COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 4, No. 1, 2014

TOLERANCE ANALYSIS AND OPTIMIZATION OF LINEAR PERIODICALLY TIME-VARIABLE CIRCUITS BASED ON THE FREQUENCY SYMBOLIC METHOD

Yuriy Shapovalov, Bohdan Mandziy, Dariya Bachyk
Lviv Polytechnic National University, Lviv, Ukraine
shapov@polynet.lviv.ua, bmandziy@polynet.lviv.ua, dariya.bachyk.smal@gmail.com

Abstract. The paper presents the procedures of tolerance analysis and optimization of linear periodically time-variable circuits, based on the frequency symbolic method and realized by the system of functions MAOPCs in an environment MATLAB. The peculiarity of tolerance analysis and circuit optimization consists in the fact that previously the assessment of asymptotic stability of the investigated linear periodically time-variable circuit within the given limits of changes of circuit parameters is carried out and the permissible areas of their change in which the circuit is stable are determined. Exclusively in these areas tolerance analysis and optimization is realized.

Key words: frequency symbolic method, tolerances analysis, optimization, linear periodically time-variable circuits.

1. Introduction

The paper presents the procedures of tolerance analysis and optimization of linear periodically time-variable (LPTV) circuits based on the calculation of circuit parametric transfer functions by the frequency symbolic method (FS-method) [1]. The parametric transfer function $W(s,t,x_1,...,x_n)$, where s is a complex variable, t is time, is approximated according to the FS-method by the Fourier trigonometric polynomial, in which some or all parameters $x_1,...,x_n$ of circuit elements are given by symbols. These parameters act as variables $x_1,...,x_n$ while performing procedures mentioned.

The peculiarity of tolerance analysis and optimization consists in the fact that the previously the assessment of stability (asymptotic) of investigated LPTV circuit within the prescribed limits of changing parameters $x_1,...,x_n$ is carried out and the permissible areas of their change in which circuit is stable are determined. Exclusively in these areas tolerance analysis and optimization occurs.

The assessment of circuit stability is carried out by the real parts of the denominator roots $\Delta(s,x_1,...,x_n)$ of a normal parametric transfer function $G(s,\xi,x_1,...,x_n)$ [2], which also is defined by the FS-method in the form of approximation by the Fourier trigonometric polynomials (ξ-moment of submission a delta function to the input of circuit).

2. Tolerance analysis

For the tolerance analysis of LPTV circuits the method of moments [3] is chosen, by which the relative deviation of the absolute value of the transfer function of the LPTV circuit has the form:

$$\delta\left|W(s,t,x_1,...,x_n)\right| \cong \sum_{i=1}^{n} S_{x_i}^{\left|W(s,t,x_1,...,x_n)\right|}\delta x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} S_{x_i,x_j}^{\left|W(s,t,x_1,...,x_n)\right|}\delta x_i \delta x_j , \quad (1)$$

where δx_i, δx_j are the relative parameter deviations of elements x_i and x_j from the nominal values, respectively; n is a number of element parameters of the LPTV circuit; $S_{x_i}^{\left|W(s,t,x_1,...,x_n)\right|}$ and $S_{x_i,x_j}^{\left|W(s,t,x_1,...,x_n)\right|}$ are functions of sensitivity of the first and second orders, respectively.

The following procedure of tolerance analysis of LPTV circuits is proposed on condition that the control of their asymptotic stability is performed.

Step 1. By the FS-method the denominator $\Delta(s,x_1,...,x_n)$ of the normal transfer function $G(s,\xi,x_1,...,x_n)$ is formed by symbolic values s and parameters $x_1,...,x_n$ of all elements of the circuit.

Step 2. The stability of the given circuit is estimated within the change limits of each parameter with a given step $x_1 = x_{\min} : x_{step} : x_{\max}$,..., $x_n = x_{\min} : x_{step} : x_{\max}$ by the real parts of the denominator roots $\Delta(s,x_1,...,x_n)$ resulting in forming a so-called "table of stability". The limits and change steps of each parameter are determined by the researcher. The limits for each parameter within which a given circuit is stable, are converted to relative changes $\delta x_1,\ldots,\delta x_n$ of parameters $x_1,...,x_n$, respectively.

Step 3. The parametric transfer function $W(s,t,x_1,...,x_n)$ of the given circuit is formed according to the FS-method by the symbolic values of the same parameters $x_1,...,x_n$.

Step 4. Sensitivity functions of the first order $S_{x_1}^{|W(s,t,x_1,...,x_n)|}$,..., $S_{x_n}^{|W(s,t,x_1,...,x_n)|}$ and the second order, $S_{x_1,x_1}^{|W(s,t,x_1,...,x_n)|}$, $S_{x_1,x_n}^{|W(s,t,x_1,...,x_n)|}$,..., $S_{x_n,x_n}^{|W(s,t,x_1,...,x_n)|}$ are determined by the function $W(s,t,x_1,...,x_n)$,.

Step 5. The relative deviation $\delta|W(s,t)|$ of the real part of the transfer function $W(s,t)$ is calculated by the equation (1), relative changes $\delta x_1,\dots,\delta x_n$ and functions of sensitivity, determined in steps 2 and 4, respectively,.

The procedure of tolerance analysis of LPTV circuits is realized in the system of functions MAOPCs in an MATLAB environment [4].

3. Optimization of linear circuits with periodically time-variable parameters

For the optimization of LPTV circuits the general additive criterion of optimality [3] is selected, which represents the sum of squared deviations of output characteristics from the technical requirements. In this case the objective function $F(x_1,...,x_n)$ (the additive criterion of optimality) has the form [3]

$$F(x_1,...x_n)=\sum_{i=1}^{p}\sum_{j=1}^{q}\left(M_F(x_1,...x_n,\omega_i,t_j)-M_0(\omega_i,t_j)\right)^2, \quad (2)$$

where $M_0(\omega_i,t_j)$ and $M_F(x_1,...,x_n,\omega_i,t_j)$ are the function of goal and the function-characteristic, respectively, $x_1,...,x_n$ are parameters by which optimization is carried out. The goal function $M_0(\omega_i,t_j)$ is given by the researcher as a set of values for selected variables ω_i and t_j. The function of the characteristic $M_F(x_1,...,x_n,\omega_i,t_j)$ is determined by the real part of the parametric transfer function $W(x_1,...,x_n,\omega_i,t_j)$ of the circuit, which, in turn, is defined as the approximation

$$M_F(x_1,...,x_n,\omega_i,t_j)=\left|W(x_1,...,x_n,\omega_i,t_j)\right|=$$
$$=\left\|\begin{matrix}W_{\pm0}(x_1,...,x_n,\omega_i)+\\ \sum_{i=1}^{k}\begin{pmatrix}W_{-i}(x_1,...,x_n,\omega_i)\cdot\exp(-j\cdot i\cdot\Omega\cdot t_j)+\\ W_{+i}(x_1,...,x_n,\omega_i)\cdot\exp(+j\cdot i\cdot\Omega\cdot t_j)\end{pmatrix}\end{matrix}\right\|, \quad (3)$$

which is formed applying the FS-method with symbolic parameters $x_1,...,x_n$ and the same values of ω_i and t_j [1].

This paper proposes the optimization procedure of the LPTV circuit with two varied parameters of elements x_1, x_2. The assessment of stability in this case is carried out by the denominator $\Delta(s,x_1,x_2)$ of the normal parametric transfer function $G(s,\xi,x_1,x_2)$. The results of this assessment are the areas of stability, whose limits are described mathematically, and which, being expressed in the form of inequalities, are the conditions for carried optimization.

The following optimization procedure for LPTV circuits is proposed on condition that the control of their asymptotic stability is performed.

Step 1. The FS-method being applied, the denominator $\Delta(s,x_1,x_2)$ of the normal parametric transfer function $G(s,\xi,x_1,x_2)$ is formed with the symbolic values of parameters x_1 and x_2.

Step 2. For each value x_1 from a group of values within a given range the limiting value x_2 is determined, at which stability changes into instability and the dependence $x_{2\lim}=f(x_1)$ is formed. As a result of these actions the so-called "map of stability" is constructed in coordinates x_1 and x_2.

Step 3. Dependences determined in step 2 are approximated by, for example, power polynomials and the areas of stability in the form of inequalities $x_2 < f(x_1)$ are defined.

Step 4. For given values of variables ω_i and t_j the goal function $M_0(\omega_i,t_j)$ is selected.

Step 5. The FS-method for the given values of variables ω_i and t_j being used, the function of the characteristic $M_F(x_1,x_2,\omega_i,t_j)$ of the circuit with the symbolic parameters x_1, x_2 is determined in the form (3).

Step 6. For given values of variables ω_i and t_j, the objective function $F(x_1,x_2)$ as the surface in coordinates of parameters x_1, x_2 is formed according to equation (2).

Step 7. The minimum value of the goal function determined by one of the optimization methods, when the conditions of stability in the form $x_2 < f(x_1)$ are fulfilled, in turn determines the unknown values of parameters x_1^* and x_2^* of the LPTV circuit.

The optimization procedure of LPTV circuits is realized in system of functions MAOPCs in an MATLAB environment [4].

4. The experimental part

Task for experiment 1. To conduct the tolerance analysis of the double-circuit parametric amplifier, whose equivalent circuit is shown in Fig. 1.

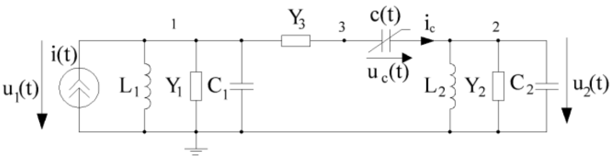

Fig. 1. Double-circuit parametric amplifier

$$i(t) = I_m \cdot \cos(\omega_s \cdot t + \varphi);\ c(t) = c_0 \cdot (1 + m \cdot \cos(\Omega \cdot t))\ ;$$

$$s = j\omega_s ; \omega_s = 2 \cdot \pi \cdot 10^8\, rad\,/\,s\ .$$

To determine the relative deviation of the real part of the parametric transfer function $Z_1(s,t) = U_1(s,t)/I(s)$ by changing the parameters of elements m, c_0, Y_1, Y_2, Y_3, C_1, C_2, L_1, L_2 on condition that the amplifier is stable.

The change limits of each parameter are as follows: $c_0 = 0.9 \cdot 10^{-12} : 9 : 1.1 \cdot 10^{-12} F$,

$m = 0.09 : 0.11$, $Y_1 = 0.91 \cdot 10^{-4} : 1.08 \cdot 10^{-4} S$,

$L_1 = 3.670422 \cdot 10^{-8} : 3.671165 \cdot 10^{-8} H$,

$L_2 = 9.311671 \cdot 10^{-9} : 9.313545 \cdot 10^{-9} H$,

$Y_2 = 0.91 \cdot 10^{-4} : 1.08 \cdot 10^{-4} S$, $Y_3 = 0.471 : 0.528 S$,

$C_1 = 67.993 : 68.012 pF$, $C_2 = 67.980 : 68.12 pF$.

Experiment 1. According to the tolerance analysis algorithm shown above we perform the following steps:

Step 1. Using the FS-method, we form the denominator $\Delta_g(m, c_0, Y_1, Y_2, Y_3, L_1, L_2, C_1, C_2, s)$ of the normal parametric transfer function $G(s,\xi)$ from current $I(s)$ to voltage on parametric capacity $U_c(s,t)$ for the amplifier (Fig.1). Since the denominator $\Delta_g(m, c_0, Y_1, Y_2, Y_3, L_1, L_2, C_1, C_2, s)$ is lengthy, we give the denominator $\Delta_g(s)$ with the symbolic value of the parameter s and $k = 1$:

$$\Delta_g(s) = -0.60 \cdot 10^{-79} s^{11} - 0.11 \cdot 10^{23} - 0.18 \cdot 10^{12} s -$$
$$-0.13 \cdot 10^{-41} s^7 - 0.93 \cdot 10^{-6} s^3 - 0.17 \cdot 10^{-23} s^5 -$$
$$-0.99 \cdot 10^{-68} s^{10} - 0.28 \cdot 10^{-98} \cdot s^{13} - 0\ .36 \cdot 10^{-122} s^{15} -$$
$$-0.63 \cdot 10^{-49} s^8 \text{-} 0.45 \cdot 10^{-60} \cdot s^9 - 0.75 \cdot 10^5 \cdot s^2 -$$
$$-0.16 \cdot 10^{-30} \cdot s^6 - 0.17 \cdot 10^{-12} s^4 - 54 \cdot 10^{-110} s^{14} -$$
$$-0.48 \cdot 10^{-87} s^{12}.$$

Step 2. According to the denominator $\Delta_g(m, c_0, Y_1, Y_2, Y_3, L_1, L_2, C_1, C_2, s)$ which, when $k = 2$, provides a satisfactory accuracy, at symbolic values of element parameters $m, c_0, Y_1, Y_2,\ \ Y_3, L_1, L_2, C_1, C_2$, we assess the stability of amplifier within the change limits for each parameter that are given in the task. The results of the stability assessment are tabulated in the "table of stability". From the "table of stability" the relative deviations of the parameters of circuit elements are determined, by which it is asymptotically stable. The results from the "table of stability" show that the double-circuit parametric amplifier is stable at the changes of parameters in the specified limits. Thus, the relative changes of the element parameters $\delta m, \delta c_0$, $\delta Y_1, \delta Y_2$, $\delta Y_3, \delta L_1, \delta L_2$, $\delta C_1, \delta C_2$ of the amplifier are as follows: $\delta m = 2\,\%$, $\delta c_0 = 1\,\%$, $\delta L_1 = 0.01\,\%$, $\delta L_2 = 0.03\,\%$, $\delta Y_1 = 3\,\%$, $\delta Y_2 = 3\,\%$, $\delta Y_3 = 5\,\%$ $\delta C_1 = 0.01\,\%$, $\delta C_2 = 0.03\,\%$

Step 3. When applying the FS-method and FS-model of the double-circuit parametric amplifier, the expression for the transfer function $Z_1(s,t) = U_1(s,t)/I(s)$ of the amplifier has the form

$$Z_1(s,t) = \frac{\left(Y_3 + Y_2 + s_i \cdot C_2 + \dfrac{1}{s_i \cdot L_2}\right) +}{\left(s_i \cdot C_1 + Y_1 + \dfrac{1}{s_i \cdot L_1}\right)\left(Y_3 + Y_2 + s_i \cdot C_2 + \dfrac{1}{s_i \cdot L_2}\right) +}$$

$$\frac{+Z(s,t) Y_3 \left(Y_2 + s_i \cdot C_2 + \dfrac{1}{s_i \cdot L_2}\right)}{+Y_3 \left(Y_2 + s_i \cdot C_2 + \dfrac{1}{s_i \cdot L_2}\right)} \qquad (4)$$

Step 4. According to symbolic expression (4) which, when $k = 2$, provides satisfactory accuracy, we determine the functions of sensitivity of the first and second order of real part of parametric transfer function $Z_1(s,t)$ by parameters $m, c_0, Y_1, Y_2, Y_3, L_1, L_2, C_1, C_2$.

Step 5. According to equation (1) and the functions of sensitivity, which were defined in step 4, we calculate the relative deviation $\delta|Z_1(t)|$ of the module parametric transfer function $Z_1(t)$ at relative changes $\delta m = 2\,\%$, $\delta c_0 = 1\,\%$, $\delta L_1 = 0.01\,\%$, $\delta L_2 = 0.03\,\%$, $\delta Y_1 = 3\,\%$, $\delta Y_2 = 3\,\%$, $\delta Y_3 = 5\,\%$ $\delta C_1 = 0.01\,\%$, $\delta C_2 = 0.03\,\%$ of the parameters of amplifier elements.

The results of the experiment 1. Formed by expression (1), the dependence of relative deviation $\delta|Z_1(t)|$ of the real part of the parametric transfer function $Z_1(t)$ from time t is shown in Fig. 1.

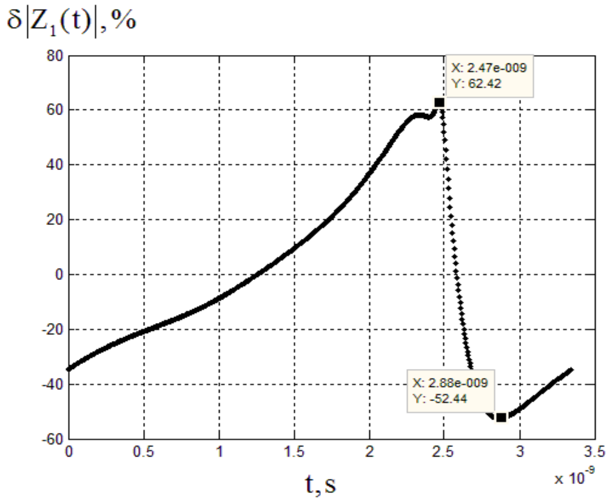

Fig. 2. The time dependence $\delta\left|Z_1(t)\right|$ *of module of* $Z_1(t)$ *at a relative change of parameters of elements.*

From the conducted computational experiment it follows that the relative deviation $\delta\left|Z_1(t)\right|$ of the real part of the parametric transfer function $Z_1(t)$ of the double-circuit parametric amplifier is a time-dependent function, because the transfer function $Z_1(t)$ itself also depends on time. It can be seen in fig.1 that relative deviation of the real part of the transfer function $Z_1(t)$ varies in the range from -52.44 % to 62.42 %

Task for the experiment 2. To carry out an optimization and determine the values c_0^* and m^*, which provide the minimum value of the goal function $F(c_0,m)=F_{\min}$, which is formed for each frequency sample ω_i from the given range $1.95\cdot\pi\cdot 10^8 : 0.0005\cdot 10^8 : 2.05\cdot\pi\cdot 10^8\, рад/c$ and for each time point t_j from the given range $0 : 0.05\cdot 10^{-9} : 3.35\cdot 10^{-9}$ for the parametric transfer function $Z_1(s,t)=U_1(s,t)/I(s)$ of the double-circuit parametric amplifier from Fig. 1 on condition that the amplifier is stable.

Experiment 2. According to the procedure of optimization algorithm shown above we perform the following steps:

Step 1. Using the FS-method, we form the denominator $\Delta_g(c_0,m,s)$ of the normal parametric transfer function $G(s,\xi)$ from the current $I(s)$ to the voltage on the parametric capacity $U_c(s,t)$ for the amplifier (Fig.1) for symbolic values of parameters m, c_0 and complex variable s. When $k=1$, the denominator $\Delta_g(c_0,m,s)$ has the form:

$$
\begin{aligned}
&\Delta_g(c_0,m,s)=\left(0.18\cdot 10^{-86}\cdot c_0{}^3\cdot m^2-0.36\cdot 10^{-86}\cdot c_0{}^3\right)\cdot\\
&\cdot s^{15}+\left(-0.54\cdot 10^{-86}\cdot c_0{}^2+0.78\cdot 10^{-76}\cdot c_0{}^3\cdot m^2+0.9\cdot\right.\\
&\left.\cdot 10^{-87}\cdot c_0{}^2\cdot m^2-0.16\cdot 10^{-75}\cdot c_0{}^3\right)\cdot s^{14}+\left(0.26\cdot 10^{-76}\cdot c_0{}^2\cdot\right.\\
&\cdot m^2-0.24\cdot 10^{-65}\cdot c_0{}^3+0.12\cdot 10^{-65}\cdot c_0{}^3\cdot m^2-0.16\cdot 10^{-75}\cdot\\
&\left.\cdot c_0{}^2-0.27\cdot 10^{-86}\cdot c_0\right)\cdot s^{13}+\left(-0.45\cdot 10^{-87}-0.15\cdot 10^{-55}\cdot\right.\\
&\cdot c_0{}^3+0.21\cdot 10^{-66}\cdot c_0{}^2\cdot m^2-0.39\cdot 10^{-76}\cdot c_0+0.72\cdot 10^{-56}\cdot\\
&\left.\cdot c_0{}^3\cdot m^2-0.13\cdot 10^{-65}\cdot c_0{}^2\right)\cdot s^{12}+\left(-0.30\cdot 10^{-56}\cdot c_0{}^2+\right.\\
&+0.51\cdot 10^{-57}\cdot c_0{}^2\cdot m^2-0.54\cdot 10^{-67}\cdot c_0-0.39\cdot 10^{-80}-\\
&\left.-0.42\cdot 10^{-46}\cdot c_0{}^3+0.21\cdot 10^{-46}\cdot c_0{}^3\cdot m^2\right)\cdot s^{11}+\left(0.11\cdot\right.\\
&\cdot 10^{-36}\cdot c_0{}^3\cdot m^2+0.36\cdot 10^{-47}\cdot c_0{}^2\cdot m^2-0.75\cdot 10^{-57}\cdot c_0-\\
&\left.-0.22\cdot 10^{-36}\cdot c_0{}^3-0.22\cdot 10^{-46}\cdot c_0{}^2-0.90\cdot 10^{-68}\right)\cdot s^{10}+\\
&+\left(0.30\cdot 10^{-38}\cdot c_0{}^2\cdot m^2-0.66\cdot 10^{-61}-0.19\cdot 10^{-37}\cdot c_0{}^2-\right.\\
&\left.-0.27\cdot 10^{-27}\cdot c_0{}^3-0.36\cdot 10^{-48}\cdot c_0+0.13\cdot 10^{-27}\cdot c_0{}^3\cdot m^2\right)\cdot\\
&\cdot s^9+\left(-0.13\cdot 10^{-27}\cdot c_0{}^2-0.48\cdot 10^{-38}\cdot c_0-0.57\cdot 10^{-49}-\right.\\
&-0.12\cdot 10^{-17}\cdot c_0{}^3+0.60\cdot 10^{-18}\cdot c_0{}^3\cdot m^2+0.21\cdot 10^{-28}\cdot c_0{}^2\cdot\\
&\left.\cdot m^2\right)\cdot s^8+\left(-0.33\cdot 10^{-42}-0.90\cdot 10^{-30}\cdot c_0-0.72\cdot 10^{-9}\cdot\right.\\
&\cdot c_0{}^3+0.72\cdot 10^{-20}\cdot c_0{}^2\cdot m^2+0.36\cdot 10^{-9}\cdot c_0{}^3\cdot m^2-\\
&\left.-0.51\cdot 10^{-19}\cdot c_0{}^2\right)\cdot s^7+\left(-20.8\cdot c_0{}^3+0.45\cdot 10^{-10}\cdot c_0{}^2\cdot\right.\\
&\cdot m^2-0.30\cdot 10^{-9}\cdot c_0{}^2+10.4\cdot c_0{}^3\cdot m^2-0.11\cdot 10^{-19}\cdot c_0-\\
&\left.-0.15\cdot 10^{-30}\right)\cdot s^6+\left(-0.60\cdot 10^{-1}\cdot c_0{}^2-0.99\cdot 10^{-12}\cdot c_0+\right.\\
&+0.72\cdot 10^{-2}\cdot c_0{}^2\cdot m^2-0.63\cdot 10^{-24}+0.42\cdot 10^{9}\cdot c_0{}^3\cdot m^2-\\
&\left.-0.84\cdot 10^{9}\cdot c_0{}^3\right)\cdot s^5+\left(-0.16\cdot 10^{-12}+0.14\cdot 10^{19}\cdot c_0{}^3\cdot\right.\\
&\cdot m^2\ -0.28\cdot 10^{19}\cdot c_0{}^3+0.36\cdot 10^{8}\cdot c_0{}^2\cdot m^2-0.12\cdot 10^{-1}\cdot\\
&\left.\cdot c_0-0.33\cdot 10^{9}\cdot c_0{}^2\right)\cdot s^4+\left(-0.30\cdot 10^{17}\cdot c_0{}^2-0.45\cdot 10^{6}\cdot\right.\\
&\cdot c_0-0.45\cdot 10^{-6}+0.30\cdot 10^{16}\cdot c_0{}^2\cdot m^2-0.39\cdot 10^{27}\cdot c_0{}^3+\\
&\left.+0.19\cdot 10^{27}\cdot c_0{}^3\cdot m^2\right)\cdot s^3+\left(-0.14\cdot 10^{27}\cdot c_0{}^2-0.57\cdot 10^{16}\cdot\right.\\
&\cdot c_0-0.69\cdot 10^{5}-0.90\cdot 10^{36}\cdot c_0{}^3+0.45\cdot 10^{36}\cdot c_0{}^3\cdot m^2+\\
&\left.+0.10\cdot 10^{26}\cdot c_0{}^2\cdot m^2\right)\cdot s^2+\left(-0.10\cdot 10^{12}+0.45\cdot 10^{33}\cdot c_0{}^2\cdot\right.\\
&\cdot m^2-0.57\cdot 10^{34}\cdot c_0{}^2+0.20\cdot 10^{44}\cdot c_0{}^3\cdot m^2-0.69\cdot 10^{23}\cdot c_0-\\
&\left.-3.9\cdot 10^{43}\cdot c_0{}^3\right)\cdot s-1\cdot 10^{22}-9\cdot 10^{32}\cdot c_0-0.2\cdot 10^{44}\cdot c_0{}^2.
\end{aligned}
$$

Step 2. According to the denominator $\Delta_g(c_0,m,s)$ which was defined in step 1 at $k=3$, for each value c_0 from the series of values of the given range we determine a limit value $m_{\lim}=f(c_0)$, at which the circuit stability changes to instability, and build «map of stability» in the coordinates c_0 and m. «Map of stability» is shown in Fig.3 in a given area of values of parameters $c_0=9.5\cdot 10^{-12}{:}0.05\cdot 10^{-12}{:}11\cdot 10^{-12}$ F and $m=0.04{:}0.005{:}0.2$

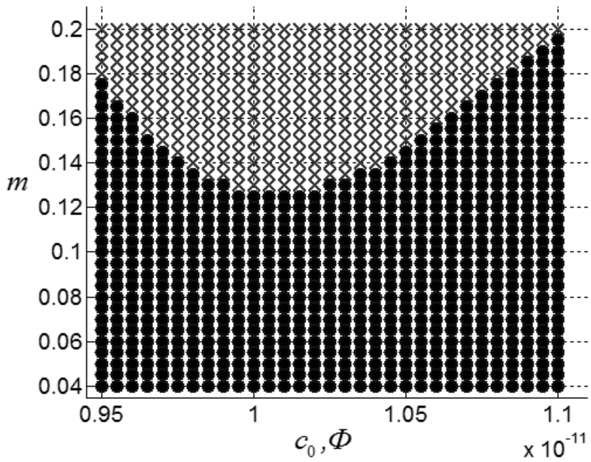

Fig. 3. "Map of stability" in the coordinates c_0 and m.

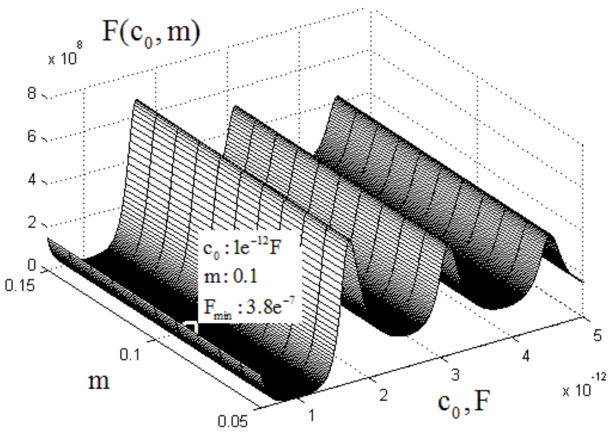

Fig. 4. Objective function $F(c_0, m)$ in coordinates c_0 and m.

Step 3. We approximate the dependence $m_{cp} = f(c_0)$, determined at step 2, by the fourth degree exponential polynomial and obtain an analytical expression for the areas of stability as:

$$m < \begin{pmatrix} -0.1073\cdot 10^{51}\cdot c_0^4 + 0.429\cdot 10^{39}\cdot c_0^3 - \\ -0.6337\cdot 10^{27}\cdot c_0^2 + 0.409\cdot 10^{15}\cdot c_0 - 96.83 \end{pmatrix}. \quad (5)$$

Step 4. We set the goal function $M_0(\omega_i, t_j)$ as the real part of the parametric transfer function of the amplifier

$$Z_1(\omega_i, t_j) = \cfrac{\left(0.5001 + s_r 68\cdot 10^{-12} + 1/s_r L_2\right) +}{\left(s_r 68\cdot 10^{-12} + 10^{-4} + 1/s_r L_1\right)\cdot} \quad \cfrac{+Z(\omega_i, t_j)\cdot}{\cdot\left(0.5001 + s_r 68\cdot 10^{-12} + 1/s_r L_2\right) + \cfrac{\cdot 0.5\cdot\left(10^{-4} + s_r 68\cdot 10^{-12} + 1/s_r L_2\right)}{+Y_3\left(10^{-4} + s_r 68\cdot 10^{-12} + 1/s_r L_2\right)}}$$

at $c_0 = 1\cdot 10^{-12} F$, $m = 0.1$, $s_r = j(\omega \pm r\cdot\Omega)$, $k = 2$ та $\omega = \omega_i$, $t = t_j$, $L_2 = 9.312609\cdot 10^{-9} H$, $L_1 = 36.70795\cdot 10^{-9} H$.

Step 5. According to the FS-method we form the function of characteristic $M_{Z_1}(c_0, m, \omega_i, t_j)$ of the double-circuit parametric amplifier as the real part of parametric transfer function

$$Z_1(m, c_0, \omega_i, t_j) = \cfrac{\left(0.5001 + s_r 68\cdot 10^{-12} + 1/s_r L_2\right) +}{\left(s_r 68\cdot 10^{-12} + 10^{-4} + 1/s_r L_1\right)\cdot} \quad \cfrac{+Z(m, c_0, \omega_i, t_j)\cdot}{\cdot\left(0.5001 + s_r 68\cdot 10^{-12} + 1/s_r L_2\right) + \cfrac{\cdot 0.5\cdot\left(10^{-4} + s_r 68\cdot 10^{-12} + 1/s_r L_2\right)}{+Y_3\left(10^{-4} + s_r 68\cdot 10^{-12} + 1/s_r L_2\right)}}$$

at $s_r = j(\omega \pm r\cdot\Omega)$, $k = 2$ та $\omega = \omega_i$, $t = t_j$, $L_2 = 9.312609\cdot 10^{-9} H, L_1 = 36.70795\cdot 10^{-9} H$..

Step 6. According to equation (2) we form the objective function $F(c_0, m)$ for given values ω_i and t_j: $1.95\cdot\pi\cdot 10^8 - 2.05\cdot\pi\cdot 10^8 rad/s$ with the step $0.0005\cdot 10^8 rad/s$ and $0 - 3.35\cdot 10^{-9} s$ with the step $0.05\cdot 10^{-9} s$, respectively, in the form:

$$F(c_0, m) = \sum_{i=1}^{5}\sum_{j=1}^{6}\left(M_{Z_1}(c_0, m, \omega_i, t_j) - M_0(\omega_i, t_j)\right)^2, \quad (6)$$

Fig. 3 shows a graphical view of the objective function $F(c_0, m)$.

Step 7. According to the function «patternsearch» of optimization, while performing determined condition of stability of the amplifier (5), we carry out the optimization and define the parameters c_0 and m, that provide the minimum value of the objective function $F(c_0, m) = F_{\min}$, which is formed for every frequency point ω_i from a given range $1.95\cdot\pi\cdot 10^8 : 0.0005\cdot 10^8 :$ $: 2.05\cdot\pi\cdot 10^8 rad/s$ and for every time point t_j from a given range $0 : 0.05\cdot 10^{-9} : 3.35\cdot 10^{-9} s$ for the parametric transfer function $Z_1(s,t) = U_1(s,t)/I(s)$ of the double-circuit parametric amplifier from Fig. 3.

The results of the experiment 2. Applying the function of optimization «patternsearch», while satisfying the defined stability conditions for amplifier (5) and at randomly chosen initial values $c_0 = 1.2\cdot 10^{-11} F$, $m = 0.15$ the varied parameters for 524 iterations, determines a minimum of $F_{\min}$ at $c_0^* = 1\cdot 10^{-12} F$ and $m^* = 0.1$ which, in Fig. 4, is marked by symbol ▣ .

5. Conclusion

1. The practice of applying the described procedures of tolerance analysis and optimization of LPTV circuits proved the effectiveness of applying the parametric transfer functions formed by the FS-method.

2. Performing the procedures of tolerance analysis and optimization of LPTV circuits, while assessing the asymptotic stability, provides adequacy of the results and increases the efficiency of further design stages.

References

[1] Yu. Shapovalov, B. Mandziy, and S. Mankovsky, "The peculiarities of analysis of linear parametric circuit performed by frequency-symbolic method", *Przeglad Elektrotechniczny*, vol. 86, no 1, pp. 158-160, 2010.

[2] Yu. Shapovalov, B. Mandziy, and S. Mankovsky, "Peculiarities of frequency-symbolic method applied to parametric circuit analysis", *Przeglad Elektrotechniczny*, vol.87, no 5, pp. 155–159, 2011.

[3] V. N Ilin and V.T. Frolkin, *Avtomatizatsiya skhemotekhnichieskoho proiektirovaniya*. Moscow, Russia: Radio i sviaz, 1987. (Russian)

[4] Yu. I. Shapovalov, D. R. Bachyk, and S. V. Mankovsky, "The system MAOPCs for the multivariate analysis and optimization of linear parametric circuits at the enviroment of MATLAB", *Journal on Radioelectronics and telecommunications,* no 766, pp. 28–34, 2013.

АНАЛІЗ ДОПУСКІВ ТА ОПТИМІЗАЦІЯ ЛІНІЙНИХ ПАРАМЕТРИЧНИХ КІЛ НА ОСНОВІ ЧАСТОТНОГО СИМВОЛЬНОГО МЕТОДУ

Юрій Шаповалов, Богдан Мандзій, Дарія Бачик

Розглянуто процедури аналізу допусків та оптимізації лінійних параметричних кіл, які основані на частотному символьному методі та реалізовані системою функцій MAOPCs у середовищі MATLAB. Особливість аналізу допусків та оптимізації полягає у тому, що попередньо проводиться оцінка стійкості (асимптотичної) досліджуваного лінійного параметричного кола в заданих межах зміни параметрів та визначення областей допустимої їх зміни, у яких коло є стійким. Виключно у цих областях відбувається аналіз допусків та оптимізація..

Bohdan Mandziy – Ph. D., Sc. D., professor of the Department of Theoretical Radio Engineering and Radio Measurement, Lviv Polytechnic National University, Ukraine. Author of over 250 scientific and educational works, including 2 monographs and 6 textbooks. The research area includes methods of automated analysis of nonlinear electronic circuits and ensuring reliability of radio electronic devices and telecommunication equipment.

Yuriy Shapovalov – Ph. D., Sc.D., professor of the Department of Radioelectronic Appliances and Systems, Lviv Polytechnic National University, Ukraine. The author of over 30 publications. His research area is frequency symbolic analysis of linear periodically time-variable (LPTV) circuits.

Dariya Bachyk - Ph. D., Assistant Professor of the Department of Radioelectronic Appliances and Systems, Lviv Polytech-nic National University, Ukraine. Research interests – investigation of stability of LPTV circuits by frequency symbolic method, investigation of frequency symbollic models, multivariate analysis and optimization LPTV circuits.