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# PROCESSING OF THE LENGTH MEASURING RESULTS DURING COMPARISONS OR CALIBRATIONS THE DISTANCE METERS AND TOTAL STATIONS ON A FIELD COMPARATOR

A method has been developed for the adjustment of the results of measurements of length during calibration of a field comparator for verification (calibration) of distance meters and distance metric parts of total stations. The method of processing the results of comparisons of distance meters to the field comparator using the least squares method (LSM) was also developed on its basis. The additive biases of length measurements by each distance meter are evaluated according to the LSM, as well as the biases that are entered to the results of length measurements by each reflector. Multiplicative degrees of equivalence of the distance meters are also calculated. During calibrations of field comparators, the biases and degrees of equivalence, obtained during the comparisons of distance meters, should be used as corrections. According to the LSM, the uncertainty is evaluated by type A of the value of the length of the field comparator lines, as well as the biases of measurements by distance meters.

*Key words*: measuring standard, comparison, calibration, distance meter, totalstation, field comparator, additive biases, multiplicative degree of equivalence, uncertainty.

## Introduction

The large range of measurements of length (from several meters to several kilometers) by distance meters, that are the measuring standards, complicates the task of their calibration and comparison. Such works are possible only with special measuring standards named by field comparators. Improvement of the effectiveness of calibration and comparison of distance meters is possible to ensure using the universal method for processing measurement results, which makes the task of development of such technique very important.

## *Foreword*

According to International vocabulary of metrology (VIM) [JCGM 200:2012] the “**2.43 metrological traceability to a measurement unit”** is “**metrological traceability** where the reference is the definition of a **measurement unit** through its practical realization”. Geodetic distance meters shall directly and independently implement the definition of the unit of length - meter through the speed of light and measured time [Kostetskaya, 1986]. Realization of the **“Metrological traceability chain”** (2.42 VIM [JCGM 200:2012]) for the distance meters begins with its comparison to field comparator and continues “…through a documented unbroken chain of calibrations, …” (from 2.41 VIM [4]). The important part of the traceability chain are the methods of processing measurement results, when comparisons or calibrations are performed.

Adjustment by least square method of the length measurements, when comparisons or calibrations are performing, is the optimal method of estimation of the comparison reference values of the field comparators line length and parameters of distance meters.

It should be noted that the distance meters, that are the measuring standards, can be either a separate device or be a part of an electronic total station. Hereinafter the term distance meter will be understood as all these devices. In sources the field comparators, that are the measuring standards, are often called geodetic bases, linear polygons, etc. [Braun, et al. 2014; Jokela, et al., 2009, 2010; JRP SIB 60; Lawson, & Henson, 1986; Kravchenko, & Neezhmakov, 2004; Kupko, et al., 2004; Rajshmann, 2010; Trevoho, et al., 2004, 2010; Trevoho, & Tsiupak, 2014]. The authors will use only the term field comparator.

The developed and presented method for processing the measurements can be used to process the results of key, regional, and additional comparisons of distance meters, as well as during calibration of field comparators using distance meters that have been verified. The method is also suitable for processing the results of interlaboratory comparisons of length measurements by distance meters and their calibrations.

The measurement of distance between the distance meter and the reflector should be carried out from the point of intersection of the horizontal and vertical axes of rotation of the distance meter to the point of intersection of the similar axes of the reflector. Instrument manufacturers and repair and service companies are trying to design, manufacture, and adjust the device taking into account the above requirements. However, due to the mechanical errors in the manufacture of distance meters and reflectors, the point of intersection of the axes does not coincide with the point from which the distance meter measures the distance [Kostetskaya, 1986]. The distance meter has a certain internal electron-optical delay of its work, which should, after adjustment, ensure the coincidence of the vertical axis of its rotation with the zero of its distance metric scale. The delay cannot be set unambiguously and can change with time, which causes the need for adjusting, and then calibrating and comparing.

This delay for the distance meter is adjusted to a specific reflector, but another reflector may have different geometrical dimensions, and the delay on it may be different. This difference of delays, which, theoretically, is necessary to know individually for each reflector, the authors of the article interpret as a bias of the reflector, which is to be determined during calibration and comparisons.

### *Analysis of the sources*

Usually, research of the measurements accuracy of the distance meters are fulfilled using specially built measuring standards – the field comparators. Such comparators are built in many countries of the world. From comparators placed in Europe of note are the field comparators in Nummela, Finland [Jokela, et al., 2009, 2010], Braunschweig, Germany [Pollinger, et al., 2012], Krissern, Switzerland [Rajshmann, 2010]. Austria, Czech Republic, Estonia, Lithuania, Belarus and others have similar field comparators. Ukraine has field comparators in Kharkov [Kupko, et al., 2004], Kyiv [Samoilenko, & Berezan, 2008], Lviv [Trevoho, et al., 2004, 2010; Trevoho, & Tsiupak, 2014] and in others cities.

Creation and calibration of the field comparators are described in [Braun, et al. 2014; Pollinger, et al., 2012; Jokela, et al., 2009, 2010; Kupko, et al., 2004; Lawson, & Henson, 1986; Kravchenko, & Neezhmakov, 2004; Kupko, et al., 2004; Trevoho, et al., 2004, 2010; Trevoho, & Tsiupak, 2014]. The main method for processing of the length measurement results of the field comparator is the least squares method (LSM). This method is described, for example in [ISO 17123-4:2012]. In [Pollinger, et al., 2012] it is described using a similar processing method of the length measurements of the field comparator lines. In [Jokela, et al., 2010] the field comparator line lengths were received by adjustment of the geodetic network obtained from measurements using the field comparator. The disadvantage of methods described in [Pollinger, et al., 2012] and [Jokela, et al., 2010] is that according to adjustment length of the lines between the points or point coordinates of the field comparator is determined without evaluation of the possible systematic errors of the distance meters (additive constants [Pollinger et al., 2012] or zero-point corrections [ISO 17123-4:2012]. In [Pollinger, et al., 2012 the additive constant of pair distance meter reflectors was determined separately by combination of the measurements using the method designed by Rueger J. In [ISO 17123-4:2012] by adjustment of the lines length measurements results, the zero-point correction was determined only for distance meter.

In this article we are proposing the adjustment of the lines length measurement using the field comparator by LSM to obtaining the additive constant and multiplicative degree of equivalence simultaneously of the several distance meters and reflectors.

From analyzed sources we can make a conclusion, that in the world basically the linear type field comparators are created. The advantage of the linear type field comparators is undoubtedly the possibility of combining measurements from different points of the field comparator to all the others in order to improve the accuracy of determining the lengths of its lines. The use of such a combination is the basis of the method proposed in this work.

By combining it is meant the well-known method for determining the systematic error (bias or correction) of the distance meter-reflector set (5.4 and fig. 2 [ISO 17123-4:2012]). According to this method, three tripods with tribraches 1, 2 and 3 are installed in the target area. The distance meter measures three lengths of lines $x^{12}$, $x^{23}$ and $x^{13}$. If the measurements were made in the opposite direction, then the averages of the two are calculated. The systematic error (bias) of the distance meter-reflector set is then equal to $d = x^{12} + x^{23} - x^{13}$.

In [ISO 17123-4:2012] the length measurements results are adjusted by least square method too, however the equations represented in [ISO 17123-4:2012], which connect measured values and evaluated parameters of the field comparator, differ from measurement models (1) and (18) proposed in this article. Both new measurement models comprise additive bias of the distance meter $d$ (in [ISO 17123-4:2012] it is named zero-point correction and is denoted $\delta$ ). But, model (1) differs from [ISO 17123-4:2012] by adding the biases for reflectors $p_m$. In case of comparison of some distance meters on the field comparator the measurement models are as in (18). It means that the multiplicative degree of equivalence for each distance meter's $b_j$ are added to (1).

New models are better to use for programing the common case of the length measurements adjustment analogically to geodetic trilateration network where the evaluationed parameters are the coordinates of the points, but not their increments.

The aim of the method of measurement adjustment, described in [ISO 17123-4:2012], is the estimation of the systematic error of the distance meter measurement results, but not formation of the traceability chain in the long length measuring sphere, however it is possible after the proposed improvement. Advantage of the proposed measurement models are presented in the test example, the data for which were taken from [ISO 17123-4:2012]. Results of the adjustment by model (1) and (18) are presented and commented in the chapter 6.

Adjustment of the results of measurements during comparisons using the least squares method isdescribed in [Nielsen, 2000; 2003; Kuzmenko, & Samoilenko, 2018] and is the mathematical basis.

## Identifying the problem

The aim of the work is to decide the problem of the development of a method for adjustment of the results of measurements performed by distance meters at the field comparator during their comparison or calibration of the field comparator.

According to the method proposed by the authors, during the key comparisons, the additive biases that are entered into measured distances, separately, distance meters and reflectors, multiplicative degrees of equivalence of distance meters and reference values of the length of the field comparator lines from joint integrated processing of all measurement results shall be determined by all distance meters by the least squares method [Kuzmenko, & Samoilenko, 2018].

In the length traceability chain, the additive biases and multiplicative degrees of equivalence of distance meters are then used as initial (constant) during calibrations of field lower-level comparators, as well as during regional and additional comparisons.

At the field comparator, which took part in various comparisons, or was calibrated, the additive biases of reflectors and reference values of lines lengths are used during the calibrations of distance meters to determine their additive biases and multiplicative degrees of equivalence.

## Main project scope

### *1. Method of measurements by distance meters at field comparator*

Let us consider the method of measuring by distance meters during their comparison at a linear type field comparator. It consists of several reinforced concrete pillars – measurement points laid in the range (see Fig. 1) and Fig. 3 from [ISO 17123-4:2012].

Among the requirements that must be met by the field comparator, the following ones shall be noted. The recommended total length of the field comparator would be at least one kilometer. The construction of the top part of the pillars should ensure the forced centering of distance meters and reflectors. Direct visibility should be provided between all points of the field comparator. The fulfillment of the last condition allows to measure the distances between the columns of the field comparator in all combinations, that is, from each column to each other.

For the optimal scheme of measurements, the scheme in Fig. 4 from [ISO 17123-4:2012] is accepted. According to it the distant measurements are performed from comparator point with number zero to all other points, from points with number one to all other points greater than one, from points with number two to all other points greater than two, and so on. It is possible to not measure all points or to measure each distance both in the forward direction and in the opposite direction.

According to the authors' experience in creating high precision engineering and geodetic networks [Samoilenko, et al., 2008], during performing distance measurements, it is not recommended to enter the meteorological parameters of the atmosphere, temperature, pressure, and air humidity into the measuring standard if it is a modern electronic total station.

It is better to fix or synchronize the time of measurement of distances and meteorological parameters by automatic weather stations, and to enter appropriate corrections in the distances when processing, taking into account the readings of all weather devices and the heights of the comparator points relative to weather stations. This method is described in detail in [Samoilenko, & Berezan, 2008].

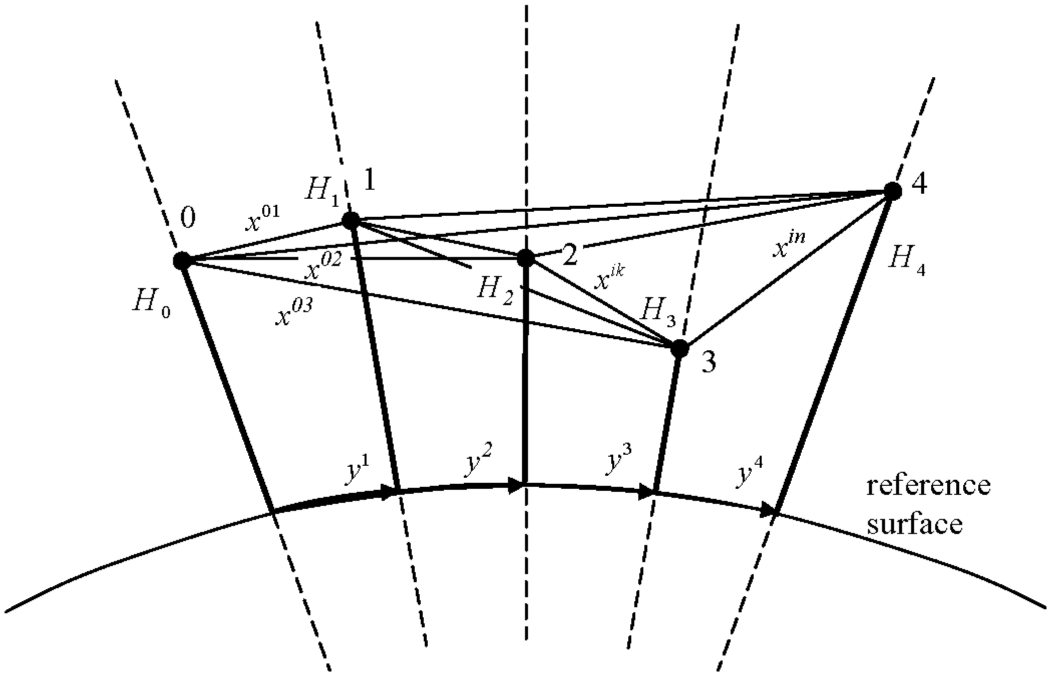


*Fig. 1. Principle scheme of measurements on a field comparator*

There should be as many tribraches as there are field comparator points. They should be installed the same at the same points for the entire calibration or comparison time.

It is important that distance meters and reflectors, between measurements at different points, rotate exclusively around the vertical axis. If the pipes of the total stations and reflectors rotated around the horizontal axis (through the zenith), then the necessary biases cannot be calculated.

During processing the results of all measurements by all distance meters, it is necessary to perform the reduction of all measured distances to the surface ratio (Fig. 1). The sphere of the radius which corresponds to the average radius of curvature of the Earth's ellipsoid at the location of the field comparator is recommended to be chosen as the form of surface ratio. Reduced-to-surface distance measurement ratios can be compared with each other and used for further joint processing.

After reduction, the actual increments of the length of the arcs $x_m^{ik}$ on the spherical surface are subject to further processing. The determined parameters of the field comparator during comparisons or its calibration is the length of the arcs on the sphere $y^i$ from the starting point of the field comparator to each next one (Fig. 1).

## *2. Adjustment of measurement results when calibrating a field comparator with a single distance meter*

During calibration of a field comparator using a distance meter, the measurement of distances between the columns of the field comparator shall be carried out only by one distance meter. For it, the measurement model (2.48, VIM [4]) of distance metric measurements has the following form:

$$x_m^{ik} = y_m^k - y^i + p_m - d \,, \qquad (1)$$

where $i = 0...n$ is the number of the field comparator point on which the distance meter shall be installed ($i = 1...n$ is line number of field comparator, the length of which is determined); $k = 0...n$ is the number of the field comparator point on which the reflector shall be installed; $m = 1...M$ is number of the reflector; $x_m^{ik}$ is increment of length between points $i$ and $k$ of the field comparator, obtained by the distance measured by the distance meter per reflector with the number $m$, after its reduction to the surface ratio (in the equation it is taken as the measured value); $y^i$, $y_m^k$ are evaluated unknown values of length of lines of the field comparator from the starting point each have the number zero (Fig. 1) with numbers $i$ and $k$, at $i \neq k$, between which the measurement was performed; $d$ is evaluated unknown additive bias entered by the distance meter to length measurements; $p_m$ is evaluated unknown additive bias entered by reflector with number $m$ to length measurements;

Equation (1) is transformed into the equation of corrections (2):

$$v_{x_m^{ik}} = \delta y_m^k - \delta y^i + p_m - d + l_{x_m^{ik}} \,, \qquad (2)$$

where $v_{x_m^{ik}}$ is correction to the measured value of the increments of the line length of the field comparator; $\delta y_m^k$ and $\delta y^i$ are corrections to approximate values of the line length of the field comparator; $l_{x_m^{ik}} = y_m^{0k} - y^{0i} - x_m^{ik}$ is constant term of correction equation; $y^{0i}$, $y_m^{0k}$ are initial (approximate) values of the line length of field comparator with numbers $i$ and $k$.

In [Cox, 2002], the value $d$ is called the degree of equivalence. In our case, it is better to use the term – additive bias of measurements by the distance meter $d$, since the beginning of the scale should coincide with the axis of rotation of the distance meter and we estimated the actual deviation of the zero of scale of the distance meter from the axis of rotation. The value $d$ should be determined during key or regional comparisons or during calibration at the field comparator. It should be used as the correction when calculating the measured distances in operation.

The values of the length of the field comparator lines are, in fact, the one-dimensional coordinates of its points in the conventional one-dimensional coordinate system.

The origin of the coordinate system is convenient to place at the starting point of the field comparator, which is assigned a zero number. Then, for the starting point with the zero number the equation of the corrections (2) is the following:

$$v_{x_m^{ik}} = \delta y_m^k - \delta y^0 + p_m - d + l_{x_m^{ik}} \,, \qquad (3)$$

The coordinate of this point, in the conditional coordinate system, is zero. During processing, this coordinate does not receive corrections (does not change), only one-dimensional coordinates of all other points receive corrections. In other words, $\delta y^0$=0. This condition protects the matrix of normal equations (7), (14) and (22) from degeneration.

In other words, for all measured increments of the length of the lines measured from the starting point with the zero number, equation (3) takes the form:

$$v_{x_m^{ik}} = \delta y_m^k + p_m - d + l_{x_m^{ik}} \,. \qquad (4)$$

For all increments of the length of the lines measured from any point to point with the number zero, the equation (3) takes the form:

$$v_{x_m^{ik}} = -\delta y^i + p_m - d + l_{x_m^{ik}} \,, \qquad (5)$$

where in the case of $p_m$, this unknown bias of the measured length is entered by the reflector with the number $m$, which was set at point with number zero.

## *3. The structure of the equations of corrections and normal equations*

The equation of corrections (3), (4) and (5) in the general matrix form is the following:

$$V_x = A_y \cdot \delta y + A_p \cdot p + A_d \cdot d + l \,, \qquad (6)$$

where $A_y$ is the matrix of coefficients of linear equations of corrections at unknown values of the measured length of lines of the field comparator; $A_d$ is the matrix of coefficients of linear equations of corrections at unknown bias, which is entered as the measured length by the distance meter, with the help of which the comparator is calibrated; $A_p$ is the matrix of coefficients of linear equations of corrections at biases, which are entered as the measured length by the reflectors from the field comparator set; $\delta y$ is column vector of corrections to approximate values of the length of the lines of the field comparator; $d$ is bias, which is entered as the measured length by the distance meter; $p$ is column vector of biases, which are entered into the measured length by the reflectors from the field comparator set; $l$ is column vector of constant terms of linear equation; $V_x$ is diagonal matrix of corrections to the measured values of the increments of the lengths of the field comparator lines.

In the general case, the matrix of coefficients and the vector of constant terms of normal equations are the following:

$$N = A^T WA = \begin{bmatrix} A_y^T WA_y & A_y^T WA_p & A_y^T WA_d \\ A_p^T WA_y & A_p^T WA_p & A_p^T WA_d \\ A_d^T WA_y & A_d^T WA_p & A_d^T WA_d \end{bmatrix}; \qquad (7)$$

$$L = A^T Wl = \begin{bmatrix} A_y^T Wl \\ A_p^T Wl \\ A_d^T Wl \end{bmatrix}.$$

where $W$ is the weight matrix of measured values of increments of lengths of lines of the field comparator.

$$W = \begin{bmatrix} W_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & W_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & W_i & 0 & 0 \\ 0 & 0 & 0 & 0 & ... & 0 \\ 0 & 0 & 0 & 0 & 0 & W_n \end{bmatrix}, \qquad (8)$$

where $W_0 = \begin{bmatrix} w_{x_m^{01}} & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & w_{x_m^{0k}} \end{bmatrix}$;

$$W_1 = \begin{bmatrix} w_{x_m^{11}} & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & w_{x_m^{1k}} \end{bmatrix}; \; W_i = \begin{bmatrix} w_{x_m^{i1}} & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & w_{x_m^{ik}} \end{bmatrix};$$

$$W_n = \begin{bmatrix} w_{x_m^{n1}} & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & w_{x_m^{nk}} \end{bmatrix},$$

where $w_{x_m^{ik}} = \dfrac{u^2(x^0)}{u^2(x_m^{ik})}$, where $u(x^0)$ is measurement uncertainty for which the weight of the measurement is taken as a unit; $u(x_m^{ik}) = u(d) + u(b) \cdot x_m^{ik}$ is uncertainty of measurements of length increments; $u(d)$ is the uncertainty of the additive bias of length measurements by the distance meter obtained from comparisons; $u(b)$ is the uncertainty of the multiplicative degree of equivalence of length measurements by distance meter obtained from comparisons.

For each point of the field comparator, on which the distance meter is installed and the measurements are made, the blocks of the matrix of coefficients of the correction equations will be the following:

$$A_{y^0} = \begin{bmatrix} 1 & 0 & 0 & ... & 0 \\ 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1 \end{bmatrix} = E; \; A_{y^1} = \begin{bmatrix} -1 & 0 & 0 & ... & 0 \\ -1 & 1 & 0 & ... & 0 \\ -1 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... \\ -1 & 0 & 0 & ... & 1 \end{bmatrix};$$

$$A_{y^2} = \begin{bmatrix} 1 & -1 & 0 & ... & 0 \\ 0 & -1 & 0 & ... & 0 \\ 0 & -1 & 1 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & -1 & 0 & ... & 1 \end{bmatrix}; \; A_{y^n} = \begin{bmatrix} 1 & 0 & 0 & ... & -1 \\ 0 & 1 & 0 & ... & -1 \\ 0 & 0 & 1 & ... & -1 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & -1 \end{bmatrix}. \qquad (9)$$

If the reflectors are one less than the field comparator points, the nearest reflector will change places with a distance meter, then the block of the matrix of normal equations with unknown biases of reflectors will have the form of a single matrix for each point where the distance meter is installed:

$$A_{p_m}=\begin{bmatrix}1&0&0&...&0\\0&1&0&...&0\\0&0&1&...&0\\...&...&...&...&...\\0&0&0&...&1\end{bmatrix}=E. \qquad (10)$$

If the reflectors are less than definable values of lengths of lines and they were moved to different points of the field comparator while the distance meter was on one point, then the matrix (10) will have a different structure. For example, if there are eight detectable values of length and four reflectors:

$$A_{p_m}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}. \qquad (11)$$

That is, during the measurements from the starting point, the first reflector shall be installed on the first and fifth points, the second one on the second and sixth points, and so on.

The matrix of coefficients with an unknown bias of measurements of distances by the distance meter at each point will be the following:

$$A_d=\begin{bmatrix}-1\\-1\\-1\\...\\-1\end{bmatrix}. \qquad (12)$$

Considering (9), (10) and (12), the whole matrix of normal equations in a block form can be obtained after multiplying the matrices:

$$A^TWA=\begin{bmatrix}E&A^T_{y_1}&A^T_{y_2}&...&A^T_{y_n}\\E&E&E&...&E\\A^T_d&A^T_d&A^T_d&...&A^T_d\end{bmatrix}W\begin{bmatrix}E&E&A_d\\A_{y_1}&E&A_d\\A_{y_2}&E&A_d\\...&...&...\\A_{y_n}&E&A_d\end{bmatrix}. \qquad (13)$$

After multiplying (13), the matrix of coefficients of normal equations and constant terms will be simplified with respect to (7):

$$N=A^TWA=\begin{bmatrix}W_0+\sum_{i=1}^{n}A^T_{y_i}W_iA_{y_i}&W_0+\sum_{i=1}^{n}W_iA_{y_i}&0\\W_0+\sum_{i=1}^{n}W_iA^T_{y_i}&\sum_{i=0}^{n}W_i&\sum_{i=0}^{n}W_iA_d\\0&A^T_d\sum_{i=0}^{n}W_i&\sum_{i=0}^{n}\sum_{k=1}^{n}w_{x_m^{ik}}\end{bmatrix}; \qquad (14)$$

$$L=A^TWl=\begin{bmatrix}W_0l_{x_m^{0k}}+\sum_{i=1}^{n}A^T_{y_i}W_il_{x_m^{ik}}\\\sum_{i=0}^{n}W_il_{x_m^{ik}}\\\sum_{i=0}^{n}\sum_{k=1}^{n}w_{x_m^{ik}}l_{x_m^{ik}}\end{bmatrix}=\begin{bmatrix}L_y\\L_p\\L_d\end{bmatrix}. \qquad (15)$$

In general, normal equations will be:

$$N\cdot\begin{bmatrix}\delta y\\p\\d\end{bmatrix}+L=0. \qquad (16)$$

The above measurement processing method is calculated in case when the task is to check, according to the results of the calibration of the field comparator, the additive bias of the distance meter. But, in this case, the matrix of coefficients of normal equations (13) has an incomplete rank, therefore, to obtain all the unknowns, it is necessary to obtain a pseudoinverse matrix [Lawson, & Henson, 1986] with respect to the matrix of normal equations:

$$\begin{bmatrix}\delta y\\p\\d\end{bmatrix}=-N^+\cdot L=-Q\cdot\begin{bmatrix}L_y\\L_p\\L_d\end{bmatrix}, \qquad (17)$$

where $Q=N^+$ matrix is pseudoinverse to the matrix of normal equations.

An additive bias $d$ and a multiplicative degree of equivalence of measurements by the distance meter $b$ from comparison, must necessarily be used to calculate the corrections to the measured values of the distances before processing during calibration.

Then, in the above equations there will be no unknown $d$. In this case, the matrix of normal equations will have full rank.

### *4. Adjustment of measurement results during comparisons of distance meters at the field comparator*

During the comparisons of distance meters at the field comparator, the processing method is complicated in relation to the calibration method of the comparator. First, each of the distance metric devices has its own additive bias, which must be evaluated by comparison, secondly, the distance meters measure the length within their range with a multiplicative (proportional to the measured distance) systematic error, for reasons that are not analyzed in this study. The estimation of this coefficient of proportionality, obtained from the results of comparisons, is a typical multiplicative degree of equivalence of the standard considered, for example, in [Kuzmenko, & Samoilenko, 2018].

The measurement model (2.48, VIM [JCGM 200:2012]) of distance metric measurements will have a slightly different, complicated form:

$$x_{jm}^{ik} = y_m^k - y_j^i + p_m - d_j + x_{jm}^{ik} \cdot b_j\,, \quad (18)$$

where $i = 0...n$ is the number of the field comparator point on which the distance meter shall be installed ($i = 1...n$ is the number of a line of field comparator, the length of which shall be determined); $k = 0...n$ is the number of the field comparator point on which the reflector shall be installed; $m = 1...M$ is number of reflector; $j = 1...J$ is the number of the distance meter that takes part in the comparisons; $x_{jm}^{ik}$ is the increment of length between points $i$ and $k$ of a field comparator, obtained by distance on the reflector with the number $m$, measured by distance meter with the number $j$, after its reduction to the surface ratio (is the measured value in the equation); $y_j^i$, $y_m^k$ are evaluated unknown values of lengths of the field comparator lines from the starting point with the number zero in each (Fig. 1) with numbers $i$ and $k$, at $i \neq k$, between which the measurements were taken; $d_j$ is evaluated unknown additive bias entered by distance meter with number $j$ as length measurements; $p_m$ is evaluated unknown additive bias entered by reflector with number $m$ as length measurements; $b_j$ is the multiplicative degree of equivalence of the measuring standard – distance meter with the number $j$ (can be interpreted as a difference, in relative measure, between the value of measurement unit that were reproduced by the specific measuring standards with respect to value of measurement Unit averaged (estimated) from the results of the comparisons).

Equation (18) is transformed into the equation of corrections (19):

$$v_{x_{jm}^{ik}} = \delta y_m^k - \delta y_j^i + p_m - d_j + b_j \cdot x_{jm}^{ik} + l_{x_{jm}^{ik}}\,, \quad (19)$$

where $v_{x_{jm}^{ik}}$ is correction to the measured value of the increment of the line length of the field comparator; $\delta y_m^k$, and $\delta y^i$ are corrections to approximate values of the line length of the field comparator taken as reference values; $l_{x_{jm}^{ik}} = y_m^{0k} - y_j^{0i} - x_{jm}^{ik}$ is constant term of correction equation; $y_j^{0i}$, $y_m^{0k}$ are approximate values of the line length of field comparator with numbers $i$ and $k$.

For adjustment of the key comparisons results the following additional equation are added to the system of equations (19):

$$\sum_{j=1}^{k} b_j = 0\,. \quad (20)$$

Adding equation (20) to the system of equations (19) let us average the value of measurement unit of a length reproduced by the specific measuring standards of the participants.

The equations of corrections (19) in the matrix form:

$$V_x = \overline{A}_y \cdot \delta y + \overline{A}_p \cdot p + \overline{A}_d \cdot d + \overline{A}_b \cdot b + l\,. \quad (21)$$

If each distance meter that takes part in the comparisons carries out the measurement at the field comparator using the same program, then the coefficient matrices (21) will consist of the corresponding blocks of equation (6):

$$\overline{A}_y = \begin{bmatrix} A_y \\ A_y \\ \cdots \\ A_y \end{bmatrix}; \overline{A}_p = \begin{bmatrix} A_p \\ A_p \\ \cdots \\ A_p \end{bmatrix}; \overline{A}_d = \begin{bmatrix} A_d & 0 & 0 & 0 \\ 0 & A_d & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & A_d \end{bmatrix};$$

$$\overline{A}_b = \begin{bmatrix} A_b & 0 & 0 & 0 \\ 0 & A_b & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & A_b \end{bmatrix}, \text{ where } A_b = \begin{bmatrix} x_{jm}^{i1} \\ x_{jm}^{i2} \\ \cdots \\ x_{jm}^{in} \end{bmatrix}. \quad (22)$$

Matrix of normal equations and the vector of constant terms in general:

$$N=\begin{bmatrix}\overline{A}_y^T W\overline{A}_y & \overline{A}_y^T W\overline{A}_p & \overline{A}_y^T W\overline{A}_d & \overline{A}_y^T W\overline{A}_b \\ \overline{A}_p^T W\overline{A}_y & \overline{A}_p^T W\overline{A}_p & \overline{A}_p^T W\overline{A}_d & \overline{A}_p^T W\overline{A}_b \\ \overline{A}_d^T W\overline{A}_y & \overline{A}_d^T W\overline{A}_p & \overline{A}_d^T W\overline{A}_d & \overline{A}_d^T W\overline{A}_b \\ \overline{A}_b^T W\overline{A}_y & \overline{A}_b^T W\overline{A}_p & \overline{A}_b^T W\overline{A}_d & \overline{A}_b^T W\overline{A}_b\end{bmatrix}; \quad (23)$$

$$L=\overline{A}^T l=\begin{bmatrix}\overline{A}_y^T Wl \\ \overline{A}_p^T Wl \\ \overline{A}_d^T Wl \\ \overline{A}_b^T Wl\end{bmatrix}.$$

In formulas (23), the weight of measurements can be calculated by the formula similar to formulas (8), with certain refinements:

$u(x_{jm}^{ik})=u(d_j)+u(b_j)\cdot x_{jm}^{ik}$ is uncertainty of measurements of length increments by distance meter with number $j$; $u(d_j)$ is the uncertainty of the additive bias of the length measurement by the distance meter declared by the comparison participant; $u(b_j)$ is the uncertainty of the multiplicative degree of equivalence of length measurement by the distance meter declared by the comparison participant.

As described in Chapter 3, the matrix of coefficients of normal equations (23) has an incomplete rank, therefore, to obtain all the unknowns, it is necessary to obtain a pseudoinverse matrix [Lawson, & Henson, 1986] with respect to the matrix of normal equations by analogy with (17). The rank of the matrix (23) will be less than the number of normal equations to the number of distance meters that participated in the comparisons.

## *5. Uncertainty evaluation measurement of length by A type*

Let us consider the uncertainty evaluation of the length measurement by A type. The uncertainty evaluation of the length measurement by B type is necessary to discuss separately.

Least-squares method of uncertainty estimation by A type is not described in [JCGM 100:2008]. However, in the development of [JCGM 100:2008; ISO 17123-4:2012] and according [JCGM 102:2011] we present evaluations of uncertainty in two stages. At the first stage, this procedure will affect the calibration of the field comparator using a single distance meter (sections 2 and 3). At the second, estimation of the uncertainty of the results of comparisons of distance meter at the field comparator (section 4) is required. The procedure developed for the first stage has important independent significance during the execution of the second one.

During processing the field comparator calibration results, after solving a system of normal equations and finding unknown reference values of the length of lines, additive biases of the distance meter and reflectors, we shall find the value of the standard deviation of the measured value of the length, the weight of which is equal to one by the formula:

$$S_j=\sqrt{\frac{\sum_{i=1}^{n}\sum_{k=1}^{n} w_{x_m^{ik}}\cdot v_{x_m^{ik}}^2}{r}}, \quad (24)$$

where $v_{x_m^{ik}}$ is calculated after solving the normal equations using formulas (2)–(5).

The maximum number of degrees of freedom in the formula (24), provided that the distances in all possible combinations were measured, is calculated by the formula:

$$r=n\cdot(n-1)-M-1. \quad (25)$$

In the formula (23) the subscript $j$ indicating the number of the totalstation is after designation of the standard deviation $S$. At the first stage of processing the results of comparisons, it is possible to use the same methodology used for processing the results of calibration of the field comparator, including the calculation of the standard deviation.

After solving a system of normal equations and finding the unknown reference values of the length of lines, the additive biases of the distance meters and reflectors, as well as the multiplicative degrees of equivalence of distance meters, we shall find the value of the standard deviation of the measured value of the length, the weight of which is equal to one by the formula:

$$S=\sqrt{\frac{\sum_{j=1}^{J}\sum_{i=1}^{n}\sum_{k=1}^{n} w_{x_{jm}^{ik}}\cdot v_{x_{jm}^{ik}}^2}{r}}, \quad (26)$$

where $v_{x_{jm}^{ik}}$ is calculated after solving the normal equations using formulas (19).

The maximum number of degrees of freedom in the formula (26), is calculated by the formula:

$$r = n \cdot (n-1) \cdot J - M - 2 \cdot J\ . \qquad (27)$$

Uncertainties by type A of the field comparators and distance meters in tame of their comparisons and calibrations is calculated using the formulas:

$$u_A(y_i) = S \cdot \sqrt{Q_{y_{ii}}}\ ; u_A(p_m) = S \cdot \sqrt{Q_{p_{mm}}}\ ; \quad (28)$$

$$u_A(d_j) = S \cdot \sqrt{Q_{d_{jj}}}\ ;\ u_A(b_j) = S \cdot \sqrt{Q_{b_{jj}}}\ , \quad (29)$$

where $Q_{y_{ii}}$, $Q_{p_{mm}}$, $Q_{d_{jj}}$ and $Q_{b_{jj}}$ are the diagonal terms of the first, second, third, and fourth diagonal blocks of the matrix of pseudoinverse [Lawson, & Henson, 1986] to the matrix of normal equations (23).

$u_A(y_i)$ is the uncertainty of the reference values of the lengths of the lines of the field comparator; $u_A(p_m)$ is the uncertainty of biases that a reflector makes to measurements of length; $u_A(d_j)$ is the uncertainty of additive measurement biases with distance meters; $u_A(b_j)$ is the uncertainty of the multiplicative degree of equivalence of distance meters.

Evaluating the uncertainty of reproduction of the beginning and scale of the length scale by all distance meters by all points of the scale that took part in the comparisons will be performed through the compilation of weight functions:

$$F_{\bar{d}} = \frac{\sum_{j=1}^{J} d_j}{J}\ ; \qquad F_{\bar{b}} = \frac{\sum_{j=1}^{J} b_j}{J}\ . \qquad (30)$$

Calculate the inverse weights of these functions is carried out using the formulas:

$$Q_{\bar{d}} = f_{\bar{d}} \cdot Q \cdot f_{\bar{d}}^T\ ; \quad Q_{\bar{b}} = f_{\bar{b}} \cdot Q \cdot f_{\bar{b}}^T\ , \qquad (31)$$

where $f_{\bar{d}} = [1/J \ \ ... \ \ 1/J \ \ ... \ \ 1/J]$ is the vector of partial derivatives of the first function (30) according to additive biases of distance meters;

$f_{\bar{b}} = [1/J \ \ ... \ \ 1/J \ \ ... \ \ 1/J]$ is the vector of partial derivatives of the second function (30) according to multiplicative degrees of equivalence.

The uncertainty of the reproduction of the beginning of scale and the value of measurement unit of the length by all distance meters by all points of the scale in which measurements were made during comparisons:

$$u_A(\bar{d}) = S \cdot \sqrt{Q_{\bar{d}}}\ ; \quad u_A(\bar{b}) = S \cdot \sqrt{Q_{\bar{b}}}\ . \qquad (32)$$

Thus, each participant receives the most likely value of the additive bias and multiplicative degree of equivalence of his distance meter and the uncertainty of these values, as well as the uncertainty of the beginning of scale of the length and the value of measurement unit for comparisons in general.

### *6. Example of the length measurement results adjustment that was performed by the field comparator*

In the Tables 1 and 2 are presented the test example of the adjustment of the line length measurement results of the field comparator, when it calibrated, for tasting data from [ISO 17123-4:2012] by methods that is presented in Sections 2, 3 and 4 of this article. For this comparator it is necessary to obtain the length of the six lines from points with number 0 (zero) to points that were denoted from 1 to 6. This length of line extend from $y^1$ to $y^6$. On the field comparator all 21 length increments $x_m^{ik}$ were measured between points in the one direction. In the second column of the Table 1 the measuring values of increment lengths of field comparator lines are presented. For the test of adjustment all weights of the measurement results were equal to unit as in [ISO 17123-4:2012].

To demonstrate of the adjustment effect for the same data three measurement models were used. The first model is maximally simplified toward the measurement model (1):

$$x_m^{ik} = y_m^k - y^i\ . \qquad (33)$$

In this model it is absent of all biases of the distance meter measurement. From adjustment only six lines length quantity of the field comparator are determined (columns 3 – 8 in Table 1).

The second model includes the bias that consists of the sum of systematic errors of the distance meter and reflectors (column 9 in Tables 1 and 2):

$$x_m^{ik} = y_m^k - y^i - d\ . \qquad (34)$$

As a result of adjustment very substantive bias was obtained: $d$ = -1.3 mm with A type uncertainty $u_A(d)$ = 1.44 mm. These results exactly correspond to results of adjustment from [ISO 17123-4:2012] in chapter B.3.

The third model of measurement corresponds to formula (1). For two reflectors with numbers 1 and 2 the measurement model (1) is the following:

$$x_1^{ik} = y_1^k - y^i - d + p_1; \qquad (35)$$

$$x_2^{ik+1} = y_2^{k+1} - y^i - d + p_2;$$

$$\ldots.$$

Coefficients of the correction equations and normal equations, that correspond to the reflectors with number 1 and 2 from equations (35), are given in columns 10 and 11 of Table 1.

Adjustment by the third model divided the bias, that was obtained by second model, between distance meter and the two reflectors: $d$ = 0.6 mm; $p_1$ = -2.7 mm; $p_2$ = =2.1 mm.

In this case the effect from this division is very meaningful. Inclusion of these biases to the measurement model allowed to reduce the standard deviation of the length measurement from $S$ = 3.23 mm to $S$ = 1.67 mm.

*Table 1*

**Measuring values of the field comparator line lengths, the coefficients and the constant terms of the linear equation of corrections, the corrections $v_{x_m^{ik}}$ to the measured value of the length $x_m^{ik}$ of the field comparator, number of degrees of freedom and standard deviations of measurement for measurement models 1, 2, 3 and 4**

| Number of comp-line | Value of the line length, m | Coefficients of the equation of corrections for: measurement model 3 | | | | | | | | | The constant terms, mm | Corrections to the measured value of the length for measurement model, mm | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | measurement model 2 | | | | | | | | | | | | | |
| | | measurement model 1 | | | | | | | | | | 1 | 2 | 3 | 4 |
| $ik$ | $x_m^{ik}$ | $y^1$ | $y^2$ | $y^3$ | $y^4$ | $y^5$ | $y^6$ | $d_1$ | $p_1$ | $p_2$ | $l_{x_m^{ik}}$ | $v_{x_m^{ik}}$ | $v_{x_m^{ik}}$ | $v_{x_m^{ik}}$ | $v_{x_m^{ik}}$ |
| 01 | 50.801 | 1 | | | | | | -1 | 1 | | 0.0 | 3.9 | 2.9 | 0.1 | 0.1 |
| 02 | 162.806 | | 1 | | | | | -1 | 1 | | 0.0 | 2.9 | 2.3 | 0.1 | 0.1 |
| 03 | 335.904 | | | 1 | | | | -1 | | 1 | 0.0 | -1.3 | -1.5 | 1.0 | 1.0 |
| 04 | 478.407 | | | | 1 | | | -1 | | 1 | 0.0 | -6.0 | -5.8 | -3.5 | -3.5 |
| 05 | 559.810 | | | | | 1 | | -1 | | 1 | 0.0 | -1.6 | -1.0 | 1.9 | 1.9 |
| 06 | 580.098 | | | | | | 1 | -1 | 1 | | 0.0 | 2.1 | 3.1 | 0.4 | 0.4 |
| 12 | 112.007 | -1 | 1 | | | | | -1 | | 1 | -2.0 | -3.0 | -3.9 | -0.6 | -0.6 |
| 13 | 285.096 | -1 | | 1 | | | | -1 | 1 | | 7.0 | 1.9 | 1.3 | -0.3 | -0.3 |
| 14 | 427.594 | -1 | | | 1 | | | -1 | 1 | | 12.0 | 2.1 | 2.0 | 0.3 | 0.3 |
| 15 | 509.004 | -1 | | | | 1 | | -1 | 1 | | 5.0 | -0.4 | -0.2 | -1.3 | -1.3 |
| 16 | 529.292 | -1 | | | | | 1 | -1 | 1 | | 5.0 | 3.3 | 3.8 | 2.0 | 2.0 |
| 23 | 173.091 | | -1 | 1 | | | | -1 | 1 | | 7.0 | 2.9 | 1.9 | -0.2 | -0.2 |
| 24 | 315.592 | | -1 | | 1 | | | -1 | | 1 | 9.0 | 0.1 | -0.4 | 2.1 | 2.1 |
| 25 | 396.999 | | -1 | | | 1 | | -1 | 1 | | 5.0 | 0.6 | 0.4 | -1.3 | -1.3 |
| 26 | 417.295 | | -1 | | | | 1 | -1 | | 1 | -3.0 | -3.7 | -3.5 | -1.2 | -1.2 |
| 34 | 142.494 | | | -1 | 1 | | | -1 | 1 | | 9.0 | 4.3 | 3.4 | 1.2 | 1.2 |
| 35 | 223.904 | | | -1 | | 1 | | -1 | 1 | | 2.0 | 1.7 | 1.2 | -0.4 | -0.4 |
| 36 | 244.200 | | | -1 | | | 1 | -1 | | 1 | -6.0 | -2.6 | -2.8 | -0.3 | -0.3 |
| 45 | 81.409 | | | | -1 | 1 | | -1 | | 1 | -6.0 | -1.6 | -2.5 | 0.8 | 0.8 |
| 46 | 101.697 | | | | -1 | | 1 | -1 | 1 | | -6.0 | 2.1 | 1.6 | -0.7 | -0.7 |
| 56 | 20.293 | | | | | -1 | 1 | -1 | | 1 | -5.0 | -1.3 | -2.2 | -0.3 | -0.3 |
| The sum of the constant terms and corrections $\sum l_{x_{jm}^{ik}}$ and $\sum v_{x_{jm}^{ik}}$, mm | | | | | | | | | | | 33.0 | 6.4 | 0.0 | 0.0 | 0.0 |
| The sum of the squares of the constant terms and corrections $\sum l^2_{x_{jm}^{ik}}$ and $\sum v^2_{x_{jm}^{ik}}$, mm | | | | | | | | | | | 629.0 | 154.7 | 146.4 | 33.6 | 100.8 |
| Number of the degrees of freedom | | | | | | | | | | | | 15 | 14 | 12 | 50 |
| Standard deviations $S$, mm | | | | | | | | | | | | 3.21 | 3.23 | 1.67 | 1.42 |

*Table 2*

**Corrections to initial (approximate) values of the line lengths and their uncertainties by type A**

| Initial (approximate) values | | Measurement model 1 | | Measurement model 2 | | Measurement model 3 | | Measurement model 4 | |
|---|---|---|---|---|---|---|---|---|---|
| | | Correc-tions | Uncer-tainty | Correc-tions | Uncer-tainty | Correc-tions | Uncer-tainty | Correc-tions | Uncer-tainty |
| $y^1$, mm | 50801 | 3.9 | 1.7 | 4.2 | 1.8 | 3.4 | 0.9 | 4.0 | 0.46 |
| $y^2$, mm | 162806 | 2.9 | 1.7 | 3.6 | 1.9 | 3.4 | 1.0 | 5.0 | 0.49 |
| $y^3$, mm | 335904 | -1.3 | 1.7 | -0.2 | 2.1 | -0.5 | 1.1 | 2.9 | 0.56 |
| $y^4$, mm | 478407 | -6.0 | 1.7 | -4.5 | 2.4 | -4.9 | 1.2 | -0.1 | 0.65 |
| $y^5$, mm | 559810 | -1.6 | 1.7 | 0.3 | 2.7 | 0.5 | 1.4 | 6.1 | 0.73 |
| $y^6$, mm | 580098 | 2.1 | 1.7 | 4.3 | 3.0 | 3.8 | 1.6 | 9.6 | 0.82 |
| $d_1$, mm | 0 | – | – | 1.3 | 1.4 | 0.6 | 0.5 | 0.4 | 0.51 |
| $d_2$, mm | 0 | – | – | – | – | – | – | 0.4 | 0.51 |
| $d_3$, mm | 0 | – | – | – | – | – | – | 0.4 | 0.51 |
| $p_1$, mm | 0 | – | – | – | – | -2.7 | 0.4 | -3.0 | 0.28 |
| $p_2$, mm | 0 | – | – | – | – | 2.1 | 0.5 | 1.8 | 0.30 |
| $b_1$, ppm | 0 | – | – | – | – | – | – | -10 | 1.5 |
| $b_2$, ppm | 0 | – | – | – | – | – | – | -20 | 1.5 |
| $b_3$, ppm | 0 | – | – | – | – | – | – | 30 | 1.5 |

The fourth model of measurement corresponds to the formulas (18)–(23) and models the comparison of the three distance meters on the same field comparator. The measurements were fulfilled to the two same reflectors with number 1 and 2. In Table 1 all coefficients of the correction equations for model 4 are not given because Table 1 must be done widely and repeated three times. For calculation of the multiplicative degrees of equivalence of the distance meters the lines length is used from second column of Table 1.

In order to verify the correctness of calculation of the multiplicative degrees of equivalence of the distance meters it was accepted that distance meters fulfilled the measurements with multiplicative systematic errors $b_1^{\text{mod}}$ = 0 ppm, $b_2^{\text{mod}}$ = -10 ppm, $b_3^{\text{mod}}$ = 40 ppm. In the real measurements these errors are not known. Other errors relative the model 3 were not modelled. Due to this, the corrections to results of the length measurements in model 4 accurately correspond to corrections from model 3 for all three distance meters (the last two columns of the Table 1).

The corrections to initial (approximate) values of the lines length and the distance meters parameters from adjustment results for model 4 and their standard uncertainties by type A are presented in the last two columns of Table 2. Adjusted values of the multiplicative degrees of equivalence of the distance meters from the Table 2 are equal to $b_1$ = -10 ppm, $b_2$ = -20 ppm, $b_3$ = 30 ppm, that is, the condition (20) is executed after adjustment. The difference between corrections to initial values of the lines' length of the field comparator for models 3 and 4 from Table 2 is 10 ppm. So, the use of the measurement model (18) is averaging the measurement unit of length by the results of comparison.

At adjustment for measurement models 1 and 2 the inverse of the matrixes of normal equations was performed, but for adjustment measurement model 3 and 4 the pseudoinversion was performed.

For practice use is necessary to calculated the constant corrections to complex of the distance meter and each reflector, for example, $c_1 = d + p_1 =$ = -3.3 mm and $c_2 = d + p_2$ = 1,5 mm.

## Conclusions

1. When applying the proposed method of adjustment under LSM, using the measurement results by distance meters, during their comparisons at the field comparator allowed assessing the

additive bias separately for distance meters and reflectors as well as the multiplicative degree of equivalence for distance meters (distance meters parameters) and their uncertainty by type A.

2. The adjusted parameters of the distance meters can be used for creation a reasonable traceability scheme of the length measurement results. The distance meters parameters that were estimated from key comparison are used in calculating the corrections when performing regional and additional comparisons and/or calibrations of both field comparators and distance meters of a lower degree in the scheme.

3. During adjustment of the results of key comparisons arose the problem of an incomplete rank of the matrix of coefficients of normal equations. The solution is proposed for use of the pseudoinverse method of the matrix of normal equations.

4. The multiplicative degree of equivalence of the specific distance meter can be interpreted as a difference, in relative measure, between the value of the measurement unit that was reproduced by the specific measuring standards with respect to the value of the measurement unit averaged (estimated) from the results of the comparisons. The uncertainty of reproducing the value of the measurement unit by all distance meters at all measurement points in the field comparator range according to the results of comparisons is also estimated.

5. The key comparison reference values for the lines length of the field comparator are not distorted, in the range of estimated uncertainty, by the distance meter's measurements systematic errors so how the multiplicative degree of equivalence and the additive bias, separately for the distance meters and the reflectors, are assessing from adjustment.

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## ОПРАЦЮВАННЯ РЕЗУЛЬТАТІВ ВИМІРЮВАНЬ ДОВЖИНИ ДЛЯ ЗВІРЕНЬ АБО КАЛІБРУВАНЬ ВІДДАЛЕМІРІВ І ТАХЕОМЕТРІВ НА ПОЛЬОВОМУ КОМПАРАТОРІ

Розроблено методику зрівнювання результатів вимірювань довжини під час калібрування польового компаратора для повірки (калібрування) віддалемірів та віддалемірної частини тахеометрів. На її основі також створено методику опрацювання результатів звірень еталонних віддалемірів на польовому компараторі за методом найменших квадратів (МНК). За МНК оцінюють адитивні систематичні зміщення вимірювань довжини кожним віддалеміром та систематичні зміщення, які вносить у результати вимірювань довжини кожний відбивач. Також визначають мультиплікативні ступені еквівалентності віддалемірів. Під час калібрувань польових компараторів систематичні зміщення та ступені еквівалентності, одержані під час звірень віддалемірів, потрібно використовувати як поправлення. За МНК оцінюють невизначеність за типом А значень довжини ліній польового компаратора, а також систематичних зміщень вимірювань віддалемірами.

*Ключові слова*: еталон, звірення, калібрування, віддалемір, тахеометр, польовий компаратор, адитивні зміщення, мультиплікативні ступені еквівалентності, невизначеність.

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