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APPLICATION OF PARTICLE SWARM OPTIMIZATION METHOD FOR SOLVING THE PROBLEMS OF APPROXIMATION OF ELECTROMECHANICAL SYSTEMS' PARTS

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Abstract: This paper deals with the application of particle swarm optimization method to solving the typical problems of electromechanics, where the parts are described by models of fractional order with the number of unknown parameters 3 ÷ 5. The examples show the effectiveness and high accuracy of the proposed method as a means of the approximation of transfer functions of integral order of electromechanical models by models of fractional order and vice versa.

Key words: method of particle swarm optimization, electromechanical systems, approximation of the transfer function of fractional order.

1. Introduction

An effective approach to finding optimal or suboptimal solutions to multi-function purpose is the method of particle swarm optimization (PSO). Creating PSO algorithms [1] was the attempt to simulate natural communication processes of personal knowledge among certain groups, including bees, which emerge when social elements in this group (in a swarm) migrate in order to achieve certain optimal properties, such as configuration, position and so on. Particle swarming occurs while searching for the best solution, based on previous iterations, with the intent to meet the best solution in this process and eventually stop at a certain minimum of its error.

This method was applied in the theory of automatic control of electromechanical systems [2] while solving the problem of choosing the parameters of the fractional PID controller.

2. Theoretical basis of PSO method

If we apply the PSO method for N particles moving in D-dimensional search space, each particle is characterized by random position and velocity. At each iteration each particle changes its trajectory basing on personal and group experience.

The i^{th} particle is denoted as

$$X_i = (x_{i1}, x_{i2}, ..., x_{iD}).$$

Its previous best solution (state, position), indicated as *pbest*, is described as follows:

$$P_i = (p_{i1}, p_{i2}, ..., p_{iD}).$$

Current velocity (the rate of position changing) is described as

$$V_i = (v_{i1}, v_{i2}, ..., v_{iD}).$$

The best solution of a swarm at the moment (*gbest*) is given as

$$P_g = (p_{g1}, p_{g2}, ..., p_{gD}).$$

At every time span, each particle moves towards the position *pbest* and *gbest*.

In [1] it was proposed to evaluate the performance (efficiency) of particle motion to the desired solution with a fitness function that establishes a relationship between the position and velocity of i-th particle for D-dimensional array by the following equations:

$$\begin{aligned} v_{id}(t+1) = \omega \cdot v_{id}(t) + c_1 \cdot \varphi_1 \cdot (p_{id}(t) - x_{id}(t)) + \\ + c_2 \cdot \varphi_2 \cdot (p_{gd}(t) - x_{id}(t)) \end{aligned}, \quad (1)$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1), \quad (2)$$

where: c_1 and c_2 are two positive constants which are respectively called local and global weighting factor; φ_1 and φ_2 are two random functions in the range [0,1]; ω denotes the weight coefficient of inertia (inertia weight) of swarm particles (a constant).

3. Improvement of PSO method

For the improvement of the algorithm [1] of particle swarm optimization method the following changes are introduced:

- the initial distribution of the swarm is done by dividing the search range of each unknown parameter by the number of swarm items in a row and equal distribution of the swarm elements in the space of solution search within the given range;
- it is recommended to set search limits of each unknown parameter, based on the estimated values of the parameters (but not to fix unreasonably wide search range kp from 1 to 500, as it was traditionally set in well-known literature source [2]);
- increasing the range of searching for optimal solutions according to a certain criterion by the particle swarm method is necessary only when no satisfactory result occure;

- it is recommended to make the refined calculation of the parameters found when the error exceeds an acceptable value by reducing search limits in the area of previously obtained results;
- it is suggested to evaluate the proposed approximation error at each iteration to speed up the approximation procedure and, with the desired accuracy of approximation established, to terminate this procedure;
- it is suggested to choose constants c_1 and c_2 basing on the users' experience, as they determine the behavior and effectiveness of the method in general.

The author has developed the algorithms of particle swarm optimization method to solve the following approximation problems in electromechanics by using fractional models with three or five variable parameters, namely:

- approximation of the parts of electromechanical systems by their transition functions (processes) using the models of fractional order;
- approximation of the high order transfer function (TF) using low order fractional models.

The solution to these problems is illustrated by the following examples:

4. Example 1.

The approximation of the parts of electromechanical systems by their transition functions using the fractional order models is shown by the example of the 3rd order binomial TF approximation (3)

$$\frac{1}{s^3 + 3s^2 + 3s + 1} \tag{3}$$

using the fractional second order model (4)

$$W(s) = \frac{k}{a_2 s^{\alpha_2} + a_1 s^{\alpha_1} + 1}. \tag{4}$$

The performance of the novel method of particle swarm optimization for five parameters (k, a_2, α_2, a_1, α_1) and such approximation is demonstrated below.

The original location of the swarm in the two-dimensional coordinates (first iteration) is performed by a uniform distribution of swarm elements in the search space.

After the second iteration in the two-dimensional coordinates the pattern of swarm location in two-dimensional coordinates appears (the projections of other coordinates appear in the plane) (Fig.1.) The picture shows the beginning of the swarm movement towards the bottom right corner of the pattern. It means that one swarm element or a group of them has found a location close to the optimal position (*gbest*) and other elements of the swarm begin to move in this direction, continuing their search for better location (*pbest*) on the way.

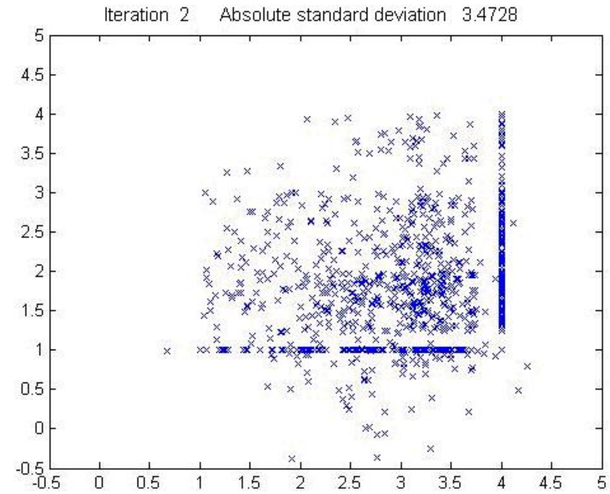

Fig. 1. Swarm location after the 1st iteration.

In Fig.2 the result of the fifth iteration shows swarm grouping within the range of optimal solution.

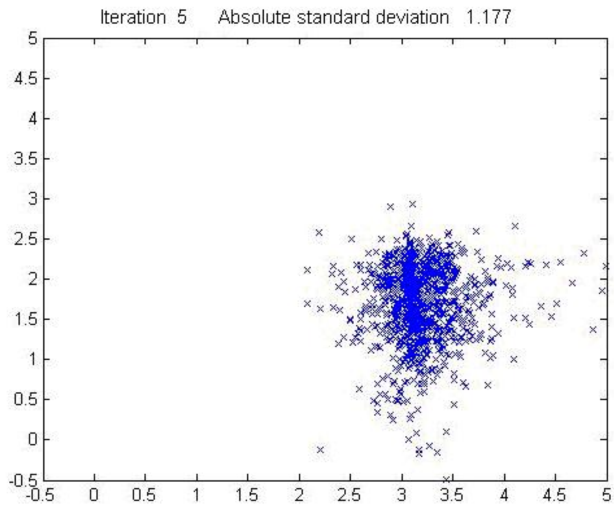

Fig. 2. Swarm location after the 5th iteration.

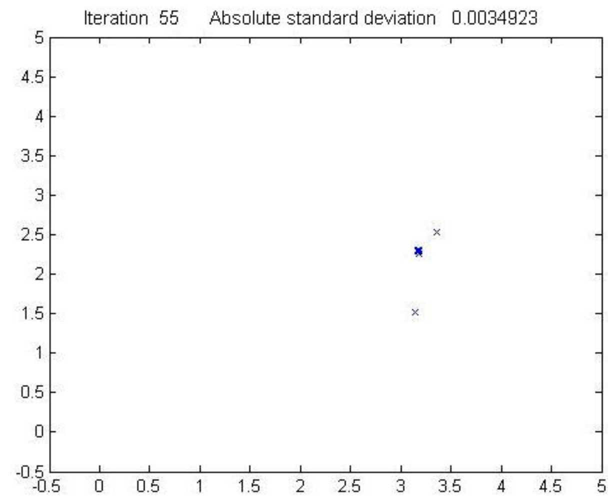

Fig. 3. Swarm location after the 55th iteration.

Fig. 3 shows the result of the 55-th iteration, that is, final swarm grouping within the range of the optimal solution.

As a result of the 55-th iteration (Fig. 3) an approximating fractional order TF is obtained, being described as follows:

$$W(s) = \frac{0.9803}{3.0446 s^{2.3542} + 3.5732 s^{1.0904} + 1}. \quad (5)$$

The transition function of such TF is shown in Fig.4, curve 2. It is laid on the transition function of the binomial form (curve 1, (3)).

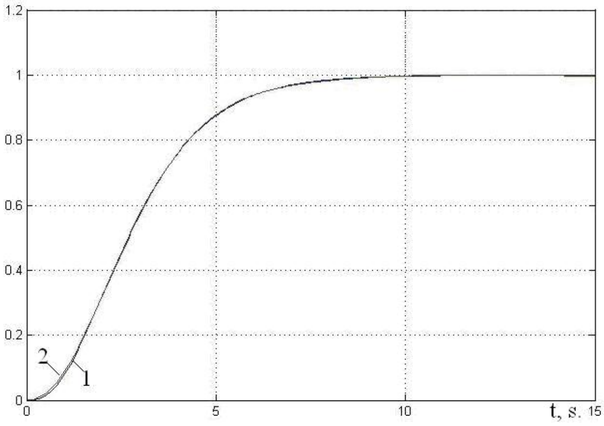

Fig.4. Transition function of approximating TF (5) "2" laid on the transition function of TF (4) "1".

To assess the accuracy of the approximation the following parameters of transitional characteristics have been used:

- absolute standard deviation calculated by the formula:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - y_{ie})^2}, \quad (6)$$

where: y_i is the value of approximating transition function at the i-th point; y_{ie} is the value of the high order transition function of the level at the i-th point; n = 401 denotes the number of points of transition process;

- relative error of approximation calculated by the following formula:

$$\delta = \frac{\sigma}{y_y} \cdot 100\%; \quad (7)$$

where: y_y is the established value of the transition function of a high order part.

For this version of the binomimal form approximation of the TF (3) with the fractional order TF (5) the errors are: $\sigma = 0.0028$, $\delta = 0.28\%$.

In order to speed up the approximation procedure it is proposed to calculate the error of approximation at the each iteration and to terminate the approximation procedure with the desired accuracy established. Fig. 5 shows the dependence of approximation error σ on the number of iterations of the particle swarm optimization method by the example of the 3rd order binomial form (5).

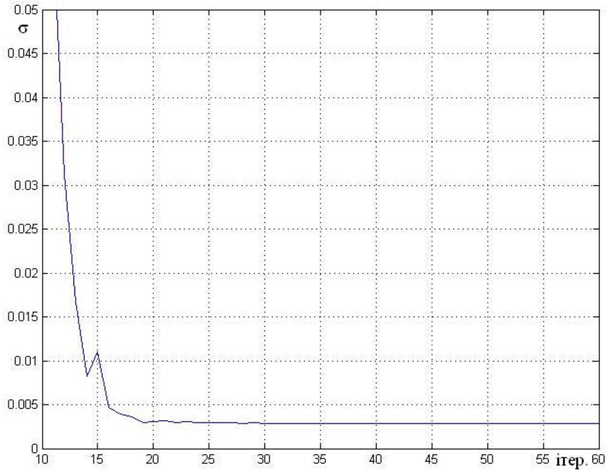

Fig. 5. Dependence of the approximation error on the number of iterations of the particle swarm optimization method by the example of the binomial form of 3rd order (5).

The analysis of the dependence (Fig. 5) clarifies that the approximation procedure with the desired accuracy could be completed after the 20-th iteration.

5. Example 2

The approximation of the high order transfer function (TF) with fractional low order models is carried out using the results of experimental studies, according to which [3] the numerical form of the TF of an induction generator (IG) with self-excitation power $W_{IG}(p)$ equal to 30 kW in the channel of excitation current has been obtained:

$$W_{IG}(p) = \frac{83p^3 + 4851p^2 + 59929p + 101674}{105p^3 + 2934p^2 + 15878p + 19395}. \quad (8)$$

As a result of the TF approximation of the IG power $W_{IG}(p)$ (8) due to the proposed particle swarm optimization method for 3 parameters, i.e. using the fractional model

$$W(s) = \frac{k}{a_1 s^{\alpha_1} + 1}, \quad (9)$$

a simplified TF of the IG has been obtained.

$$W(s) = \frac{5.5060}{0.2662 s^{0.7866} + 1}. \quad (10)$$

Fig. 6 shows the transition functions, whereas Fig. 7 demonstates logarithmic amplitude-frequency and phase-frequency characteristics (the Bode diagram) obtained from the TF (8) - curve 1, compared with the relevant characteristics obtained by the TF of the IG (10) - curve 2.

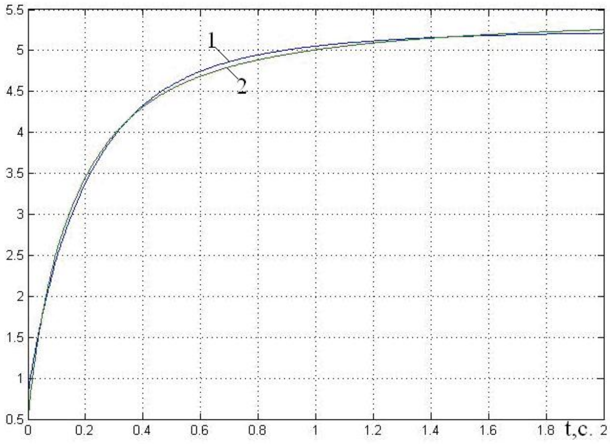

Fig. 6. Voltage transition processes at IG terminals in the mode of hopping self capacitance.

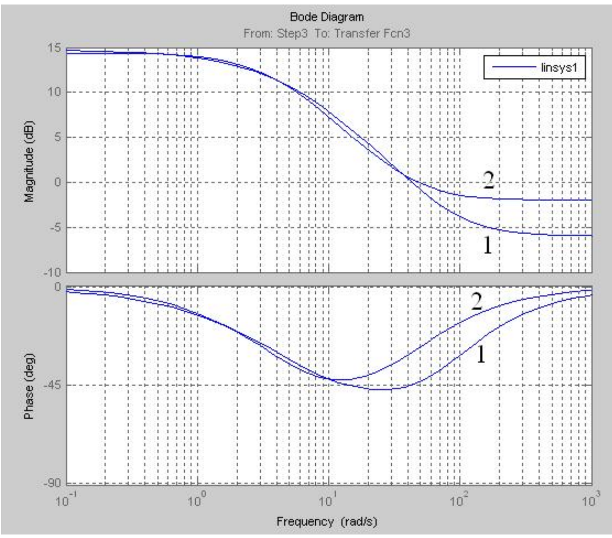

Fig. 7. Bode diagram for the TF of the IG $W_{IG}(p)$.

Based on the analysis of the above characteristics, the accuracy of the approximation is evaluated:

- approximation error for the transition functions: $\sigma = 0{,}049$, $\delta = 0{,}93\%$;
- error of the cutoff frequency of the Bode diagram – 5,7 rad/s;
- error of the Bode diagram by phase - 17 grad.

6. Conclusion

- application of the particle swarm optimization method to approximating high order transfer functions enables us to obtain fractional models of lower-order transfer functions that reproduce the behavior of the given systems;
- conducted research, including examples given in this article, proves that it is sufficient to use fractional models with three or five variable parameters to solve approximation problems of the parts of electromechanical systems for their transition functions while providing relative error of 1%.

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УДОСКОНАЛЕННЯ МЕТОДУ РОЮ ДЛЯ РОЗВ’ЯЗКУ ЗАДАЧ АПРОКСИМАЦІЇ ЛАНОК ЕЛЕКТРОМЕХАНІЧНИХ СИСТЕМ

Богдан Копчак

Розглянуто удосконалення методу рою частинок, а також адаптацію цього методу для розв’язку типових задач електромеханіки, у яких ланки описуються моделями дробового порядку з кількістю невідомих параметрів 3–5. На прикладах показано ефективність і високу точність запропонованого методу як засобу для апроксимації передаточних функцій повного порядку ланок електромеханічних систем дробовими моделями і навпаки.

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