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GENERATION OF A CIRCULAR MAGNETIC FIELD IN A THREE-PHASE ASYNCHRONOUS MOTOR AT SINGLE-PHASE POWER SUPPLY

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Abstract. One of the important problems of electromechanics is choosing the capacitance required for switching three-phase asynchronous motors (AM) into a single-phase network. In fact, the problem comes down to the choice of a capacitance value, at which an operational magnetic field of the motor will be as close to circular as possible. The solution to this problem by using mathematical models of symmetrical electric machines needs setting a number of simplifying assumptions and does not ensure the reliability of calculation results. In the article, we propose a method and an algorithm for the determination of a capacitance value in the winding of a three-phase asynchronous motor powered by a single-phase network, at which a circular magnetic field is generated. The method is based on the mathematical model of AM, which considers the magnetic saturation and current displacement in the rotor rods, and the projection method for solving the boundary problem for the system of electromagnetic equilibrium equations composed in the three-phase axes. The mathematical model developed can serve as a basis for microprocessor control of the motor starting process.

Key words: three-phase asynchronous motor, single phasing, capacitor, circular magnetic field, boundary problem, static characteristic, magnetic system saturation, current displacement.

1. Introduction

When operating three-phase asynchronous motors, there often arises a need of their using at single-phase supply. In addition, they can be used as capacitor motors for low-power electric drives in various branches of industry and agriculture, where there is not a three-phase network available, for any reason. The presence of capacitors in the stator winding causes its asymmetry, and hence their performance characteristics significantly differ from those of symmetrical classical-design motors that are powered by a symmetrical three-phase voltage system. Therefore, in practice, it is important that the characteristics of asynchronous motors with the capacitors in any one phase be as close to the classical three-phase motors of general purpose as possible.

In most cases, using either capacitors in one of the phases or additional resistance results in generating an elliptical magnetic field in an air gap, which can be divided into two fields rotating in opposite directions. A reverse magnetic field induces a decelerating torque, which can cause such crevasses in the electromagnetic torque curve that starting the motor with the required load will be impossible. Thus, there emerges a problem of creating a circular or close to circular operational magnetic field of a three-phase AM powered by a single-phase network using capacitors. For this, it is necessary to have a mathematical model, which would adequately reflect the electromagnetic processes in AM with the capacitors in one of the phases, and would enable, by a mathematical experimental game, the choice of their capacity value to be optimized. The importance of this problem is evidenced by numerous publications, whose number continues to grow [1-5]. In most publications, this problem is solved by using mathematical models of symmetrical electric machines that requires a number of assumptions to be set, including the neglect of change in the parameters resulted from the magnetic core saturation and skin effect phenomenon. The well-known method of symmetrical components [4] and the theory of two rotating fields [1] are based on the method of superposition, which is appliable only for linear objects. For the research to be done, using the orthogonal coordinate systems [5] cannot be considered effective because, given the dynamic nature of the processes caused by the asymmetry of the stator circle of AM, an adequate description of the processes in the transformed coordinates is impossible .

The choice of a capacitance based on the use of the simplified mathematical models of AM, as well as the ones adapted to specific conditions of equivalent circuits [6] cannot be reliable, and requires field research, and the incorrect choice of capacitors may lead to creating a significant reverse rotating magnetic field and, consequently, crevasses in an electromagnetic torque curve. Meanwhile, using the adequate dynamic mathematical models of the motors requires the imployment of the appropriate mathematical methods of calculation.

The purpose of the article is to develop a method and an algorithm of mathematical modelling of the performance of a three-phase AM, which is powered by a single-phase network that would enable a capacitance value to be determined, and in this way ensure the generation of a circular magnetic field in any operational conditions.

2. Equation of electrical equilibrium

There are many types of connection of the capacitors with windings of AM with a three-phase motor is switched into a single-phase network [2]. To concretize, let us choose one of the schemes in which the stator winding is Y-connected. The investigation of AM processes requires a sufficiently precise determination of the motor parameters, including resistance, internal and mutual inductance of electric circuits. These parameters depend on magnetic core saturation and current displacement in squirrel-cage rotor rods, and therefore, reliable calculation results, which make it possible to refuse natural experiments, can be obtained only on the basis of mathematical models of high adequacy motors [7, 8].

The important issue is choosing a coordinates system to describe electromagnetic connections between electric circuits of AM. Since DE of electromagnetic equilibrium of AM in the three-phase physical coordinates has, due to the rotor spinning, periodic coefficients that greatly complicates their solving, research into the AM processes with the capacitors switched into any one phase is carried out by using the AM mathematical model developed in the stationary three-phase axes [7]. For the stator phases, these axes are real, material, and for the rotor phases, they are stationary, three-phase, whose position and direction coincides with the axes of the stator phases. Practice shows that this coordinate system does not lead to a significant loss of the calculation accuracy, but allows you to take into account the asymmetry of the stator winding. For the current displacement to be taken into account, each rod of the squirrel-cage rotor is divided into n elementary elements with respect to height [8, 9], causing the squirrel-cage rotor winding to be represented by n three-phase windings between which there is a mutual inductive coupling. As the calculations showed, in most cases n = 4–5 is adequate. As a result, the system of differential equations (DE) of electrical equilibrium, which describes the dynamic mode of AM with the capacitors switched in series into one of the phases, has the form

$$\frac{dy_A}{dt}-\frac{dy_B}{dt}=-r_A i_A + r_B i_B - u_{kA};$$

$$\frac{dy_B}{dt}-\frac{dy_C}{dt}=-r_B i_B + r_C i_C + \sqrt{3}U_m \sin(w_0 t + p/6);$$

$$i_A + i_B + i_C = 0;$$

$$\frac{dy_{a1}}{dt}-\frac{dy_{b1}}{dt}=-r_{a1}i_{a1}+r_{b1}i_{b1}-a\left(y_{b1}-2y_{c1}+y_{a1}\right);$$

$$\frac{dy_{b1}}{dt}-\frac{dy_{c1}}{dt}=-r_{b1}i_{b1}+r_{c1}i_{c1}-a\left(y_{c1}-2y_{a1}+y_{b1}\right);$$

$$i_{a1}+i_{b1}+i_{c1}=0;$$

$$\vdots$$

$$\frac{dy_{an}}{dt}-\frac{dy_{bn}}{dt}=-r_{an}i_{an}+r_{bn}i_{bn}-a\left(y_{bn}-2y_{cn}+y_{an}\right);$$

$$\frac{dy_{bn}}{dt}-\frac{dy_{cn}}{dt}=-r_{bn}i_{bn}+r_{cn}i_{cn}-a\left(y_{cn}-2y_{an}+y_{bn}\right);$$

$$i_{an}+i_{bn}+i_{cn}=0;$$

$$\frac{du_k}{dt}=\frac{i_A}{C}; \qquad (1)$$

where y_x, i_x, r_x $(x = A, B, C, a_1, b_1, c_1, ..., a_n, b_n, c_n)$ represents the linkage, currents and loop resistance; where $a = w_0(1-s)/\sqrt{3}$; s is the slip; U_m, w_0 are the phase voltage amplidude and motor line frequency двигуна; u_k stands for the capacitor voltage with a capacity C.

3. Boundary problem

If there is a constant slip caused by the asymmetry of the system stator ring, DE (1) describes the dynamic periodic mode. In mathematical terms, this is a boundary problem for a system of nonlinear first-order DE with periodic boundary conditions, and a resulting solution is the dependences of its coordinates for the period of process repetability, which is determined by the supply variation frequency.

According to the literature sources, the methods of projection are among the most effective methods for solving boundary problems. Their essence is to develop a DE space onto some other subspace, which is defined by a set of so-called basic functions. These functions can be trigonometric functions or different type polynomials. However, due to the inherent flaws of this approximation, these functions have not become widespread. Essentially different properties are inherent to the three-order spline approximations, whose use contributed to the development of an algorithm for solving boundary problems for the first-order DE with periodic boundary conditions [10]. Their use enables a DE system to be algebraized in the period of the process repeatability, i.e. its algebraic analogue to be created. Solving the resulting nonlinear algebraic system makes it possible to obtain an approximate solution in the form of functional periodic dependencies of state variables.

Calculation of the capacity required to obtain a circular magnetic field can be most effectively achieved by calculating for each slip s a static characteristic as a dependence of the mode coordinates on the value of capacitance. Let us consider the method for obtaining this characteristic. For this, to make the material presentation short, let DE system (1) be given in vector representation

$$\frac{\partial\vec{y}(\vec{x},t)}{\partial\vec{x}} = \vec{f}\,(\vec{y},\vec{x},\vec{u},t) \qquad (2)$$

where the corresponding vectors consist of the following elements:

$$\vec{y}=(\vec{y},u_k)^*;\quad \vec{x}=(\vec{i},u_k)^*;\quad \vec{u}=(0,u_{BC},0,...,0)^*;$$

$$\vec{i}=(i_A,i_B,i_C,i_{a1},i_{b1},i_{c1},...,i_{an},i_{bn},i_{cn})^*;$$

$$\vec{y}=(y_A,y_B,y_C,y_{a1},y_{b1},y_{c1},...,y_{an},y_{bn},y_{cn})^*.$$

The spline approximation of the coordinates on the grid *+1* of the period nodes having been performed by the three-order splines as specified in with [10], we will obtain an algebraic analoque of system (2) of the *m*=3(*n*+1)-th order as an nonlinear algebraic equation of the *Nm*-th order

$$H\vec{Y}(\vec{X})=\mathrm{F}(\vec{Y},\vec{X}), \qquad (3)$$

where H is the $Nm\acute{}Nm$ matrix, whose elements are defined by the period node grid [10];

$$\vec{Y}=(\vec{y}_1,\vec{y}_N)^*;\quad \vec{X}=(\vec{x}_1,\vec{x}_N)^*;\quad \vec{F}=(\vec{f}_1,\vec{f}_N)^*$$

are the vectors, the components of each one are *N* of the vectors of node values of the respective variables..

The unknown variable in system (3) is the vector $\vec{X}$, which consists of the vectors of node values of the AM loop currents and the voltage on the capacitor.

4. Problem solving algorithm

To investigate the dependence of the coordinates of the sustainable mode of the motor on the capacitantce at a given slip value, we will make use of the differential method for the calculation of static characteristics of periodic processes [7]. For this, algebraic differential equation (3) is differenciated with respect to C that will lead to a new DE system of the argument C

$$\left(\left(H-\frac{\partial\vec{Z}}{\partial\vec{Y}}\right)\frac{\partial\vec{Y}}{\partial\vec{X}}-\frac{\partial\vec{Z}}{\partial\vec{X}}\right)\frac{d\vec{X}}{dC}=\frac{\partial\vec{Z}}{\partial C}, \qquad (4)$$

In which $\left.\frac{\partial\vec{Y}}{\partial\vec{X}}\right|_j = diag\left(\boxed{\begin{array}{c|c} L_j & 0 \\ \hline 0 & 1\end{array}}\right)$, (*j* = *1*,...,*N*), and $L_j=\frac{\partial\vec{y}_j}{\partial\vec{i}_j}$ is the full matrix of the differential motor inductances [7], which must be set for each nodal point. This can be done by using the main characteristic of magnetization and dependences of linkages of stator and rotor leakage on the currents of stator and rotor, respectively.

As an example, Fig. 1 shows the calculation results of time dependences of the currents of the stator phases and hodograph of the vector of the motor linkage 4A160M6Y3 (P = 15kVt, U = 220 V, I = 29,9 A; p = 3) for two values of the capacitances in the phase A during the starting ($s=1{,}0$).

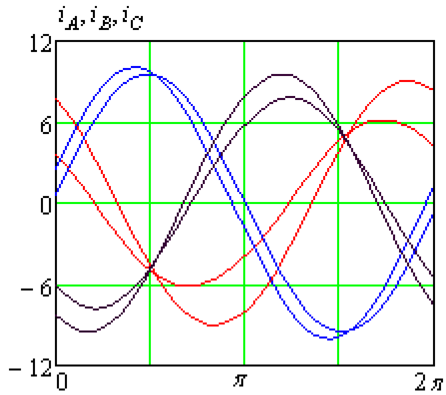

Fig. 1. Periodical dependences of relative values of phase currents of motor for two values of capacitor capacitance at $s = 1{,}0$.

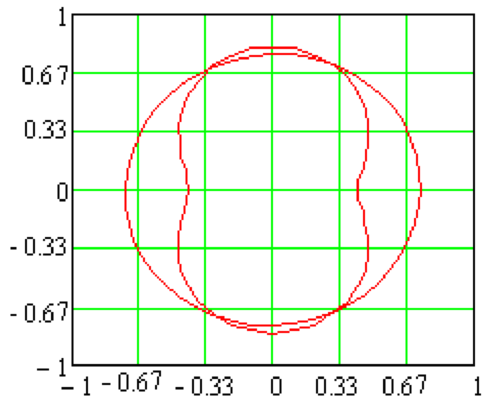

Fig. 2. Hodographs of a depicted vector of linkage at the shown in Fig.1 phase currents.

5. Conclusion

1. The method and algorithm for the calculation of a capacitor capacitance of a three-phase AM, which is necessary to generate a circular magnetic field when the motor is switched to a single-phase network.

2. The algorithm is based on the projection method for solving the boundary problem for a DE system of electrical equilibrium of AM loops and the differential method for the calculation of static characteristics.

3. The problem is solved in the sationary three-phase system of coordinate axes taking into account the saturation of the magnetic system of the motor and

current displacement in the squirrel-cage rods, which ensures the adequacy calculation results.

4. For the saturation to be taken into account, we use the characteristics of magnetization by the primary magnetic flux and the fluxes of leakage of stator and rotor windings. The current displacement is taken into account by dividing the rods including the linkage rings with respect to height into several layers resulting in the representation of a linkage winding in an AM mathematical model by several three-phase ones.

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УТВОРЕННЯ КРУГОВОГО ОБЕРТОВОГО МАГНІТНОГО ПОЛЯ У ТРИФАЗНОМУ АСИНХРОННОМУ ДВИГУНІ ЗА ОДНОФАЗНОГО ЖИВЛЕННЯ

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Запропоновано метод і алгоритм визначення значення ємності конденсаторів в обмотці трифазного асинхронного двигуна, що живиться від однофазної мережі, за якого утворюється кругове магнітне поле. Розроблена математична модель дає змогу на основі розв’язування задачі як крайової для системи рівнянь електромагнітної рівноваги розраховувати значення ємності конденсаторів для будь-якого значення ковзання ротора з метою створення кругового магнітного поля. У математичній моделі двигуна враховується явище насичення магнітопроводу та витіснення струму в стержнях ротора. Використовуючи розроблену математичну модель, можна визначати закон зміни величини ємності коденсаторів впродовж усього процесу пуску, що дає змогу здійснювати мікропроцесорне керування процесом пуску двигуна.

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