КОНФЕРЕНЦІЇ

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Report of Sub-commission 5
HYDROSTATIC/ISOSTATIC EARTH'S REFERENCE MODELS
of IAG Special Commission 1 for the period 1999-2003

by
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According to the program of activity of the Sub-commission 5 of the IAG Special Commission 1, a research was dedicated to the following 5 items.

1. Construction of piecewise radial density profiles. The recovery of density from seismic velocity was considered in (Lelgemann and Marchenko, 2001). The solution is based on three (differential) equations, which restrict the possible solution domain in such a way that a reasonable solution will be the result. The density function can be separated into a hydrostatic main part and in an additional small part due to chemical/phase inhomogeneities or superadiabatic temperatures. The radial (1D) density model was based on a remarkable description of the seismic velocities by generalized Darwin's functions without singularity at the origin. General consideration of such famous laws as Legendre-Laplace, Roche, and Darwin density distributions with respect to Clairaut, Williamson-Adams, and Poisson equations was done in (Marchenko, 1999). The parameterization of the hydrostatic/adiabatic 1D density distribution was considered in the cases of Roche's law (Marchenko, 2000a, 2000b), Gauss' law (Marchenko 2000b, 2000c), and Legendre-Laplace law (Marchenko 2000d). It was found that the density at the origin and seismic data at the boundaries of the Earth's density jumps are sufficient to derive the coefficients of piecewise Gauss model. The recursive formulae were found for the solution of direct and inverse problems. The constructed piecewise models for 1D density distribution are in a good agreement with the PREM-density model. Gauss law was used for the interpretation of radial density distribution of some planets of the solar system (Marchenko, 2000e).

2. Stable determination of parameters of radial density models. This problem is considered in (Lelgemann and Marchenko, 2001), where was noted that gravity is the pivot of used technique to stable recover density from seismic velocity data. In the paper (Marchenko, 2000f) Legendre-Laplace, Roche and Gauss continuous radial density distributions representing the global trend of piecewise density profile were used for the creation of simplest stabilizers for the construction of radial density distribution. On this ground, the operational approach was studied that leads to the determination of density model parameters in the frame of least squares collocation/regularization method. A choice of regularization parameter was considered in (Abrikosov, 2000b; 2001), which is responsible only for accuracy of the initial operator. Numerical tests of this regularization algorithm were fulfilled by (Abrikosov, 2000a) for the Earth's piecewise density models, based on Roche, Gauss, and Legendre-Laplace laws.

3. Reproducing kernels and solution of variational problem. The interpretation of some reproducing kernels by means of simplest singularities of density was considered in (Marchenko and Lelgemann, 2001). It was shown that the set of all suitable kernel functions can be set up as finite sums of two point singularities (pole, dipole) as well as straight line singularities with density functions of the form $\nu = \ell^{A-1}$. A first connection to the Darwin law, as a tool to describe the volume density of radial layered global Earth models, is of interest in view of the use of collocation for an interpretation instead of simple approximation of the disturbing potential. Three different principles (misclosure, quasi-solution and smoothing functional, respectively) were considered in (Abrikosov, 2000b; 2001) for the determination of an upper limit of regularization parameter in the variational problem of data processing. Note here that this fact (upper limit) was omitted in the mentioned papers and the necessary explanation can be found partly in (Marchenko and Tartachynska, 2002; 2003). An optimum point model of the global gravity field was compiled in (Avdev et al., 2000) on the basis of absolute values of gravity data. The field of the optimum point model is close in values to the field of the reference ellipsoid (the normal gravity field), and that of the residual anomalies - to the free-air anomalies. Singular point harmonic functions (radial multipole potentials and the corresponded kernel functions) from 1 to 8 degree were used successfully for the geoid construction from airborne (AGMASKO EU-project) and marine gravimetry data in the Skagerrak (Marchenko et al., 2001; 2002). Two solutions for gravity anomalies inverted from GEOSAT, ERS-2, and TOPEX/POSEIDON altimetry are evaluated by (Marchenko and Tartachynska, 2002; 2003) in the Black sea area by the collocation and regularization approaches based on kernel functions, which are described by singular point harmonic functions as well. Comparison with independent marine gravimetry data indicates a better accordance of the inverted gravity anomalies by means of the regularization method. A good quantitative agreement of gravity anomalies inverted from ERS1&ERS2 altimetry in the Antarctic area with other solutions (Marchenko et. al., 2003) has a certain theoretical meaning: reproducing kernel of point singularities was used successfully in the frame of such instable problem as the inversion of SSH data leading to a more stable process of gravity anomalies recovery by the regularization method.

4. Compressible fluid Earth and gravitational-viscoelastic perturbations. In three papers (Wolf and Kaufmann, 2000; Martinec et al., 2001; Wolf and Li, 2001), the problem of load-induced, gravitational-viscoelastic perturbations of a compressible earth initially in hydrostatic equilibrium is considered. Whereas Wolf and Kaufmann (2000) are concerned with the plane-earth approximation of the problem, Wolf and Li (2001) derive an explicit solution of the perturbation equations for a spherical earth consisting of a compositionally homogeneous mantle surrounding a fluid core. The density stratification is given by Darwin's law, which can be shown to satisfy the field equations governing the initial state. The generalized problem for a spherical earth consisting of compositionally homogeneous shells is solved by Martinec et al. (2001) using propagator matrices. Novel features of the earth models considered in the three papers are the following: (a) the initial density stratification applies to compositionally homogeneous layers or shells and is consistent with the assumption of compressibility (previous analytic solutions of the perturbation equations apply to earth models with layers or shells of homogeneous density), (b) the gravitational-viscoelastic perturbations are assumed to be compressible. The perturbed state is thus consistent with the initial state and no singularities result in the solutions to the perturbation equations, (c) the sole approximation admitted is that the perturbations are constrained by the assumption of local incompressibility. This approximation is almost perfect near the fluid limit of the material but less satisfactory near the elastic limit.

5. Earth's tensor of inertia and the corresponding space/time density distribution. An exact closed solution for the determination of the Earth's principal axes of inertia was derived by (Marchenko and Abrikosov, 1999; 2001) as the corresponding solution of (a) time-independent and (b) time-dependent eigenvalues-eigenvectors problem in the canonical form. In the last case this canonical quadratic form is defined by temporal variations of the harmonic coefficients and always remains finite, even within infinite time interval. A hyperbolic model for harmonic coefficients of the 2nd degree was constructed instead of the standard linear model. A quadratic form described the Earth's dynamical figure was used by (Marchenko and Abrikosov, 2000a; 2000b) for the creation of an auxiliary function represented by the product of a generalized quadratic form and some power function regarding any real degree m of a current radius. 3D density distribution and Darwin's law of (1D) radial density with a certain real m were obtained under the assumption that this auxiliary function can fulfill Poisson's equation. Such distribution may be treated now as 3D model of Darwin's kind with a singularity at the origin. Application of the geodetic fundamental constants together with Lauricella's approach to 1D Darwin's law leads to the global trend of the Earth's radial density without such singularity. ***Mechanical and geometrical parameters of the Earth*** associated with the degree 2 coefficients of the geopotential and the Earth's inertia tensor were estimated by (Marchenko and Schwintzer, 2002; 2003) from the least-squares adjustment of gravitational harmonic coefficients of second degree of four global gravity field models (JGM-3; EGM96; GRIM5-S1; GRIM5-S1CH1) and six different values for the dynamical ellipticity (Williams, 1994; Souchay & Kinoshita, 1996; Hartmann et al. 1997; Bretagnon et al. 1998; Roosbeek & Dehant, 1998; Mathews, 2000) all transformed to common value of precession constant. Closed exact expressions for the determination of these parameters including relationships for a rigorous error propagation are developed and based on the exact solution of the eigenvalue-eigenvector problem. These formulae are applied to determine (a) static components and accuracy of the Earth's tensor of inertia at epoch and (b) the variation with time of the Earth's tensor of inertia and its accuracy, based on given information on the secular and periodic variations of the second degree harmonic coefficients. Results of this simultaneous adjustment at epoch (1997) of the mentioned parameters are given in the zero-frequency-tide system (see, also http://people.polynet.lviv.ua/sc5/). ***The evolution with time*** of the dynamic figure of the Earth was found in (Marchenko and Schwintzer, 2002; 2003) from the mean pole path and the observed secular rate of change in the second-degree zonal coefficient J_2. The secular change of the adjusted value of dynamical ellipticity was estimated from the observed secular variation of J_2. 3D mass density distribution corresponded to the time-dependent Earth's inertia tensor is constructed through these adjusted geodetic and astronomical fundamental constants by (Marchenko and Yarema, 2003) applying the well-known solution of the Cartesian moments problem (see, Grafarend et al., 2000) of the mass density distribution given by Prof. Meshcheryakov in 1970-1980 for a static case. The estimation of an influence of uncertainties of principal fundamental constants on accuracy of 3D density distribution of the spherical and ellipsoidal Earth was made on the ground of the developed formulae for the corresponding rigorous error propagation. A solution of the time-dependent Cartesian moments problem of the 3D density distribution together with the mentioned consistent set of the Earth's mechanical and geometrical parameters is applied to determine (a) 3D density variations with time within the spherical Earth (Marchenko, 2003) and (b) 3D density variations with time within the ellipsoidal Earth (Marchenko and Yarema, 2003).

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