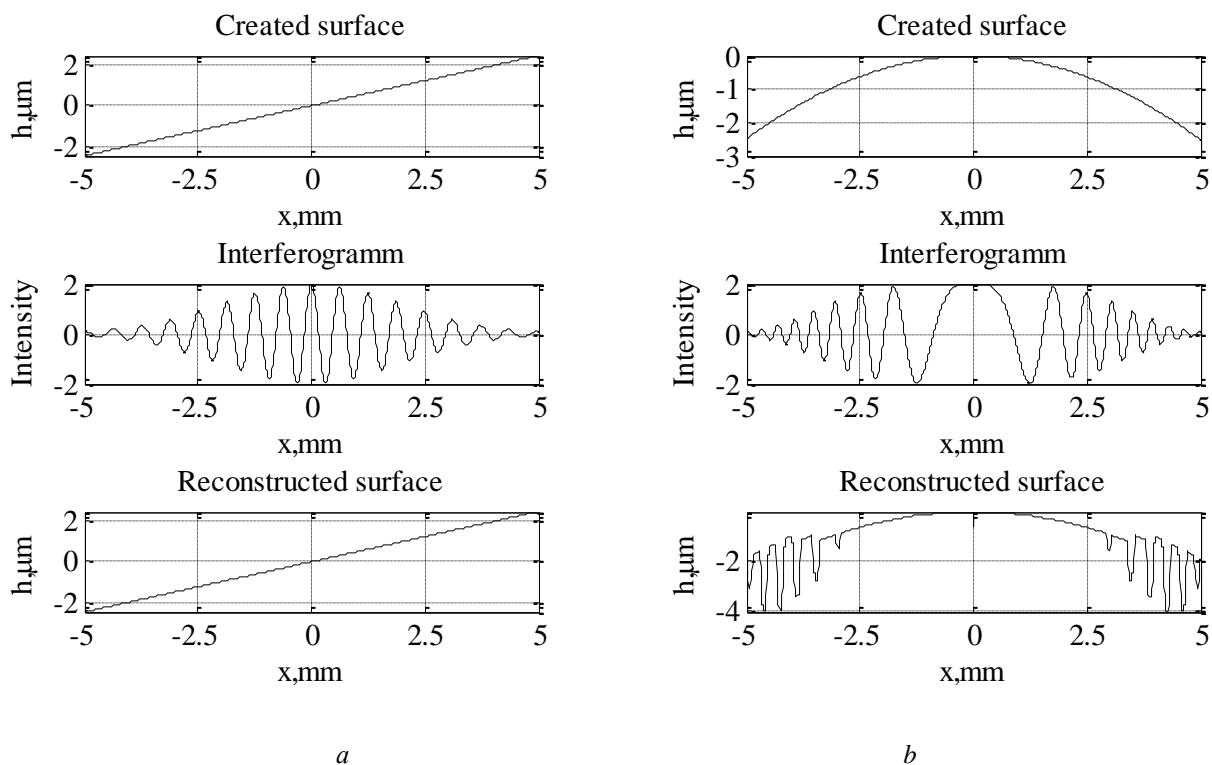


Fig. 6. The results of usage the quadrature demodulation method: a - tilted, - b spherical surface

Conclusions

In this paper was considered the possibility of surface topology reconstruction from white light interferogram using quadrature signal. The basic theoretical principles, including the structure and principle of white-light interferometer, the dependence of the nature and form of interferogram from the parameters of the radiation source. Also a mathematical model of interferogram was analyzed. A method for surface topology detection based on the phase demodulation using quadrature signal that is obtained from the Hilbert transform was proposed. The verification was carried out on simulated data for two cases - linear and spherical surfaces. The obtained results show that the proposed method is effective only for linear surfaces. In addition, there are other limitations, such as the need to identify and eliminate the constant component, using integer periods, and so on. Therefore, it is reasonable to further research aimed at finding new approaches for the analysis of nonlinear spherical and other surfaces.

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