CARTOGRAPHY AND AERIAL PHOTOGRAPHY

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INVESTIGATION OF THE ACCURACY OF PLANS OF LVIV IN 1844 AND 1931

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Purpose of this work is the accuracy assessment of plans of Lviv in 1844 and 1931. **Methodology**. The main criterion for evaluation of the authenticity of the old plans is the study of their geometric accuracy, which is determined by the magnitudes of errors that arise when measuring lengths, angles, coordinates, and other cartometric attributes. For the investigation of accuracy, quantitative method used direct measurements of the lengths of lines and angles between two directions in the oldert and modern plans and analysis of these measurements based on the statistical theory of errors using the Gauss and Bessel formulas. **Results**. According to this methodology, for each plan, the mean square error of the distance, the angle of rotation, and the mean square error of the directional angles were determined. These characteristics made it possible to compare the errors of linear and angular variables of two time-varying and multi-scale plans of Lviv to each other and concluded that the plan of Lviv in 1931 is more accurate with respect to the angular measurements. Distortion distances are characteristic for both Lviv’s plans. Nevertheless, there was less relative mean square error is in the plan of 1931, despite its smaller scale. The calculated distortion coefficients of the lengths of lines and the amplitude of fluctuations of the boundary values of the scale denominator made it possible to carry out the analysis of distortion in different parts of the plan and to continue the study of the accuracy of the above-mentioned cartographic materials using graphical research methods that allowed us to evaluate and illustrate spatial variations of errors, as well as to detect and understand the facts and technical aspects of creating these cartographic works. **Scientific novelty and practical significance**. The determination of the geometric accuracy of the 1844 plans of Lviv, as a tool for scientific research, is relevant since it enables us to evaluate the cartographic publication as a result of human activity more objectively and fully. Numerical data obtained during such a research made it possible to compare the cartographic, documentary, and content values of old plans. The quantitative technique chosen in the study of Lviv’s old plans, based on direct measurements of the lengths of lines and angles between two directions, made it possible to compare the errors of linear and angular variables of time-varying and multi-scale plans of Lviv. Having been calculated in this research, this methodology can be used to evaluate the accuracy of other plans of Lviv and any other city.

Key words: old plan, theory of errors, length distortion, mean square error, Gauss formula, directional angle.

Introduction

Modern plans and maps constructed under the mathematics laws on the basis of precise geodetic measurements, with the usage of the latest technologies and equipment, reflect the real spatial relationships with sufficient accuracy. However, such precise works appeared not suddenly, but were the result of the gradual evolutionary development of cartography and some related to it sciences such as astronomy, geodesy, mathematics, and many others.

Ancient plans and maps of Lviv in historical terms have not only recorded the development of the city's economic, social, and cultural traditions for centuries, which are important for researchers of the history of the city and for the urban planning, but they show the history of the cartography development. They provide us an opportunity to follow the development of ways to display the terrain, the constant improvement of methods and ways of creating plans from the planimetric and altitude reasoning, and topographical survey to the printing of plans and atlases [Sossa et al., 2016].

Quite often ancient maps and plans serve as an information source in various scientific research, in the verification or rejection of scientific hypotheses, since they provide very important information about the location, properties, size, and time changes of many elements of the environment and the development of socio-economic and cultural conditions. Non-critical attitude to information provided by old cartographic works may sometimes lead to false conclusions. The degree of reliability of data obtained from the older map is different and it depends on the overall level of the map’s accuracy [Wolski, 2012].

Scientific novelty and practical significance

An ancient plan as a scientific tool and source of research should be investigated in terms of its accuracy and reliability. The answer to the question on how accurate the ancient plan is makes it possible to more objectively and thoroughly evaluate a particular plan as a result of human activity. Numerical data obtained during such investigations make it possible to compare the cartographic, documentary, and content values of ancient plans. Such considerations make it relevant to study the accuracy of cartographic representation of ancient plans, which in many cases is important for cartometry works.

Many scientists in Switzerland, the Czech Republic, Poland, and other countries were engaged and still are engaged in investigations of the accuracy of ancient maps, which are carried out mainly in the territory of their countries [Jenny et al., 2007], [Zimová et al., 2006], [Petkiewicz, 1995], [Szeliga, 1993]. In particular, it is necessery to mention the work of Polish scientists whose research was initiated by Henryk Merczyng in 1913 [Ostrowski, 2014].

Nowadays, in the study of the accuracy of ancient maps, GIS tools are widely used [Manzano-Agugliaro, 2012], [Wolski, 2012]. The main objective of the study [Affek, 2013] is therefore to introduce principles for the georeferencing of historical maps for their further usage in historical GIS. Two alternative georeferencing methods for maps based or not based on a geodetic network are described, and the georeferencing of archival maps is discussed further by reference to the First, Second, and Third Military Surveys of Galicia conducted by the MGI (Militärgeographisches Institute), and completed in 1783, 1863, and 1879 respectively.

The article [Jenny et al., 2007] describes the steps leading to visualizations of a map’s planimetric accuracy using a new software application MapAnalyst. It illustrates local map distortion and additionally computes the old map’s scale and rotation, as well as statistical indicators summarizing the map’s geometric accuracy.

In Ukraine, this direction of cartometric investigations failed to develop due to the complexity of access to the originals of old maps and their digital images in the past. Therefore, the subject of our study, in addition to its pioneering nature for Ukraine, is important from the view point of the development of a methodology for conducting such studies and initiating the accumulation of results of the definite accuracy of certain past cartographic works. We begin this research with the plans of Lviv.

The first work in the study of ancient plans of Lviv in terms of their development and, partly, of their geometric precision was the work of Ye. Gavrylova "The map of Lviv and its development" [Havrylova, 1956]. The study of geometric accuracy consisted of calculating several (5-10) distances between the identified objects (mainly in the central part of the ancient plan) and comparing them with the corresponding distances on the photoplan.

The research of Lviv's old plans was carried out primarily in the source, archaeological, and historical aspects and found their reflection in the papers of Polish researchers [Czerner, 1997] and in the first atlas of the history of Lviv [Kapral, 2014].

The continuation of this direction of historical and cartographic research and the opportunity for the general public to get acquainted with the originals and copies of the old plans of Lviv was the exhibition "Cartographic publications of Lviv" [Sossa et al., 2016].

We consider the thorough investigation of the accuracy of Lviv's plans to be relevant, and that made it possible to evaluate this cartographic publication more objectively and more fully.

Purpose

The main purposes of the article are the accuracy assessments of the plans of Lviv in 1844 and 1931 using a quantitative method that uses direct measurements of the lengths of lines and angles in the old and in the modern plans, and analysis of measurement results based on the statistical theory of errors.

Methodology

The main criteria of assasement of the authenticity of old plans is the study of their geometric accuracy, which is determined by the magnitudes of errors that appears during linear and angular measurements. Obviously, the greatest influence on the accuracy can be made by the increased scale of the plan.

The methodology of accuracy assessment is based on cartometric and geometric analyses of sets of identical points in the old map and of a reference map, using the diverse technique and transformation [Bayer et al., 2009], [Manzano-Agugliaro et al., 2012], [Jenny & Hurni, 2011].

Methods of investigation of the accuracy of old plans can be divided into two groups:

- quantitative methods;
- graphical methods.

Quantitative methods characterize the accuracy of the plan using numerical values that express a number of errors. They are based on the use of cartometric techniques (measurement of the size of same elements on the old and modern plans), comparing the results obtained and their processing according to the theory of errors. The difference between a numerical value of the element on the old plan and on the modern one is considered to be an error of the plan being studied. Based on several dozen individual errors, according to the statistical theory of errors, the root mean square error (RMSE) is calculated using the known Gauss formula [Szeliga, 1993], [Pietkiewicz, 1995].

Graphical methods clearly illustrate and supplement the numerical values obtained with the help of quantitative methods and provide an opportunity to immediately see more and less accurate parts of the plan [Pietkiewicz, 1975]. *Graphical methods* include:

- construction of a deformed cartographic grid (called the grid of distortions) which is a projection of the grid of the modern plan transferred to the plan under investigation [Jenny et al., 2007];
- overlay of the selected elements of the content of the plan under investigation, to the modern one. These elements are represented on the same plane with different signs and vectors, thus, visualizing the differences in the length, distance and direction of these elements on both plans [Jenny & Hurni, 2011].

To study the accuracy of the Lviv's plans, a quantitative method was chosen, that uses direct measurements of the selected elements in the old and modern plans and the analysis of the measurement results based on the statistical theory of errors. In the absence of cartographic grid on the plans, the study of geometric precision was carried out by the comparison of the distances between the objects as well as of the angles between the two directions.

Before the study, one should calculate the partial scales of the plan by comparing the lengths of lines by the equality:

$$M_i = \frac{\tilde{l}_i}{L_i}, \; i = 1,...,n \tag{1}$$

where $\tilde{l}_i$ is the length of the i-th line in the old plan, L_i is the real length of the same line, which is determined from the modern plan, n is the number of measurements.

It should be noted that the values of scales M_i have different values in different parts of a plan, therefore the amplitude of fluctuations of scale values is one of the characteristics of the accuracy of the plan: the less accurate the plan is, the greater the amplitude between the boundary values is.

Further, for each of the lines, the distortion of the lines' lengths is determined as the ratio of the partial scale M_i to the main one M, which is indicated on the plan:

$$\mu_i = \frac{M_i}{M}, \; i = 1,...,n. \tag{2}$$

According to the distortion coefficient of the lines lengths and the amplitude of fluctuations of the boundary values of the denominator of the scale, the analysis of distortion of distances can be carried out in different parts of the plan.

The following comparable element that gives a numerical characteristic of the accuracy of the old plan (map) is the distance between the two objects. Differences in the distance between the results obtained for the old and modern plans are treated as absolute errors (deviations) of distances $\Delta_i = l_i - L_i$. These deviations can be positive as well as negative. On the basis of several dozen absolute errors of the lengths of the lines, their mean errors are calculated:

$$l_{mean} = \frac{\sum_i |\Delta_i|}{n},$$

and subsequently the mean square error of the lengths of lines according to the known Gauss formula [Gauss et al., 1995]:

$$m_l = \pm\sqrt{\frac{\sum_i \Delta_i^2}{n}}. \tag{3}$$

In addition, to compare the distance errors in various old plans, it is convenient to use the relative mean square error, which is determined by the formula:

$$M_l = \pm\sqrt{\frac{\sum_i \left(\frac{\Delta_i}{L_i}\right)^2}{n}} \cdot 100\,\%,$$

or through the length distortion coefficient

$$M_l = \pm\sqrt{\frac{\sum_i (\mu_i - 1)^2}{n}} \cdot 100\,\%. \tag{4}$$

This error can be interpreted as the actual error of the line length in a conventional distance of 100 m, and thus, to compare the errors of time-varying and multi-scale plans.

Numerical results that outline the accuracy of the plan also give a measurement and comparison of the angles between two directions or directional angles relative to the chosen zero direction. The angle differences between the results obtained for the old and modern plans are treated as deviations of angles $\delta_i = \alpha_i - A_i$. Note that if among the differences δ_i only positive or negative values are dominated, this indicates the systematic error of the angles, namely the rotation of the plane relative to the main (northern) direction of the coordinate axis. In this case, according to the statistical theory of errors, the rotation angle is determined as the arithmetic mean of the resulting differences of angles:

$$\theta = \frac{\sum_i \delta_i}{n}. \tag{5}$$

Excluding from the differences the systematic error of the plan rotation, we obtain the corrected differences $\delta_i' = \delta_i - \theta$ and find the mean square error of the angles for the old plan according to the Bessel mean square error formula [Zazuliak et al., 2007], [Yonghe, 2015]:

$$m_\alpha = \pm\sqrt{\frac{\sum_i (\delta_i')^2}{n-1}}. \tag{6}$$

These values can also be used to evaluate the accuracy of time-varying cartographic works.

Excluding from the differences the systematic error of the plan rotation, we obtain the corrected differences $\delta_i' = \delta_i - \theta$ and find the mean square error of the angles for the old plan according to the Bessel mean square error formula [Zazuliak et al., 2007], [Yonghe, 2015]:

The plan *«Lviv with its suburbs in 1844»* (Fig. 1) is engraved on the stone by the cadets Kratochwill and Radoicsich, and signed by Brankowich. The authentic title of the plan is *«Lemberg mit seinen Vorstaedten im Jahre 1844», auf Stein gest. von Cadeten Kratochwill und Radoicsich, geschrieben Brankowich* [Lemberg mit seinen Vorstaedten…, 1844] This plan is stored in Stefanyk National Science Library. The size of the plan sheet is 73 × 76 cm, which is pasted on the canvas and cut into 16 parts. The scale of the plan is 1: 7200.

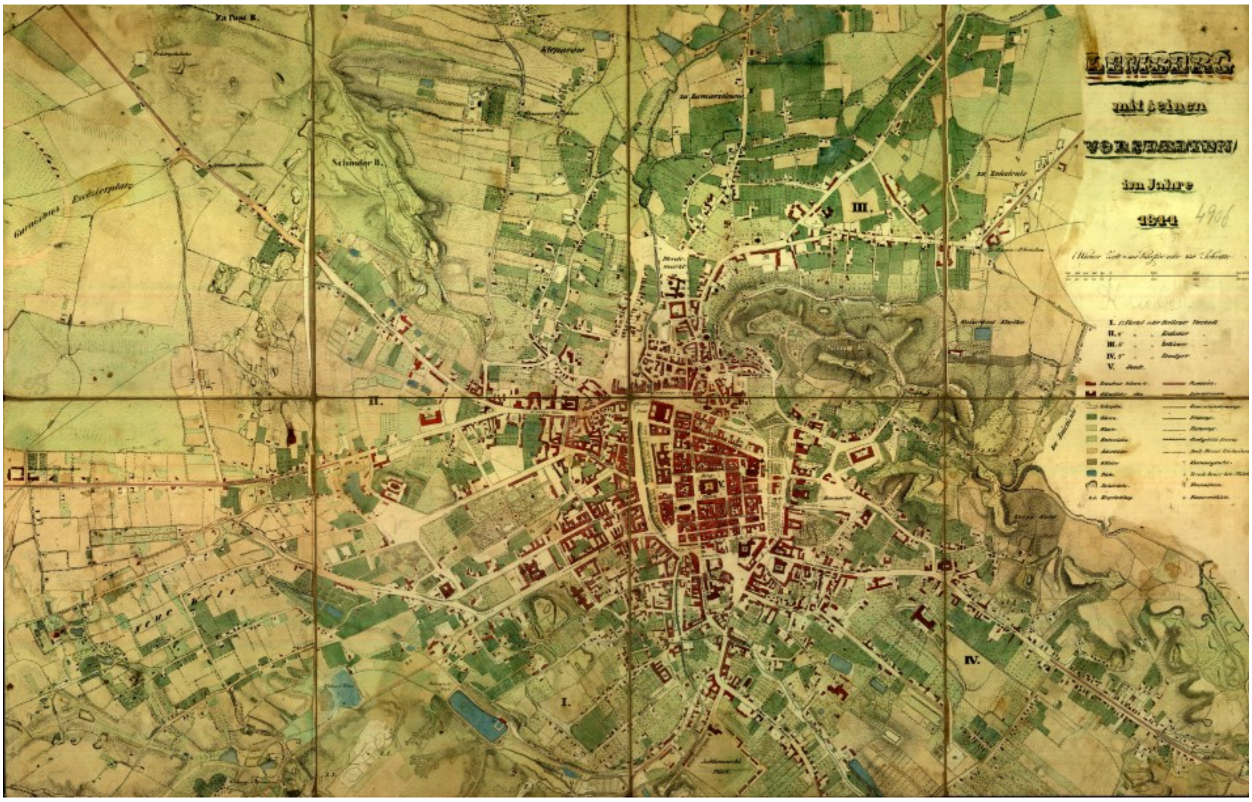

Fig. 1. Fragment of the plan "Lviv with its suburbs in 1844"

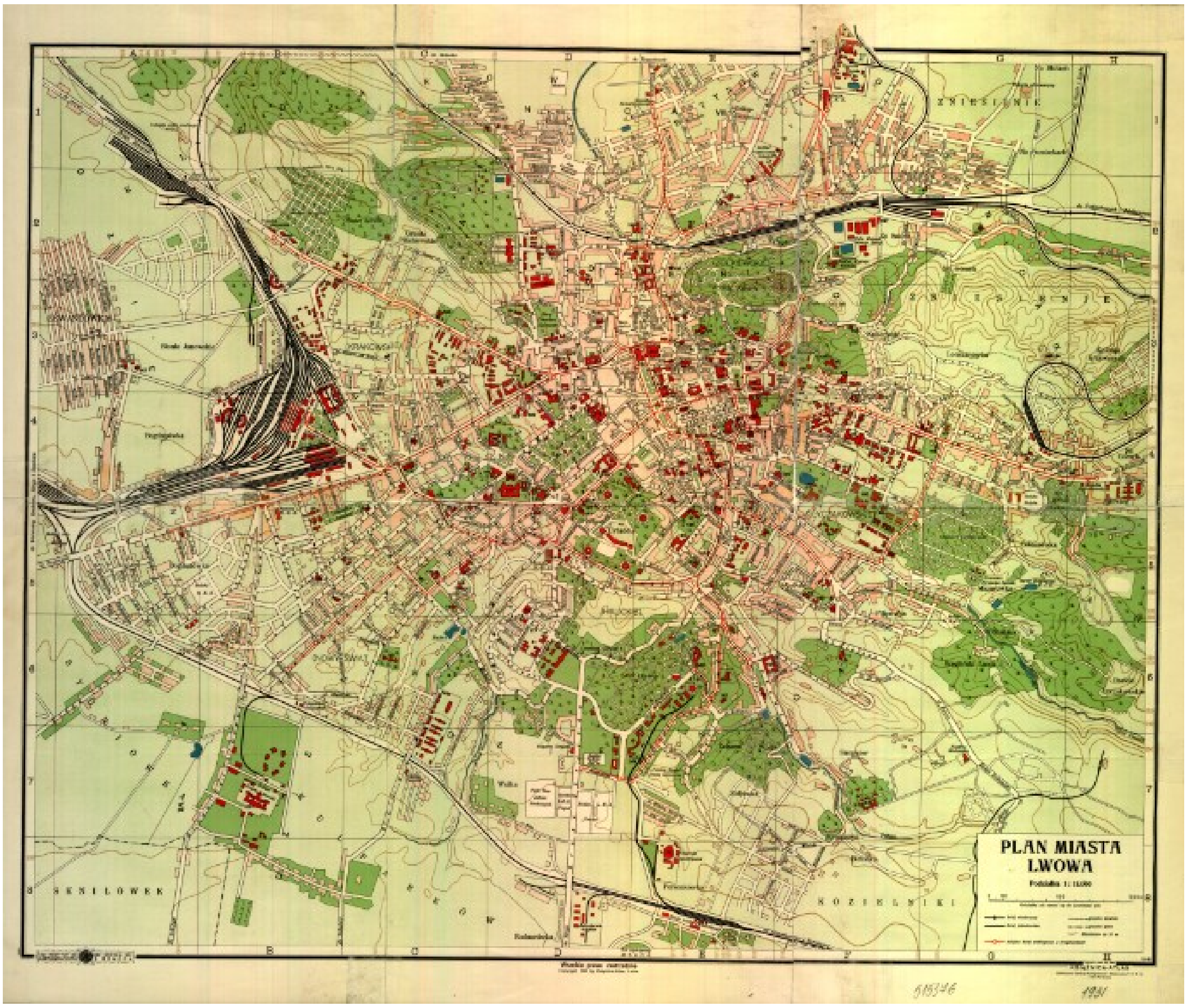

Fig. 2. The plan of the city of Lviv, 1931

The heading is located in the north-eastern corner, below the headline the linear scale is presented and a table of conventional signs is provided, including 21 items. In the south-west corner there is a list of prominent suburban buildings, and in the south-eastern corner there is a list of prominent buildings of the city. The plan is oriented to the north with a slight deviation.

«The plan of the city of Lviv», 1931 (Fig. 2), was fulfilled by E. Romer Cartographic Institute and printed by the «Książnica-Atlas» corporate enterprise. An authentic name is *«Plan miasta Lwowa»* [Plan miasta Lwowa. Instytyt Kartograficzny…,1931]. This plan is stored in Stefanyk National Science Library. The size of the sheet is 46 × 56 cm. The scale of the plan is 1:15 000.

During the 1920's, E. Romer Cartographic Institute in Lviv prepared several "Plans of Lviv" at a scale of 1:15 000 at a good cartographic level. The first one appeared in 1922. Further, the known cartographic plans of the city were in 1926 and in 1929; and our last plan dates up to 1931. By appointment, they are tourist plans with the designation of the routes of urban transport, civil, and sacral objects. The lists of streets and the lists of objects and prominent buildings of the city were added to the plans. Especially elaborated is the edition of 1931, in which a number of new military, sports and recreational facilities appeared such as military barracks, stadiums, parks, and power stations. Administrative, industrial and public buildings are indicated in bright red. The plan is decorated in a beautiful color scheme. On the back of the plan is a list of the main objects shown and numbered on it. Among them are objects of transport structure, administrative bodies, medical and cultural-educational establishments, churches, and sacred objects. The plan is supplemented with a labeling of the names of the streets and squares of Lviv, as well as with a separate list of streets for the suburbs of Cleparive, Zamarstyniv, Levandivka, and Znesiniya [Sossa et al., 2016].

The study of the accuracy of Lviv's plans consisted in measuring of the lines' lengths and directional angles in the raster images of old plans and the same elements in the modern digital vector plans of Lviv at scale 1: 2 000. The accuracy of the reference modern plan corresponds to the accuracy of the topographic plan of scale 1: 2 000. For the measurements, the software package ArcGis was used.

When identifying control points in both old and modern plans, we have selected corners of the churches, temples, cathedrals, and old buildings of the city center, since they have survived to this day and their location is permanent in the analyzed time horizon.

After georeferencing the old plans' scans to the reference modern plan, using a set of control points and affine transformation, we obtained a mean RMSE (root mean square error) for two selected old plans to the data frame equal to 9.81 m (for plan in 1844) and 16.1 m (for plan in 1931). The smalles value of RMS location errors for all residuals are 2.98 m and 2.4 m for plans in 1844 and 1931 respectively. The largest value of RMSE for all residuals are 14.99 m and 27.3 m for the respective plans.

Appropriate lines of both plans are constructed between recognized objects (Fig. 3, 4).

Further, using the Calculate Geometry tool and the Python programming language, the lengths of lines, the rumba, and their directional angles were determined.

The following step was to calculate the directional angles of the constructed directions, relative to those chosen in the modern plan as a "reference point", whose directional angle was 351.170246°. The value of this angle was chosen as zero since it was closest to the northern direction of the modern plan, and in the old plans there was no cartographic grid.

The resulting attribute tables, which contain information on measuring the lengths of lines and angles of the old and modern plans, were exported to xls format and all further processing and mathematical calculations were performed in Excel spreadsheet editor (Fig. 5).

To determine the accuracy of the plan of Lviv in 1844, lengths and directional angles for 40 directions were measured and partial scales were found for each direction. The results of measurements and calculations for the plan "Lviv with its suburbs in 1844" are presented in Table 1.

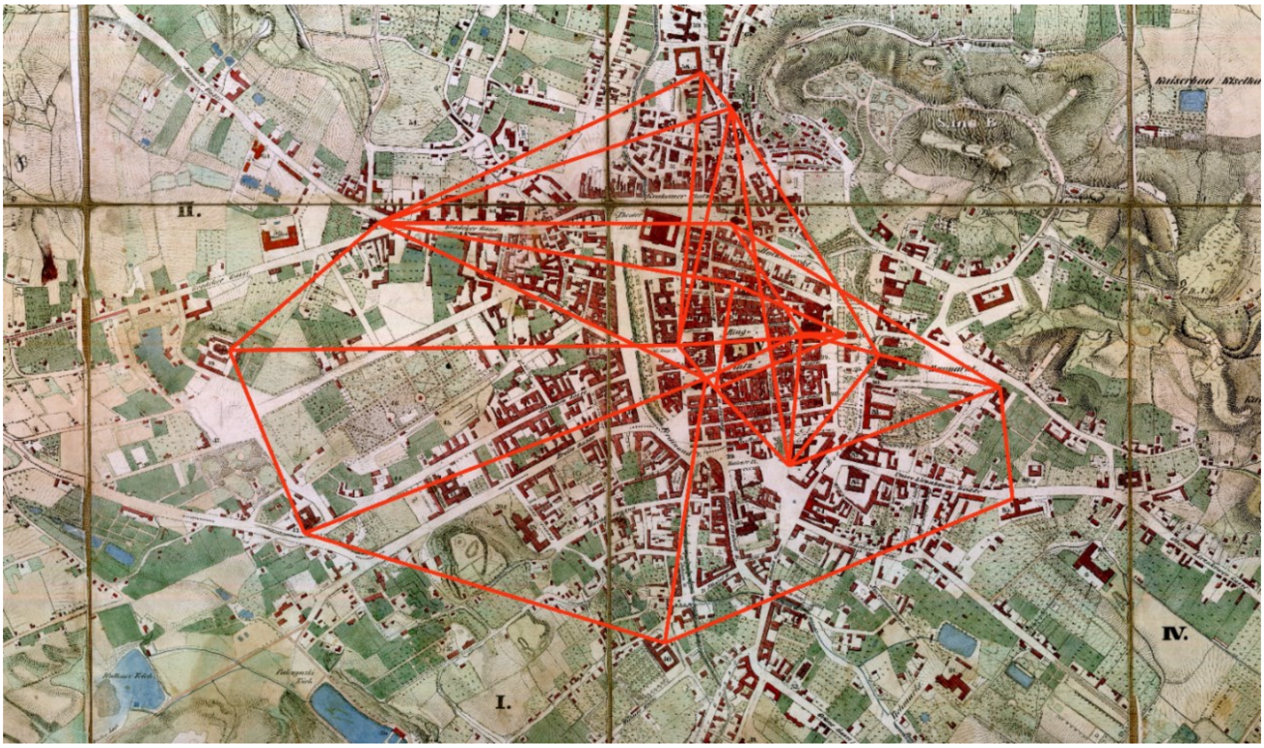

Fig. 3. The lines constructed in the old plan

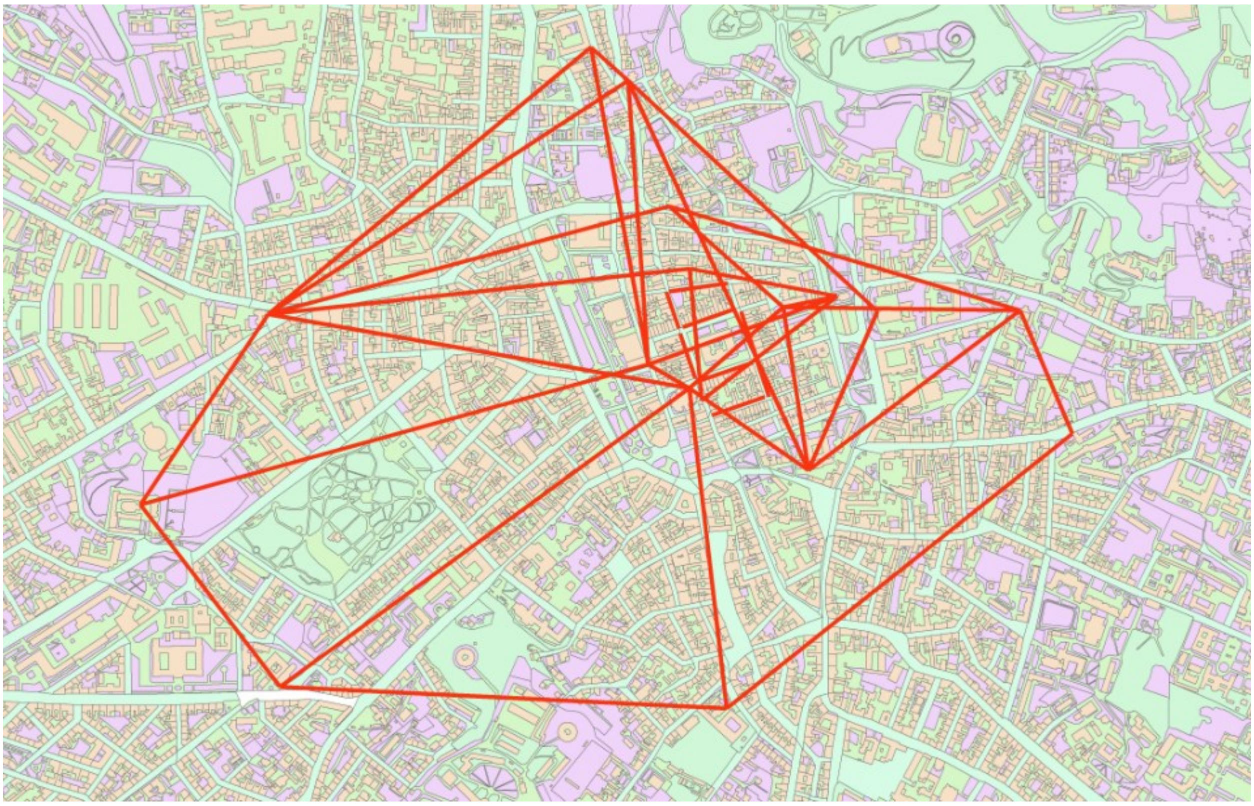

Fig. 4. The lines constructed in the modern plan

Table 1 shows that the smallest denominator 6657 of the scale differs from the largest 7802 by 1145. Thus, the amplitude of the fluctuation of the scale of the plan attains 16 %. With the help of the calculated partial scales, we determine the distortion coefficient of the lengths of lines μ_i, in accordance with the formula (2). For some lines this value is almost equal to unit (these are the directions 6, 13, 16, 22, 25, 29, 32. 40), so the distortion of the lengths is minimal. However, there are directions where μ_i varies from 4 to 8 %, there are more than ten of them, and these are mostly short distances. The predominance of relative distortion coefficients with minus signs indicates a distortion of the lengths of lines in the direction of compression. It is also possible to note that the most inaccurate lines are concentrated in the area of the Rynok Square.

The mean square error of the length of line and the relative error of length are calculated according to the formulas (3)–(4). The mean square error of the distance in the plan of 1844 is $m_l = 8.5$ m, and the relative error is determined by the relative distortion coefficient of the lengths $M_l = 3.2$ %.

To determine the angle of rotation of the plan and the error of the angles, the difference between the directional angles of the old and modern plans, which were measured relative to the chosen zero direction (line 4, table 2), was calculated.

Having completed all the necessary calculations, we determine the angle of rotation of the plan by the formula (5), which is 15° in the north-easterly direction in relation to the modern plan. The magnitude of the mean square error of the angles, according to (6), is $m_\alpha = 1.7^\circ$.

To investigate the accuracy of the plan of the city of Lviv in 1931, the lengths and the directional angles for 32 directions were measured. Following the algorithm described above, we obtained the results of the study of errors of the lengths of lines (Table 3) and of the errors of directional angles (Table 4).

The smallest denominator of the partial scale is 11841, and the largest one is 15373. Thus, the amplitude of fluctuations of the scales is 23%. By means of the calculated partial scales, the distortion coefficient of the lengths of lines μ_i is calculated.

Table

Lviv_1884_last

FID	Shape *	Id	Length	Length_m	Direction	X_Start	Y_Start	X_End	Y_End	Rumb	Dir_b
0	Polyline	1	5,20996	952,797465	64,806621	11,879362	13,418732	16,593731	15,63648	64,806621	73,636375
1	Polyline	2	5,293115	968,004819	115,705119	11,879362	13,418732	16,648661	11,122899	64,294881	124,534873
2	Polyline	3	1,769694	323,641545	136,722594	16,648661	11,122899	17,861841	9,834485	43,277406	145,552348
3	Polyline	4	1,96456	359,278781	5,326242	17,861841	9,834485	18,044205	11,790563	5,326242	14,155996
23	Polyline	5	2,089748	382,173074	218,599392	19,165575	11,46768	17,861841	9,834485	38,599392	227,429146
4	Polyline	6	4,228745	773,352846	149,477584	17,0179	15,11045	19,165575	11,46768	30,522416	158,307338
5	Polyline	7	5,40985	989,353343	71,777336	11,879362	13,418732	17,0179	15,11045	71,777336	80,60709
6	Polyline	8	0,675742	123,579661	141,118785	16,593731	15,63648	17,0179	15,11045	38,881215	149,948539
7	Polyline	9	3,580672	654,833255	192,421002	17,0179	15,11045	16,247721	11,613589	12,421002	201,250756
8	Polyline	10	2,437207	445,716437	86,205843	16,247721	11,613589	18,679586	11,774864	86,205843	95,035597
9	Polyline	11	3,001202	548,859907	105,372937	18,044205	11,790563	20,938027	10,994942	74,627063	114,202691
10	Polyline	12	4,600103	841,266907	301,623401	20,938027	10,994942	17,020979	13,406932	58,376599	310,453155
11	Polyline	13	4,037744	738,422643	4,915929	16,247721	11,613589	16,593731	15,63648	4,915929	13,745683
12	Polyline	14	1,568373	286,824036	9,337603	16,796594	10,979112	17,051065	12,526704	9,337603	18,167357
13	Polyline	15	5,248069	959,766856	279,786231	17,051065	12,526704	11,879362	13,418732	80,213769	288,615985
14	Polyline	16	1,913018	349,852712	147,664562	17,020979	13,406932	18,044205	11,790563	32,335438	156,494316
15	Polyline	17	5,141631	940,301433	270,131497	17,020979	13,406932	11,879362	13,418732	89,868503	278,961251
16	Polyline	18	3,287792	601,271373	69,331569	17,861841	9,834485	20,938027	10,994942	69,331569	78,161323
17	Polyline	19	5,343036	977,134494	170,911973	17,0179	15,11045	17,861841	9,834485	9,088027	179,741727
18	Polyline	20	2,044231	373,848885	247,091098	18,679586	11,774864	16,796594	10,979112	67,091098	255,920852
19	Polyline	21	0,633664	115,884465	320,747949	16,648661	11,122899	16,247721	11,613589	39,252051	329,577703
20	Polyline	22	1,547035	282,921836	64,432328	16,648661	11,122899	18,044205	11,790563	64,432328	73,262082
21	Polyline	23	1,793695	328,031012	294,781358	18,679586	11,774864	17,051065	12,526704	65,218642	303,611112
22	Polyline	24	0,635575	116,234009	271,415383	18,679586	11,774864	18,044205	11,790563	88,584617	280,245137
24	Polyline	25	6,545852	1197,105446	269,337567	16,247721	11,613589	9,702306	11,53791	89,337567	278,167321
25	Polyline	26	6,283821	1149,185179	248,719348	16,648661	11,122899	10,79331	8,84227	68,719348	257,549102
26	Polyline	27	3,947475	721,914154	188,717892	16,648661	11,122899	16,050344	7,221031	8,717892	197,547646
27	Polyline	28	1,654216	302,522964	173,538652	20,938027	10,994942	21,12418	9,351234	6,461348	182,368406
28	Polyline	29	2,876989	526,143689	229,175308	11,879362	13,418732	9,702306	11,53791	49,175308	238,005062
29	Polyline	30	2,908051	531,824436	157,965396	9,702306	11,53791	10,79331	8,84227	22,034604	166,79515
30	Polyline	31	5,501348	1006,086525	107,139462	10,79331	8,84227	16,050344	7,221031	72,860538	115,969216
31	Polyline	32	5,50287	1006,364847	67,225347	16,050344	7,221031	21,12418	9,351234	67,225347	76,055101
32	Polyline	33	0,675486	123,532792	85,333369	16,824292	11,881535	17,497538	11,936491	85,333369	94,163123
33	Polyline	34	0,564496	103,235097	174,969325	17,548622	11,890951	17,598123	11,328629	5,030675	183,799079
34	Polyline	35	0,685826	125,423846	264,533007	17,53595	11,273981	16,853244	11,208641	84,533007	273,362761
35	Polyline	36	0,503204	92,025991	355,426079	16,797012	11,308433	16,756884	11,810035	4,573921	4,255833
36	Polyline	37	0,702031	128,387506	96,810441	16,785024	12,337861	17,482102	12,25461	83,189559	105,640195
37	Polyline	38	0,464145	84,882832	174,663947	16,704356	12,343414	16,74752	11,88128	5,336053	183,493701
38	Polyline	39	0,690264	126,235418	84,394888	16,894194	10,742729	17,581158	10,810148	84,394888	93,224642
39	Polyline	40	0,464854	85,012537	175,766009	17,599803	11,279926	17,634123	10,816341	4,233991	184,595763

1 (0 out of 40 Selected)

Lviv_1884_last

Fig. 5. Attributive table of the plan of the city of Lviv in 1844

Table 1

Results of studies on errors of the lengths of lines in the plan of 1844

№ lines	Length of line $\tilde{l}_i$ in the old plan, cm	Length of line l_i in the old plan, m	Length of line L_i in the modern plan, m	Partial scale denominator M_i	Distortion coefficient of the lengths of lines, μ_i	Relative distortion of the lengths of lines in %	The difference $\Delta_i = l_i - L_i$, m	Squared differences Δ_i^2
1	13.2333	952.7975	956.3600	7226.9212	0.9963	-0.37	-3.5626	12.6919
2	13.4445	968.0048	974.1991	7246.0730	0.9936	-0.64	-6.1943	38.3693
3	4.4950	323.6415	330.8311	7359.9447	0.9783	-2.17	-7.1895	51.6896
4	4.9900	359.2788	378.4233	7583.6582	0.9494	-5.06	-19.1445	366.5110
5	5.3080	382.1731	396.6734	7473.1811	0.9634	-3.66	-14.5003	210.2599
6	10.7410	773.3528	775.8468	7223.2190	0.9968	-0.32	-2.4940	6.2198
7	13.7410	989.3533	978.9713	7124.4450	1.0106	1.06	10.3820	107.7864
8	1.7164	123.5797	114.2763	6657.9659	1.0814	8.14	9.3034	86.5530
9	9.0949	654.8333	651.7217	7165.7874	1.0048	0.48	3.1116	9.6821
10	6.1905	445.7164	458.4019	7404.9173	0.9723	-2.77	-12.6854	160.9198
11	7.6231	548.8599	540.0370	7084.2606	1.0163	1.63	8.8229	77.8432
12	11.6843	841.2669	834.9587	7146.0108	1.0076	0.76	6.3082	39.7939
13	10.2559	738.4226	738.4822	7200.5808	0.9999	-0.01	-0.0596	0.0035
14	3.9837	286.8240	302.5778	7595.4598	0.9479	-5.21	-15.7538	248.1823
15	13.3301	959.7669	965.7965	7245.2331	0.9938	-0.62	-6.0296	36.3563
16	4.8591	349.8527	347.8783	7159.3674	1.0057	0.57	1.9744	3.8981
17	13.0597	940.3014	946.0882	7244.3096	0.9939	-0.61	-5.7867	33.4862
18	8.3510	601.2714	605.3362	7248.6748	0.9933	-0.67	-4.0648	16.5229
19	13.5713	977.1345	982.6132	7240.3699	0.9944	-0.56	-5.4787	30.0164
20	5.1923	373.8489	390.5988	7522.5883	0.9571	-4.29	-16.7499	280.5592
21	1.6095	115.8845	107.8154	6698.6645	1.0748	7.48	8.0690	65.1092
22	3.9295	282.9218	283.2139	7207.4332	0.9990	-0.10	-0.2921	0.0853
23	4.5560	328.0310	342.3581	7514.4678	0.9582	-4.18	-14.3271	205.2661
24	1.6144	116.2340	125.9588	7802.3918	0.9228	-7.72	-9.7248	94.5713
25	16.6265	1197.1054	1201.1970	7224.6086	0.9966	-0.34	-4.0915	16.7407
26	15.9609	1149.1852	1156.7385	7247.3236	0.9935	-0.65	-7.5533	57.0521
27	10.0266	721.9142	738.3416	7363.8387	0.9778	-2.22	-16.4274	269.8604
28	4.2017	302.5230	306.5087	7294.8601	0.9870	-1.30	-3.9857	15.8861
29	7.3076	526.1437	524.5800	7178.6013	1.0030	0.30	1.5637	2.4452
30	7.3865	531.8244	524.9129	7106.4289	1.0132	1.32	6.9116	47.7699
31	13.9734	1006.0865	1019.1851	7293.7393	0.9871	-1.29	-13.0986	171.5730
32	13.9773	1006.3648	1007.6651	7209.3029	0.9987	-0.13	-1.3003	1.6908
33	1.7157	123.5328	126.3178	7362.3211	0.9780	-2.20	-2.7850	7.7562
34	1.4338	103.2351	109.7126	7651.7643	0.9410	-5.90	-6.4775	41.9579
35	1.7420	125.4238	123.7999	7106.7776	1.0131	1.31	1.6239	2.6372
36	1.4305	102.9988	108.3847	7576.4937	0.9503	-4.97	-5.3859	29.0078
37	1.7832	128.3875	134.1486	7523.0813	0.9571	-4.29	-5.7611	33.1898
38	1.1789	84.8828	87.7411	7442.4493	0.9674	-3.26	-2.8583	8.1699
39	1.7533	126.2354	126.9521	7240.8782	0.9944	-0.56	-0.7167	0.5137
40	1.1807	85.0125	85.0843	7206.0815	0.9992	-0.08	-0.0718	0.0052

Table 2

Results of studies on errors of the directional angles in the plan of 1844

№ lines	Directional angle in the plan of 1844	Directional angle in the modern plan	Difference of directional angles δ_i	Corrected differences δ_i'	Squares of the corrected differences $\delta_i'^2$
1	73.636	59.072	14.564	-0.336	0.1129
2	124.535	108.823	15.712	0.812	0.6593
3	145.552	133.528	12.025	-2.875	8.2656
4	14.156	0.000	14.156	-0.744	0.5535
5	227.429	212.139	15.290	0.390	0.1521
6	158.307	141.716	16.592	1.691	2.8595
7	80.607	65.709	14.898	-0.002	0.0000
8	149.949	140.998	8.951	-5.950	35.4025
9	201.251	184.771	16.479	1.579	2.4932
10	95.036	78.815	16.221	1.321	1.7450
11	114.203	99.635	14.567	-0.333	0.1109
12	310.453	295.428	15.025	0.125	0.0156
13	13.746	358.626	15.120	0.220	0.0484
14	18.167	3.618	14.550	-0.350	0.1225
15	288.616	272.632	15.984	1.084	1.1751
16	156.494	140.420	16.075	1.174	1.3783
17	278.961	263.666	15.295	0.395	0.1560
18	78.161	61.587	16.574	1.674	2.8023
19	179.742	164.072	15.670	0.770	0.5929
20	255.921	241.144	14.777	-0.123	0.0151
21	329.578	308.814	20.764	5.863	34.3748
22	73.262	57.881	15.381	0.481	0.2314
23	303.611	289.355	14.256	-0.645	0.4160
24	280.245	267.306	12.939	-1.961	3.8455
25	278.167	263.331	14.836	-0.064	0.0041
26	257.549	242.553	14.996	0.096	0.0092
27	197.548	182.181	15.366	0.466	0.2172
28	182.368	166.002	16.366	1.466	2.1492
29	238.005	222.586	15.419	0.519	0.2694
30	166.795	151.490	15.305	0.405	0.1640
31	115.969	101.583	14.386	-0.514	0.2642
32	76.055	60.213	15.842	0.942	0.8874
33	94.163	79.379	14.784	-0.116	0.0135
34	183.799	170.345	13.454	-1.446	2.0909
35	273.363	259.970	13.393	-1.507	2.2710
36	4.256	350.366	13.890	-1.010	1.0201
37	105.640	90.743	14.897	-0.003	0.0000
38	183.494	170.361	13.133	-1.767	3.1223
39	93.225	79.435	13.790	-1.111	1.2343
40	184.596	170.308	14.287	-0.613	0.3758

Table 3

Results of studies on errors of the lengths of lines in the plan of 1931

№ lines	Length of line $\tilde{l}_i$ in the old plan, cm	Length of line l_i in the old plan, m	Length of line L_i in the modern plan, m	Partial scale denominator M_i	Distortion coefficient of the lengths of lines, μ_i	Relative distortion of the lengths of lines in %	The difference $\Delta_i = l_i - L_i$, m	Squared differences Δ_i^2
1	6.5764	986.4588	956.3600	14542.3215	1.0315	3.15	30.0987	905.9334
2	6.8016	1020.2397	974.1991	14323.0916	1.0473	4.73	46.0406	2119.7356
3	2.2432	336.4751	330.8311	14748.3923	1.0171	1.71	5.6440	31.8545
4	2.5755	386.3289	378.4233	14693.0468	1.0209	2.09	7.9057	62.4995
5	2.6865	402.9794	396.6734	14765.2723	1.0159	1.59	6.3060	39.7660
6	5.2197	782.9624	775.8468	14863.6791	1.0092	0.92	7.1156	50.6319
7	6.8066	1020.9970	978.9713	14382.5784	1.0429	4.29	42.0257	1766.1602
8	0.8810	132.1512	114.2763	12971.0776	1.1564	15.64	17.8750	319.5148
9	4.3441	651.6209	651.7217	15002.3202	0.9998	-0.02	-0.1008	0.0102
10	3.1392	470.8831	458.4019	14602.4084	1.0272	2.72	12.4813	155.7823
11	3.6730	550.9511	540.0370	14702.8574	1.0202	2.02	10.9141	119.1169
12	5.6481	847.2191	834.9587	14782.9294	1.0147	1.47	12.2604	150.3180
13	4.9689	745.3351	738.4822	14862.0835	1.0093	0.93	6.8529	46.9627
14	2.1164	317.4537	302.5778	14297.1025	1.0492	4.92	14.8758	221.2902
15	6.6955	1004.3184	965.7965	14424.6552	1.0399	3.99	38.5220	1483.9414
16	2.3729	355.9408	347.8783	14660.2329	1.0232	2.32	8.0625	65.0033
17	6.5161	977.4091	946.0882	14519.3270	1.0331	3.31	31.3209	981.0016
18	4.1690	625.3553	605.3362	14519.8158	1.0331	3.31	20.0190	400.7623
19	6.6878	1003.1738	982.6132	14692.5664	1.0209	2.09	20.5606	422.7392
20	2.6700	400.4958	390.5988	14629.3200	1.0253	2.53	9.8971	97.9517
21	0.9105	136.5800	107.8154	11840.9121	1.2668	26.68	28.7645	827.3990
22	1.9031	285.4695	283.2139	14881.4803	1.0080	0.80	2.2556	5.0877
23	2.2843	342.6510	342.3581	14987.1808	1.0009	0.09	0.2928	0.0858
24	0.8265	123.9680	125.9588	15240.8814	0.9842	-1.58	-1.9908	3.9632
25	8.1758	1226.3671	1201.1970	14692.1380	1.0210	2.10	25.1701	633.5352
26	7.9250	1188.7438	1156.7385	14596.1452	1.0277	2.77	32.0053	1024.3408
27	4.8028	720.4241	738.3416	15373.0607	0.9757	-2.43	-17.9175	321.0354
28	2.0292	304.3829	306.5087	15104.7613	0.9931	-0.69	-2.1258	4.5192
29	3.4877	523.1532	524.5800	15040.9097	0.9973	-0.27	-1.4268	2.0358
30	3.5286	529.2947	524.9129	14875.8198	1.0083	0.83	4.3819	19.2007
31	7.0770	1061.5515	1019.1851	14401.3516	1.0416	4.16	42.3664	1794.912
32	6.8827	1032.4111	1007.665	14640.4635	1.0246	2.46	24.7460	612.362

Table 4

Results of studies on errors of the directional angles in the plan of 1931

№ lines	Directional angle in the plan of 1931	Directional angle in the modern plan	Difference of directional angles δ_i	Corrected differences δ_i'	Squares of the corrected differences $\delta_i'^2$
1	60.9810	59.0722	1.9088	0.5526	0.3054
2	110.6109	108.8229	1.7880	0.4318	0.1865
3	134.3103	133.5275	0.7827	-0.5735	0.3289
4	0.1952	0.0000	0.1952	-1.1611	1.3482
5	212.3106	212.1390	0.1717	-1.1846	1.4033
6	143.7775	141.7158	2.0617	0.7055	0.4977
7	68.2684	65.7089	2.5595	1.2032	1.4477
8	139.5056	140.9978	-1.4922	-2.8484	8.1134
9	188.3021	184.7713	3.5308	2.1746	4.7289
10	82.2248	78.8148	3.4101	2.0539	4.2185
11	98.5327	99.6355	-1.1028	-2.4590	6.0467
12	295.1351	295.4281	-0.2930	-1.6492	2.7199
13	0.6360	358.6258	2.0102	0.6540	0.4277
14	5.6543	3.6175	2.0368	0.6806	0.4632
15	274.5648	272.6322	1.9326	0.5764	0.3322
16	141.3839	140.4197	0.9642	-0.3920	0.1537
17	265.8782	263.6658	2.2125	0.8563	0.7332
18	60.8531	61.5871	-0.7341	-2.0903	4.3694
19	165.7300	164.0715	1.6585	0.3022	0.0913
20	241.2255	241.1438	0.0817	-1.2745	1.6244
21	312.5649	308.8140	3.7509	2.3947	5.7346
22	57.9980	57.8810	0.1170	-1.2393	1.5359
23	291.0591	289.3555	1.7036	0.3474	0.1207
24	267.7783	267.3057	0.4726	-0.8836	0.7807
25	265.0599	263.3315	1.7284	0.3722	0.1385
26	245.8590	242.5531	3.3058	1.9496	3.8009
27	183.9014	182.1814	1.7200	0.3638	0.1324
28	166.4226	166.0022	0.4204	-0.9358	0.8757
29	224.6206	222.5857	2.0349	0.6787	0.4606
30	153.3238	151.4904	1.8334	0.4772	0.2277
31	102.6557	101.5828	1.0729	-0.2833	0.0803
32	61.5132	60.2128	1.9088	0.5526	0.3054

As one can see, the distortion coefficient for several directions of this plan is almost equal to unit numbers 9 and 23, which are located along the main directions of the plan, so the distortion of their lengths is minimal and the scale is kept along these directions on the plan. The lines with high distortion coefficient are concentrated in the center of the plan, in the area of the Rynok Square, routed along the streets. For the outer directions, a slight distortion of lengths less than 3 % is typical.

The mean square error of the distance in the plan of 1931 is $m_l = 20$ m, and the relative error was determined through the relative distortion coefficient of the lengths $M_l = 2.5\%$.

According to the results given in Table 4, the angle of rotation of the plan is determined and its magnitude is 1,35° in the north-easterly direction in relation to the modern plan. The magnitude of the mean square error of the angles $m_\alpha = 1.31^\circ$.

Conclusions

The determination of the geometric accuracy of the old plans of Lviv, as a tool for scientific research, is relevant since it enables us to evaluate the cartographic publication as a result of human activity more objectively and fully. Numerical data obtained during such a research make it possible to compare the cartographic, documentary, and content values of old plans.

The quantitative technique chosen in the study of Lviv's 19th and 20th century plans, based on direct measurements of the lengths of lines and angles between two directions, made it possible to compare the errors of linear and angular variables of time-varying and multi-scale plans of Lviv.

The plan of Lviv in 1931 is more precise with respect to angle measurements: the rotation angle of the plan is 1,35° in the north-easterly direction in relation to the modern plan (for the plan of 1844 this value is 15°); the mean square error of the angles is $m_\alpha = 1,31^\circ$ (for the Lviv's plan of 1844 $m_\alpha = 1,7^\circ$).

The distortion of the lengths of the lines is characteristic to both the plans of Lviv, but the plan of Lviv in 1931, with the exception of its central part, has a less relative RMSE of distance $M_l = 2.5\,\%$ unlike the plan of 1844($M_l = 3.2\,\%$).

Having been worked out by us the research methodology can be used to evaluate the accuracy of other plans of Lviv and other Ukrainian cities.

We consider it expedient to continue the study of the accuracy of the above-mentioned plans of Lviv using graphical research methods that will allow us to evaluate and illustrate the spatial variations of errors, as well as to identify and understand the technical aspects of creating these cartographic works. In particular, what surveying methods and sources were used to create a plan, or to check its geodetic reference.

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ДОСЛІДЖЕННЯ ТОЧНОСТІ ПЛАНІВ ЛЬВОВА 1844 ТА 1931 РОКІВ

Мета цієї роботи - оцінка точності планів Львова 1844 та 1931 років. **Методика**. Основним критерієм оцінки достовірності стародавніх планів є дослідження їх геометричної точності, яка визначається величинами похибок, що виникають при вимірюванні довжин, кутів, координат та інших картометричних характеристик. Для дослідження точності планів Львова 1844 та 1931 років було застосовано кількісну методику, яка базується на використанні безпосередніх вимірювань довжин ліній та кутів між двома напрямками на стародавньому і сучасному планах та аналізі цих вимірювань на основі статистичної теорії похибок з використанням формул Гаусса та Бесселя. **Результати.** Згідно з вказаною методикою для кожного плану було визначено середню квадратичну похибку відстані, відносну середню квадратичну похибку відстані, кут повороту та середню квадратичну похибку дирекційних кутів. Ці характеристики дали можливість порівняти похибки лінійних та кутових величин двох різночасових та різномасштабних планів Львова між собою та дали підстави вважати, що план Львова 1931 року є точнішим щодо кутових вимірів. Спотворення відстаней характерні для обох планів Львова, однак меншу відносну середню квадратичну похибку має все ж таки план 1931 року, незважаючи на дрібніший масштаб. Обчислені показники спотворень довжин лінії та амплітуда коливань граничних величин знаменника масштабу роблять можливим проведення аналізу спотворень відстаней у різних частинах плану та продовження дослідження точності вищезгаданих картографічних матеріалів з використанням графічних методів дослідження, які дозволили б оцінити та проілюструвати просторові варіації похибок, а також виявити і зрозуміти факти та технічні аспекти створення цих картографічних творів. **Наукова новизна та практична значущість.** Визначення геометричної точності стародавніх планів Львова як інструментарію для наукових досліджень є актуальним, оскільки сприяє об’єктивнішому і повнішому оцінюванню картографічного видання як результату людської діяльності. Числові дані, що отримані в процесі досліджень, дають можливість порівнювати картографічну, документальну та змістову цінність стародавніх планів. Вибрана кількісна методика при дослідженні стародавніх планів Львова, що базується на безпосередніх вимірюваннях довжин ліній та кутів між двома напрямками, уможливила порівняти похибки лінійних та кутових величин різночасових та різномасштабних планів Львова. Опрацьована методика дослідження може бути використана для оцінювання точності інших планів Львова та міст України.

Ключові слова: стародавній план; теорія похибок; показник спотворення довжини; середня квадратична похибка; формула Гауса; дирекційний кут.

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