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SPECTRAL-BASED APPROACH TO SUBPIXEL IMAGE FORMATION

Ivan Prudyus, Victor Tkachenko, Peter Kondratov, Leonid Lazko, Serhiy Fabirovskyy

Lviv Polytechnic National University, Ukraine

lazko_new@yahoo.com, iprudyus@polynet.lviv.ua

Abstract: Current approach to the increase of the resolution is based on receiving images from several image sensor arrays shifted by subpixel distance. [1, 2]. The analysis of the image forming process has shown that a pixel aperture works as a low-pass spatial filter and decreases spatial resolution even if several image sensors shifted by subpixel distance are used. The proposed by the authors approach uses both shifted image sensor arrays and data processing based on inverse filtering. That technique is more advantageous in comparison to existing approaches based on image sensors positioning or masks.

The main results of the investigation are: discussion of problems of increase in the resolution of remote sensing; simulation of the image forming process for estimation of the impact of a pixel aperture on the image forming process; a new technique for eliminating the influence of a pixel aperture.

The simulation performed on the basis of the proposed approach has shown that it is possible to get a restored image with the resolution almost similar to the resolution of a test high-resolution image, what indicates the effective reduction of the influence of the pixel aperture. It has been shown that for a blurred test image received by using the aperture of 8 pixels its normalized absolute error is of 0.123 and for a restored by inverse filter image its error reduces to 0.019.

Key words: image processing, spatial resolution, subpixel image processing, inverse problem.

1. Introduction

Improving the quality of satellite images is one of the most important tasks set before remote sensing. The most important parameter of satellite images that defines their quality is their spatial resolution which depends on the quantity of pixel per image and its geometric projection onto the Earth surface. The decrease in the pixel's size is limited by the current state of technology and data processing algorithms.

The decrease in the pixel's size requires an improvement in photosensitivity and increase in frequency bands for both synchronization signals and data signals. But this approach makes a significant load on an onboard processing unit.

A solution to such a problem is using the proposed method that combines subpixel data forming with inverse filtering which allows a high-resolution digital image to be produced from a series of low-resolution images taken with some spatial shift.

Subpixel image processing improves spatial resolution by using the same sensor array without changing the pixels' geometry.

The series of low-resolution images can be taken:

a) by positioning several image sensor arrays with a subpixel shift from one another [1];

b) by defocusing the images applying sensor array's shift in the focal plane;

c) by using an optical shielding mask posed before the sensor array (in the focal plane) and by defocusing the image on the sensor plane [2],

Typically, charge-coupled devices (CCD-lines) are used as sensor arrays.

2. Image formation with subpixel resolution

Usually, due to high cost of sensor arrays, only two sensor arrays are positioned with a subpixel shift from one another. The shift is made in two directions that intersect at right angle. Fig. 1 shows the relative spatial arrangement of two CCD-lines.

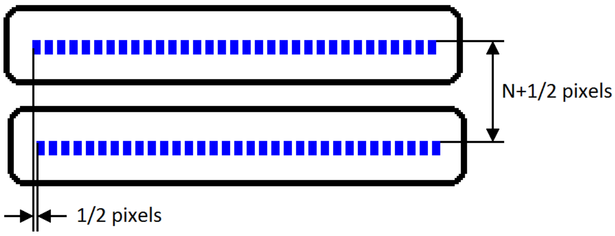

Fig. 1. Spatial arrangement of two CCD-lines.

The method of subpixel image processing presented below uses only two original low-resolution images obtained.

In Fig. 2, the spatial distribution of pixels of the original images and their disposition are shown.

When being processed, any original image pixel is divided into four resulting pixels. Owing to the application of the second image, for each row the accuracy of a pixel value is improved. For each column a pixel is duplicated to keep the proportions of the original image, however, it does not improve the quality. The algorithm increases the number of lines of the image to $2n+1$, where n is the number of lines of the original image [6].

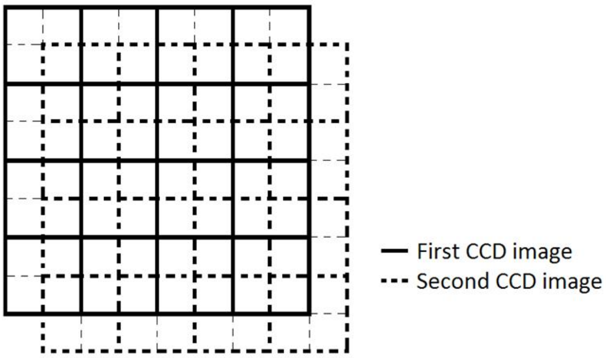

Fig. 2. Spatial distribution of pixels.

This method is robust to noise because it is based on the method of averaging.

The algorithm provides a significant improvement in the image quality. But the resulting image is of much less quality than an image with the same high resolution obtained by direct photographing of the object. The picture looks a little blurry, fuzzy [7].

However, the method can be successfully used, as it has several advantages. To gain a significant improvement, it is sufficient that only two images be used as initial data.

What concerns two other ways of obtaining and processing a series of low-resolution images, they are based on the principle of increasing the resolution of a digital image at a subpixel resolution level using defocusing.

Defocusing the images by applying sensor array's shift in the focal plane was used before the method of an optical mask was introduced.

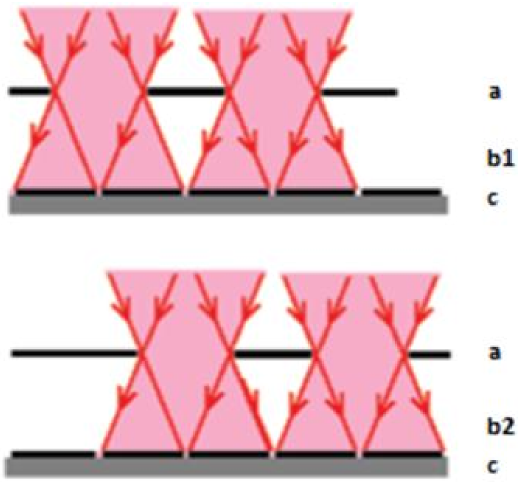

Fig. 3. One-dimensional model of a system that forms two digital images of objects with a sensor array: "a" represents the mask; "c" stands for the sensor array "b1" and "b2" are two different positions of the mask with respect to the sensor array.

The principle of the method is as follows: on a plane close to a focal one, an optical system forms sharp optical images of distant objects. In a plane of maximum focus, an opaque (for optical rays) screen is placed. The screen consists of regularly spaced diaphragms that reflect light and form a defocused image on the CCD-line of the same size. A screen with diaphragms is called a mask. Two positions of the mask are shown in Fiig. 3.

One can see that the mask can move along the direction defined by the intersection of the focal plane and the Fig. 3 plane at a distance equal to the width of the diaphragm. Fig. 3 shows the corresponding position of the sensor array with respect to the mask: b1 and b2.

The sensor signals in each of two positions of the mask give the defocused image of the fragments cut out by means of the mask from the image in the focal plane. The focal plane and the plane of sensor line must be kept at a certain chosen distance lest the out-of-focus images of neighboring fragments should overlap.

The signals from two neighboring sensors that register the defocused image of a remote object may be uniquely associated with "signals" from some virtual sensors whose transverse dimensions are two times less than those of the actual sensors positioned at the location of an appropriate diapfragm. Based on this we can get an image that has a two times better resolution.

However, the images obtained either by applying a subpixel shift or by applying an optical mask need to be processed to eliminate the influence of a pixel aperture.

3. The influence of a pixel aperture and its elimination

From mathematical point of view, the task of improving the resolution of an image can be represented as the inverse problem for an operator equation that is written as follows

$$A * z = u; \quad z \in Z, u \in U \tag{1}$$

where u is a known image of low resolution, z represents an unknown image of high resolution, A stands for the operator which causes decreasing of spatial resolution, Z and U are image spaces which are defined on the meshes with large and small pitches, respectively.

If we consider a typical image sensor, it contains pixels of some shape, usually square. The pixel shape impacts the point spread function (the response of an imaging system to a point source) significally and determines system properties at low frequencies (with respect to a spatial spectrum domain).

Therefore, the model of image forming looks like a pre-application of a low-pass spatial filter with further spatial discretization:

$$g(k, l) = D\{ f(x, y) \oplus h(x, y)\} \tag{2}$$

where $\oplus$ stands for the convolution; $D\{\cdot\}$ is the discretization operator; $f(x, y)$ stands for the initial object whose image one should receive; $h(x, y)$ is the point spread function determined by the aperture of a pixel; x, y are the continuous coordinates; k, l are discrete coordinates; $G(k, l)$ stands for the object image.

That low-pass filter limits the potential of a spatial resolution even if a fine pitch is used for the spatial discretization.

Based on the described image forming model, the relation between the possible spatial resolution and the property of the low-pass filter (caused by the pixel aperture) is evident. That means that two shifted image sensors cannot produce an image with by 2 times improved spatial resolution because of the low-pass filtering.

A large pixel aperture also causes another problem. Typically, scanning the Earth's surface is executed without overlapping instantaneous fields of view. Thus the geometric resolution is equal to the diameter of the field of view. However, when conducting a remote sensing of the Earth, its scanning is done in a way when instant fields of view are overlapped; the scan result resembles an image obtained with an unfocused lens. At a large pixel aperture in that type of survey the image is blurred, unfocused. The way out of this situation consists in reducing the pixel aperture which in turn leads to a decrease in the influence of neighboring parts of the image. A weak point of decreasing the pixel aperture is reduction in sensitivity of photosensitive element of CCD line.

There are two methods to eliminate the undesired effect of a pixel aperture without reducing the aperture itself:

- application of an aprture mask;
- inverse filtering.

Let us discuss the first method. Using an aperture mask, one can reduce the size of the pixel aperture and, as a result, increase the level of ambient light filtration. Such aperture mask is a black sheet made of a certain material with holes of specific diameter over each pixel. Thus, when one chooses optimal diameters of the holes, the pixel aperture effect can be compensated. In such a case the influence of neighboring parts of the image decreases. Each hole in the mask increases the focus depth and shifts the focal plane. That method is somewhat cost effective because it is necessary to manufacture the aperture mask.

The second method - inverse filtering - allows filtering without using the mask but its disadvantage is a significant load on the onboard processor.

Nonlinear filtering methods belong to one of the image processing methods in a frequency domain. The class of the nonlinear digital filters is very broad to be described in general terms. Some of the well-known methods belonging to the family of nonlinear digital filters are the Gold’s algorithm, the Van-Cittert method, the Lucy-Richardson method, etc.

The essence of inverse filtering for restoration (deconvolution) of images may be described in such a way.

In the discrete coordinates (*k, l*) the equation (2) takes the form:

$$g(k, l) = h(k, l) \oplus f(k, l) + n(k, l) , \qquad (3)$$

where $g(k, l)$ stands for a distorted (corrupted) image; $h(k, l)$ stands for a point spread function (a pixel aperture); $f(k, l)$ stands for a high-resolution image of the object photographed; $n(k, l)$ stands for a noise function.

After applying two-dimensional discrete Fourier transform (DFT) we obtain an equation in a spatial frequency domain

$$G(u, v) = H(u, v)F(u, v) + N(u, v) \qquad (5)$$

where G, H, F, N are the spatial spectra of g, h, f, n; u, v are spatial frequencies.

The transfer function of the inverse filter can be defined by

$$W(u, v) = 1/ H(u, v) , \qquad (6)$$

The approximate solution of (5) is:

$$\begin{aligned} \hat{F}(u, v) &= W(u, v)G(u, v) = \\ &= F(u, v) + W(u, v)N(u, v) \end{aligned}, \qquad (7)$$

where $\hat{F}$ stands for a restored image.

The restored image equals the sum of the original image and the noise of monitoring that has passed through the inverse filter. If the noise is absent, the distorted image can be restored to a high degree of accuracy using the inverse filter.

When images are restored by inverse filtering, edge effects appear which manifest themselves in the form of high-power oscillating noise that fully masks the restored image. This is a serious disadvantage of inverse filtering. Another problem is a difficulty in obtaining estimates at the points of a zero row and a zero column. The solution consists in application of Kalman one-dimensional filtering.

4. The proposed approach

The authors propose to combine subpixel image formation with data processing based on inverse filtering using Tikhonov regularization. The subpixel image formation utilizes images obtained by means of shifted image sensor arrays.

As it has been already pointed out, two shifted image sensors cannot produce a high-resolution image because of the low-pass filtering. An improvement can be achieved with applying inverse filtering which compensates the influence caused by a pixel aperture. However, since that inverse filtering works as high-pass filtering and, consequently, increases the noise, we propose to use Tikhonov regularization [3] to reduce the noise.

The image forming process looks like

$$g(k,l) = D\{f(x,y) \oplus h(x,y)\} + \\ +D\{f(x+\Delta x, y+\Delta y) \oplus h(x,y)\} \quad (8)$$

where $\Delta x, \Delta y$ are the spatial subpixel shifts.

In the frequency domain

$$G(u,v) = H(u,v)F(u,v) + H(u,v)F(u+0.5,v) + \\ +H(u,v)F(u,v+0.5) + H(u,v)F(u+0.5,v+0.5) + \\ +N(k,l) \quad (9)$$

After applying the Tikhonov regularization in the frequency domain we receive a restored image:

$$\hat{F}(u,v) = \frac{H^*(u,v)}{|H(u,v|^2 + a\,|W(u,v|^2} G(u,v) \quad (10)$$

where $W(u,v)$ is a regularization function; a is a regularization parameter.

To demonstrate the effetiveness of regularization, we have performed the restoration of an image applying an inverse filter without regularization. The restored image is shown in Fig. 5. We can see that this filter gives a residual error, evev without any noise.

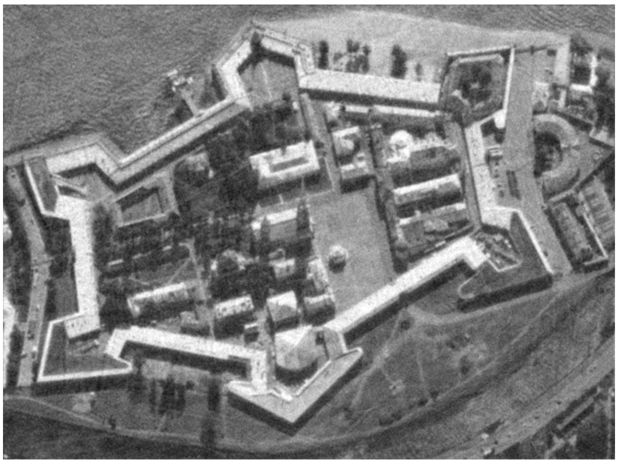

Fig. 5. A restored by inverse filter image with NAE = 9,2% and NMSE = 1,2%.

To show an effect of inverse filtering with Tikhonov regularization a software, implementing this method, has been developed. The software is written in MATLAB environment.

In the first experiment we simulated the influence of a pixel aperture by a blurred image (Fig. 6) that was obtained from a high-resolution original image by means of its convolution with the aperture function of a width of 2 pixels. The restored image received by inverse filtering with Tikhonov regularization is shown in Fig. 7.

In the second experiment, similar to the first one, the aperture function of a width of 8 pixels was used. The blurred image is shown in Fig. 8. The restored image is shown in Fig. 9.

Fig.6. A blurred image with NAE = 6,79%; NMSE = 1%.

Fig.7. A restored image NAE = 0,261%; NMSE = 0,0018%.

Fig.8. A blurred image with NAE = 12,3%; NMSE = 2,6%.

To compare the blurred and restored images their normalized absolute and normalized mean square errors have been calculated as:

$$NAE = \frac{\sum_{k=1}^{M}\sum_{l=1}^{N}\left| f(k,l) - \hat{f}(k,l)\right|}{\sum_{k=1}^{M}\sum_{l=1}^{N}\left| f(k,l)\right|} \quad (11)$$

$$NMSE = \frac{\sum_{k=1}^{M}\sum_{l=1}^{N}(f(k,l) - \hat{f}(k,l))^2}{\sum_{k=1}^{M}\sum_{l=1}^{N}(f(k,l))^2} \quad (12)$$

where *NAE* is the normalized absolute error; *NMSE* is the normalized mean sqare error; *M, N* are the image dimensions in number of pixels; $f(k,l)$ is the original high-resolution image; $\hat{f}(k,l)$ is the blurred or restored image.

Fig.9. A restored image with NAE = 1,9%; NMSE = 0,049%.

Table 1

Calculated errors

Error	Aperture function of 2 pixels		Aperture function of 8 pixels	
	Blurred image Fig. 6	Restored image Fig. 7	Blurred image Fig. 8	Restored image Fig. 9
NAE	0.0679	0.00261	0.123	0,019
NMSE	0.01	0.000018	0.026	0.00049

5. Conclusion

The analysis of image forming process by image sensors array shows that a pixel aperture works as a lowpass spatial filter and degrades spatial resolution even if several image sensors having a subpixel shift are used. The approach proposed uses the constructive placement of image sensors as well as data processing based on the inverse filtering. The technique provides better image restoration in comparison with existing approaches based on the placement of image sensors or masks.

The performed simulation shows that the restored image is almost equal to the original. Note that after being blurred by the 8 pixel width aperture, a normalized absolute error was 0.123, and after recovering the image using the inverse filter with Tikhovnov regularization, the error became 0.019.

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СПЕКТРАЛЬНИЙ ПІДХІД ДО ФОРМУВАННЯ ЗОБРАЖЕНЬ ІЗ СУБПІКСЕЛЬНОЮ РОЗДІЛЬНОЮ ЗДАТНІСТЮ

Іван Прудиус, Віктор Ткаченко, Петро Кондратов, Леонід Лазько, Сергій Фабіровський

Існуючі підходи до підвищення роздільної здатності базуються на конструктивному розміщенні декількох масивів давачів зображення, зміщених на субпіксеельну віддаль [1,2]. Аналіз процесу формування зображення масивом давачів зображення показує, що апертура пікселя працює як просторовий фільтр нижніх частот і погіршує просторову роздільну здатність, навіть якщо використовувати декілька давачів зображення, зміщених на субпіксельну відстань. Пропонований підхід використовує конструктивне розміщення давачів зображення й обробку даних на основі інверсної фільтрації. Це дає переваги в порівнянні з існуючими сучасними підходами, заснованими на розміщенні давачів зображення або використанні маски.

У результаті проведеного дослідження розглянуто проблеми підвищення роздільної здатності систем дистанційного зондування Землі та побудовано модель процессу формування зображень з урахуванням впливу апертури пікселя. До отриманого субпіксельним формуванням зображення застосовано метод інверсної фільтрації з регуляризацією Тихонова, що усуває вплив апертури пікселя.

За допомогою математичного моделювання показано, що вдається отримати відновлене зображення, яке за якістю наближається до тестового зображення з високою роздільною здатністю, що свідчить про ефективне зменшення впливу апертури пікселя. Відновлено зображення, яке було розмите апертурою шириною 8 пікселів; нормована абсолютна похибка розмитого зображення становила 0.123, а після відновлення зображення інверсним фільтром похибка зменшилась до 0.019.

Ivan Prudyus – D.Sc., professor, director of the Institute of Telecommunications, Radioelectronics and Electronic Engineering, Head of the Department of Radioelectronic Devices and Systems, Lviv Polytechnic National University, Ukraine.

His basic directions of scientific research cover antennas, antenna systems, multispectral monitoring systems and complexes.

Viktor Tkachenko - Ph.D., senior scientist, director of the Research Design Institute of Computer and Measurement Technology, Lviv Polytechnic National University, Ukraine.

His main areas of research relate to the development and construction of scanners for remote sensing, processing and transmission of objects' images.

Petro Kondratov - Ph.D., a senior research fellow of the Research Design Institute of Computer and Measurement Technology, Lviv Polytechnic National University, Ukraine.

His main areas of research relate to the development and construction of the systems for monitoring objects in infrared spectrum (infrared camera), processing and transmission of objects' images.

Leonid Lazko - Ph.D., Associate Professor of the Department of Radioelectronic Devices and Systems at Lviv Polytechnic National University, Ukraine.

His main areas of research relate to image and signal processing, remote sensing, sparse antenna arrays.

Serhiy Fabirovskyy – a postgraduate student of the Department of Radioelectronic Devices and Systems, Lviv Polytechnic National University, Ukraine.

His current research interests include image restoration, remote sensing, image processing and data fusion.