ADVANCES IN CYBER-PHYSICAL SYSTEMS
Vol. 4, No. 2, 2019

IDENTIFICATION OF THE DEFECT IN THE ELASTIC LAYER BY SOUNDING OF THE NORMAL SH-WAVE

Myron Voytko, Yaroslav Kulynych, Mykhaylo Grynenko, and Dozyslav Kuryliak

Karpenko Physico-Mechanical Institute of the NAS of Ukraine, 5, Naukova Str., Lviv, 79060, Ukraine.
Authors' e-mail*: myron.voytko@gmail.com*

Submitted on 01.05.2019

***Abstract*: The Fourier integral transform has been used to reduce the diffraction problem of the normal SH-wave on a semi-infinite rigid inclusion in the elastic layer to the Wiener-Hopf equation. Its solution is obtained by the factorization method. The analytical expressions of the diffracted displacement fields have been represented in any region of interest. The dependences of the scattered field on the parameters of the structure have been given. The properties of identification of the inclusion type defect in the plane layer have been illustrated.**

***Index Terms*: Elastic layer, inclusion, diffraction, normal SH-wave, Wiener-Hopf technique.**

I. INTRODUCTION

The study of the diffraction of elastic waves by defects located in various constructions is important for the development of new intelligent diagnostic technologies that combine the usage of various technical means for collecting and processing of information. For example, they are based on the common use of optical and ultrasonic methods [1-6]. In order to provide the theoretical basis for this technology, the problem of SH-wave diffraction from a finite crack in an elastic layer and on a crack located at the boundary of the junction of a layer with a half-space is solved [7–11] by the Wiener-Hopf technique. In these articles, the cracks were modelled by a finite slit of zero thickness without any stresses on the faces. Within the framework of this model, the resonance vibrations are analysed to obtain the maximum response. The problem of SH-wave scattering from a semi-infinite crack in a plane elastic waveguide is solved by the mode-matching technique [12]; the reflected and transmitted coefficients as functions of frequency were analysed.

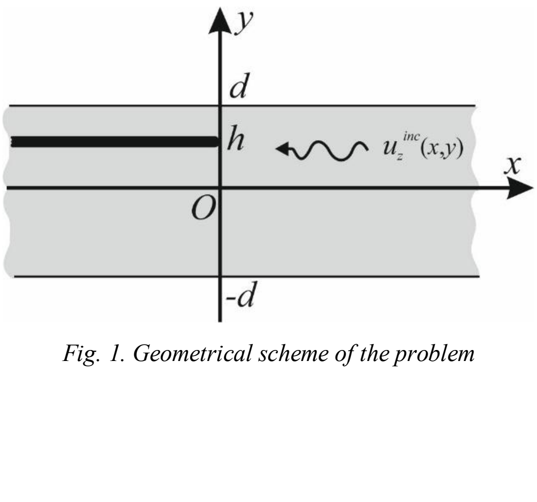


Fig. 1. Geometrical scheme of the problem

The purpose of this paper is to model the displacement field on the layer's surfaces with an internal defect for its further identification. For this purpose, the problem of SH-wave diffraction from the defect located in the elastic layer is solved. The defect is modelled by the rigid semi-infinite inclusion of zero thickness. Time factor is assumed to be $e^{-i\omega t}$ and suppressed through the paper.

II. FORMULATION OF THE PROBLEM

Let us consider the elastic layer in the Cartesian coordinate system xOy as

$$P(y):\{x\in(-\infty,+\infty),y\in(-d,+d),z\in(-\infty,+\infty)\}$$

with the rigid inclusion (Fig. 1):

$$\Gamma(h):\{x\in(-\infty,0),y=h,z\in(-\infty,+\infty)\}.$$

Let the incident normal transverse elastic wave of the horizontal polarization (SH-wave) propagates in the negative direction of the axis x as

$$u_z^{inc}(x,y)=e^{\gamma_j x}\cos(\beta_j y), \qquad (1)$$

where $\beta_j=\dfrac{\pi j}{2d}$, $\gamma_j=\sqrt{\beta_j^2-k^2}$, $j=0,2,...$; k is the wave number, $k=k'+ik''$, $k',k''>0$.

We seek the unknown diffracted field $u=u(x,y)$ from the solution of the mixed boundary value problem for Helmholtz equation

$$\partial_x^2 u(x,y)+\partial_y^2 u(x,y)+k^2 u(x,y)=0, \qquad (2)$$

with the boundary conditions on the defect

$$u^{tot}(x,h\pm 0)=0,\ x\in(-\infty,0) \qquad (3)$$

and on the elastic layer surfaces

$$\partial_y u^{tot}(x,y=\pm d)=0,\ x\in(-\infty,+\infty), \qquad (4)$$

where $u^{tot}=u+u^{inc}$ is the total field. We seek the solution in the class of functions which satisfy the radiation and the edge conditions.

III. SOLUTION OF THE PROBLEM

Let us introduce the Fourier integral transformation of the diffracted field as follows

$$U(\alpha,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}u(x,y)e^{i\alpha x}dx, \qquad (5)$$

where $\alpha=\sigma+i\tau$ (σ, τ are the real values).

Next, we represent the solution of the Equation (2) in the Fourier transform domain in the form as

$$U(\alpha,y)=\begin{cases}A(\alpha)e^{\gamma y}+B(\alpha)e^{-\gamma y},\ h<y<d,\\ C(\alpha)e^{\gamma y}+D(\alpha)e^{-\gamma y},\ -d<y<h.\end{cases} \quad (6)$$

Here, $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, $D(\alpha)$ are unknown functions, that are regular in the strip $\alpha\in\Pi$: $\{-\tau_0<\tau<\tau_0\}$, where $-k''<\tau_0<k''$; in order to satisfy the radiation condition at infinity, we find that $\operatorname{Re}\gamma>0$, where $\gamma=\sqrt{\alpha^2-k^2}$.

Let us introduce the Fourier integrals:

$$U^-(\alpha,h\pm 0)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}u(x,h\pm 0)e^{\mathrm{i}\alpha x}\mathrm{d}x, \quad (7)$$

$$U^+(\alpha,h\pm 0)=\frac{1}{\sqrt{2\pi}}\int_{0}^{+\infty}u(x,h\pm 0)e^{\mathrm{i}\alpha x}\mathrm{d}x. \quad (8)$$

Here, $U^-(\alpha,h\pm 0)$, $U^+(\alpha,h\pm 0)$ are regular functions in the overlapping half-planes $\tau<\tau_0$, $\tau>-\tau_0$ with a common stripe of regularity $\alpha\in\Pi$.

Applying the Fourier transform to the boundary condition (3), we find that

$$U^-(\alpha,h+0)=U^-(\alpha,h-0)=U^-(\alpha)$$

and

$$U^-(\alpha)=\frac{\mathrm{i}\cos(\beta_j h)}{\sqrt{2\pi}(\alpha-\mathrm{i}\gamma_j)}. \quad (9)$$

Using the notations (7)–(9), we rewrite the expression (6) as follows:

$$U(\alpha,y)=\begin{cases}\dfrac{U^-(\alpha)+U^+(\alpha)}{\operatorname{ch}(\gamma(h-d))}\operatorname{ch}(\gamma(y-d)),\ h<y<d,\\ \dfrac{U^-(\alpha)+U^+(\alpha)}{\operatorname{ch}(\gamma(h+d))}\operatorname{ch}(\gamma(y+d)),\ -d<y<h.\end{cases} \quad (10)$$

Further, using the condition of continuity of the normal stresses at $\{x\in(0,+\infty), y=h\pm 0\}$, we reduce the problem to the Wiener-Hopf equation [13,14]:

$$U^+(\alpha)M_+(\alpha)+\frac{\mathrm{i}\cos(\beta_j h)M_+(\alpha)}{\sqrt{2\pi}(\alpha-\mathrm{i}\gamma_j)}=\frac{J_1^-(\alpha)}{M_-(\alpha)},$$

$$\alpha\in\Pi. \quad (11)$$

Here,

$$J_1^-(\alpha)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}\left[\partial_y u(x,h+0)-\partial_y u(x,h-0)\right]e^{\mathrm{i}\alpha x}\mathrm{d}x$$

is the unknown function that is regular in the lower half plane $\tau<\tau_0$; $U^+(\alpha)=O(\alpha^{-3/2})$, $J_1^-(\alpha)=O(\alpha^{-1/2})$, if $|\alpha|\to\infty$ in the domain of regularity. The known functions $M_-(\alpha)$, $M_+(\alpha)$ are regular in overlapping half-planes $\tau<\tau_0$, $\tau>-\tau_0$, respectively. Outside the strip Π they have simple zeros at $\alpha=\pm i\gamma_{n1}$ and poles at $\alpha=\pm i\gamma_{n2}$, $\alpha=\pm i\gamma_{n3}$, $n=1,2,\dots$, where

$$\gamma_{n1}=\frac{1}{2d}\sqrt{(\pi n)^2-(2kd)^2},$$

$$\gamma_{n2}=\frac{1}{l_1}\sqrt{\left(\pi\left(n-\frac{1}{2}\right)\right)^2-(kl_1)^2},$$

$$\gamma_{n3}=\frac{1}{l_2}\sqrt{\left(\pi\left(n-\frac{1}{2}\right)\right)^2-(kl_2)^2},$$

$$l_{1(2)}=d\mp h;$$

$M_+(\alpha)=M_-(-\alpha)$ and $M_\pm(\alpha)=O(\alpha^{1/2})$, if $|\alpha|\to\infty$;

$$M_+(\alpha)=\sqrt{2d}\,\frac{-\mathrm{i}e^{\chi(\alpha)}(\alpha+k)\sqrt{\sin(2kd)}M_1(\alpha)}{\Lambda M_2(\alpha)M_3(\alpha)}, \quad (12)$$

$$\chi(\alpha)=-\frac{\mathrm{i}\alpha}{\pi\left(l_1\ln\left(\frac{2d}{l_1}\right)+l_2\ln\left(\frac{2d}{l_2}\right)\right)},$$

$$\Lambda=\sqrt{k\cos(kl_1)\cos(kl_2)},$$

$$M_1(\alpha)=\prod_{n=1}^{\infty}\left[1+\frac{\alpha}{\mathrm{i}\gamma_{n1}}\right]e^{\frac{2\mathrm{i}\alpha d}{\pi n}},$$

$$M_2(\alpha)=\prod_{n=1}^{\infty}\left[1+\frac{\alpha}{\mathrm{i}\gamma_{n2}}\right]e^{\frac{\mathrm{i}\alpha l_1}{\pi n}},$$

$$M_3(\alpha)=\prod_{n=1}^{\infty}\left[1+\frac{\alpha}{\mathrm{i}\gamma_{n3}}\right]e^{\frac{\mathrm{i}\alpha l_2}{\pi n}}.$$

Applying the Liouville's theorem, we arrive at the solution of equation (11) in the following form:

$$U^+(\alpha)=\frac{\mathrm{i}\cos(\beta_j h)}{\sqrt{2\pi}M_+(\alpha)}\left[\frac{M_+(\alpha)}{t-\mathrm{i}\gamma_j}-\frac{M_+(\mathrm{i}\gamma_j)}{\alpha-\mathrm{i}\gamma_j}\right]. \quad (13)$$

IV. REPRESENTATION OF FIELDS

Substituting the expressions (13) and (9) into the representation (10), we find the integral representation of the diffracted field in the form as

$$u(x,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}U(\alpha,y)e^{-\mathrm{i}\alpha x}d\alpha, \quad (14)$$

For the field analysis, we represent integral (14) through the series of the residues; for this purpose we deform the integration path into the upper/lower complex half-planes. As a result, the scattered field for each of the regions is written as follows:

I. $x>0, -d<y<d$:

$$u(x,y)=\frac{\cos(\beta_j h)M_+(i\gamma_j)}{2d}\times$$

$$\times\sum_{q=0}^{\infty}\frac{\varepsilon_q(-1)^{q+1}M_+(i\gamma_{q1})\cos\left(\frac{\pi q l_1}{2d}\right)e^{-\gamma_{q1}x}}{\gamma_{q1}(\gamma_{q1}+\gamma_j)}\times$$

$$\times\cos\left(\frac{\pi q(y-d)}{2d}\right), \tag{15}$$

where $\varepsilon_q = 1/2$, when $q = 0$ and $\varepsilon_q = 1$, when $q \geq 1$;

II. $x < 0, -d < y < h$:

$$u(x,y) = -u_z^{inc}(x,y) + \frac{\cos(\beta_j h) M_+(i\gamma_j)}{l_1^2} \times$$
$$\times\sum_{q=1}^{\infty} \frac{(-1)^q \pi(2q-1) e^{\gamma_{q2} x}}{2\gamma_{q2}(\gamma_{q2}-\gamma_j) M_+(i\gamma_{q2})} \cos\left(\frac{\pi(q-1/2)(y+d)}{l_2}\right); \tag{16}$$

III. $x < 0, h < y < d$:

$$u(x,y) = -u_z^{inc}(x,y) + \frac{\cos(\beta_j h) M_+(i\gamma_j)}{l_2^2} \times$$
$$\times\sum_{q=1}^{\infty} \frac{(-1)^q \pi(2q-1) e^{\gamma_{q3} x}}{2\gamma_{q3}(\gamma_{q3}-\gamma_j) M_+(i\gamma_{q3})} \cos\left(\frac{\pi(q-1/2)(y+d)}{l_1}\right). \tag{17}$$

V. NUMERICAL ANALYSIS

Numerical calculations of the total displacement field are represented for the layer that is shown in Fig. 1 for two different positions of the defect: $h = d/2$ and $h = 0$.

The defect is irradiated by a normal SH-wave (1) with number $j = 0$ of the unit amplitude. The dimensionless thickness of the plate is equal to $2kd$.

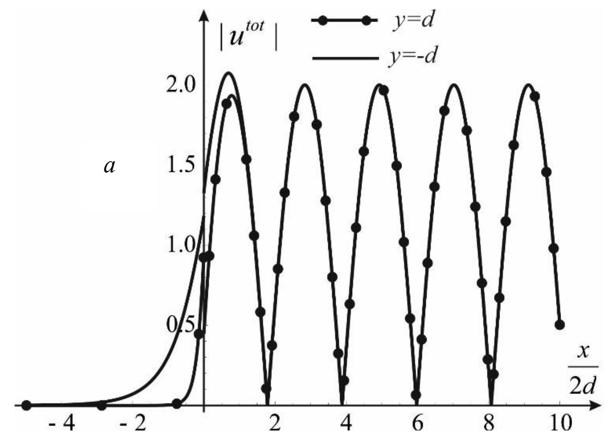
Case 1: $h = d/2$. Under such condition, all the modes are evanescent in the region $x < 0, h < y < d$, if $0 < 2kd < 2\pi$. In the region $x < 0, -d < y < h$ all the modes are evanescent, if $0 < 2kd < 2\pi/3$. In this region the first propagating mode appears, if $2\pi/3 < 2kd < 2\pi$.

Case 2: $h = 0$. In this case, the inclusion is located in the middle of the layer. In the domain $0 < 2kd < \pi$ all modes are evanescent; the first propagating mode is formed, if $\pi < 2kd < 2\pi$.

In Fig. 2 and Fig. 3 the dependences of the total displacement field $\left|u^{tot}\right| = \left|u^{tot}(x/(2d), y = \pm d)\right|$ are shown on the layer surface $P(y = \pm d)$: the defect $\Gamma(h = d/2)$ (see Fig. 2) and $\Gamma(h = 0)$ (see Fig. 3).

Here, taking into account the exponential nature of the damping of the summands (15)–(17), no more than five terms were used for the calculations.

From the Fig. 2(a) we observe, that in the region $x < 0$ the total field decays to zero on the layer surfaces $y = \pm d$. The speed of the decaying on the surface $y = d$ is higher, than on the surface $y = -d$. This happens because the waveguide area above the inclusion is narrower than the one below. If $x > 0$, the dependence $\left|u^{tot}\right|$ on the dimensionless parameter $x/(2d)$ is oscillatory in its nature. The module of the complex amplitude of the oscillation on the layer surfaces for $y = -d$ and for $y = d$ is different, if $x/(2d) < 1$; and this amplitude is twice as large as the amplitude of the primary normal mode $u_z^{inc}(x,y)$. That is, the oscillations in the incident and reflected waves occur in a phase. With the increase of thickness up to $2kd = 4.5$ (see Fig. 2(b), we see that in the region $x < 0$, $y = d$ the total field $\left|u^{tot}\right|$ saves an exponentially decreasing behavior. Here, in the region above the inclusion, all the modes are evanescent. In the region below the inclusion, one propagating mode is formed. On the surface $x < 0$, $y = -d$ the behavior of $\left|u^{tot}\right|$ decreases exponentially to the value 0.25. This corresponds to the amplitude of the scattered mode. Oscillatory behavior of $\left|u^{tot}\right|$ is different for the upper and the lower surfaces, if $x > 0$.



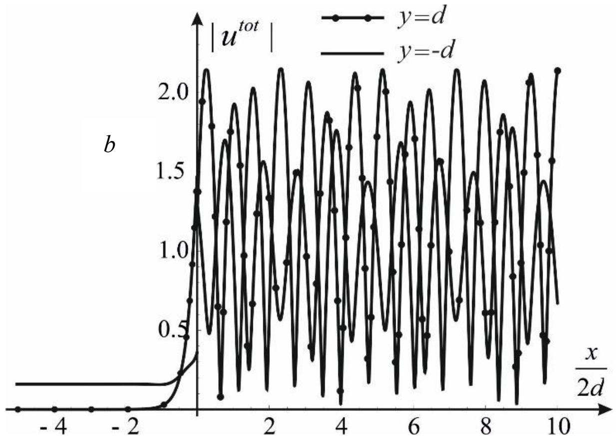


Fig. 2. The dependence of the total displacement field $|u^{tot}|$ on the normalized coordinate $x/2d$ with $h = d/2$; $a - 2kd = 1.5$; $b - 2kd = 4.5$

When the defect approaches the middle of the layer (see Fig. 3), the regions above and below the inclusion are equal in width. Obviously, for reasons of symmetry, the behavior on the sides $y = \pm d$ must be the same, as it is evidenced by overlapping of the curves in this figure.

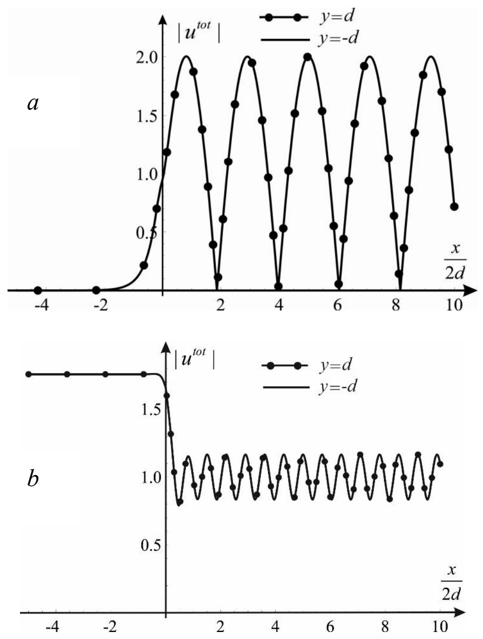


Fig. 3. The dependence of the total displacement field $|u^{tot}|$ *on the normalized coordinate* $x/2d$ *with* $h=0$*;*
a – $2kd=1.5$*;* b – $2kd=4.5$

For $2kd=1.5$ (see Fig. 3(a), all the waves are evanescent in the region $x<0$ and only the reflected waves exist in the region $x>0$. The overlapping of the reflected and the incident waves leads to the increase of the resulting amplitude twice as the amplitude of the incident wave. With the increase of the dimensionless frequency (see Fig. 3(b), $2kd=4.5$), one propagating mode with the amplitude $\left|u^{tot}\right|\approx 1.75$ exits, if $x<0$. In the region $x>0$ the value of the amplitude has an oscillatory dependence within $\left|u^{tot}\right|\approx 1\pm 0.2$.

VI. CONCLUSION

Using the Wiener-Hopf technique, we obtained the exact analytical solution of the problem of the diffraction of normal SH-wave on a semi-infinite inclusion, which is located in an elastic plane layer. The influence of the inclusion depth on the distribution of the displacement field, depending on the thickness of the layer, was investigated. It was found that the thickness of the layer and the frequencies of its probing form intense oscillations of the elastic field on the layer surfaces. The place, where the oscillation started, is an indicator of the defect which can extend to the area where oscillations are absent. The change of the frequency parameter allowed to estimate the depth of the defect.

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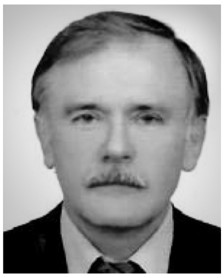
Myron Voytko was born in Svytaziv village, Sokal district, Lviv region, on June 17, 1979. He received MS degree in Physics at Ivan Franko National University of Lviv in 2001. He joined Karpenko Physico-Mechanical Institute of the National Academy of Sciences of Ukraine, Lviv, Ukraine in 2003. He served as an engineer during 2003–2007 and junior researcher during 2007–2010. Since 2010 he has been a research of the department of Physical Bases of Materials Diagnostics.

He received the PhD degree in mechanics of deformable bodies at Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of the National Academy of Sciences of Ukraine, Lviv, Ukraine. His research interests are mainly in the developing of new theoretical models for the analysis of the elastic wave scattering and diffraction from various internal and interface defects of the layered materials for their diagnostics. He was honored with an award of junior scientists of the National Academy of Scientists of Ukraine in 2013.

Yaroslav Kulynych was born in Dovhe village, Lviv region, Ukraine on February 23, 1955. He graduated from the Mathematical Department of Ivan Franko State University of Lviv, Ukraine in 1977; he worked as an engineer, junior and senior researchers at the Karpenko Physico-Mechanical Institute of the National Academy of Sciences of Ukraine, Lviv, Ukraine. In 1997 he defended his candidate-degree thesis "Mathematical modeling of signals and electromagnetic fields of marine electrical prospecting". His research interests are in the mathematical modeling of the electromagnetic and elastic fields interaction with an inhomogeneous medium. His scientific results were obtained and developed by theoretical bases for interpretation of the eddy-current and the radio-wave non-destructive testing data for the diagnostic technologies. He is the author of one monograph and above 80 journal and conference papers. He has the academic rank of Senior Researcher of Mathematical Modeling and Computational Methods.

Mykhaylo Grynenko was born in Zolochiv, Lviv region, Ukraine, on October 30, 1995. He received the BS degree in mechanics in 2016 and MS degree in applied mathematics in 2018 at Ivan Franko National University of Lviv, Ukraine. Since 2018, he is the PhD student of Karpenko Physico-Mechanical Institute of the National Academy of Sciences of Ukraine, Lviv, Ukraine. His research interests are in elastic wave propagation and diffraction from the defects.



Dozyslav Kuryliak was born in Rava-Ruska, Lviv region, Ukraine, on November 08, 1952. He received the MS degree in radio physics and electronics at Ivan Franko State University of Lviv in 1975 (honor); received the PhD and D.Sc. degrees in radio physics in 1988 and 2002 respectively at Kharkiv State University and O. Ya. Usikov Institute for Radiophysics and Electronics of the National Academy of Sciences of Ukraine, Kharkiv, Ukraine.

He joined Karpenko Physico-Mechanical Institute of the National Academy of Sciences of Ukraine, Lviv, Ukraine in 1977. He served as a senior and leading engineer in the Institute during 1977–1991, senior researcher during 1991–2003 and leading scientist during 2003–2007. Since 2007, he has been a Chair of the department of Physical Bases of Materials Diagnostics. His research interests are mainly in the functional-theoretical methods for rigorous analysis of wave diffraction from the canonical structures and their application for modeling of the electromagnetic, acoustic and elastic waves scattering from the defects. He is the author of two monographs and above 160 journal and conference papers. He is the winner of the Ukrainian State Prize in Science and Technology. Since 2006, he has been a professor of the Department of Electronic Computing Machines, Lviv Polytechnic National University. He has the academic ranks of Senior Researcher of Radio Science and Professor of Physics and Astronomy.