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DETERMINATION OF PLUMB LINES WITH USING TRIGONOMETRIC LEVELLING AND GNSS MEASUREMENTS

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The purpose of the study is to attempt to determine the deviation of vertical lines using trigonometric levelling and Global Navigation Satellite System (GNNS) measurements. For the last decades with the emergence of high-precision electronic theodolites and tacheometers, trigonometric levelling becomes a competitor of the geometric levelling of the II and III accuracy classes. This is primarily the definition of exceedances at distances up to 1–2 km for topographic surveying and the study of geodynamic processes in zones of the man-made load. Today, high precision robotic electronic tacheometers have been developed, which allow to significantly improve the accuracy of the measurement of zenith distances by automatically guiding the target to the maximum reflected signal. Such robotic tacheometers carry out measurements of anti-aircraft distances and distances without the direct participation of the observer. **The method** of achieving this goal is provided by theoretical and experimental studies to improve the accuracy of trigonometric alignment and the use of high-precision GNSS measurements. It is also important here to switch from the spatial geodetic coordinates B, L, H to local topocentric coordinates in order to provide control over the deformation of hydraulic structures and the territory of man-caused loading of the main structures in the area at the Dniester hydroaccumulation power plant (DHPP). **The main result** of the study is the possibility of taking into account the influence of the refraction and the gravitational field of the earth on the accuracy of the trigonometric levelling and the determination of the deviations of vertical lines from a two-way trigonometric levelling with short distances from 500 to 1000 m. **Scientific novelty:** the proposed approach allows to calculate the effect of the refraction and the gravitational field of the Earth on the resulting trigonometric levelling with high accuracy. In addition, using trigonometric levelling and GNSS measurements, it is possible to independently determine the deviation of the vertical lines. **Practical significance:** the proposed method makes it possible to estimate the effect of vertical lines on the results of two-way synchronous trigonometric levelling.

Key words: trigonometric levelling, GNSS-measurement, vertical refraction, the deviation of the vertical lines, the gravitational field of the Earth.

Introduction

The main characteristics of the gravitational field of the Earth are the components of the deviations of vertical lines, the height of the quasigoid (height anomalies), and normal heights. Thus, for calculating the component deviations of the vertical lines, two methods are mainly used: astronomical-geodesic and gravimetric. In the astronomical-geodesic method, the components of the deviations of the vertical lines $\xi_{a г}$ and $\eta_{a г}$, are determined from the ratio:

$$\xi_{a г} = \varphi - B \quad (1)$$
$$\eta_{a г} = (\lambda - L)\cos\varphi ,$$

where φ and λ – astronomical coordinates of the point of the physical surface of the Earth, which are obtained from astronomical observations; B and L – geodetic coordinates obtained from geodetic network processing or from satellite measurements.

Consequently, the astronomical-geodesic components of the deviations of vertical lines $\xi_{a г}$ and $\eta_{a г}$ are calculated from the known astronomical and geodetic coordinates of the points of the physical surface of the Earth, using the ratio (1).

In the gravimetric method, from the points of the physical surface of the Earth, the angle between the directions of the vectors of the real and normal acceleration of the free fall is determined, and then, respectively, the gravimetric components of the deviations of the vertical lines $\xi_{г p}$ and $\eta_{г p}$, using the Vening Meinesz's formulas:

$$\begin{Bmatrix} \xi_{г p} \\ \eta_{г p} \end{Bmatrix} = \frac{1}{2\pi} \int_0^{\pi} \int_0^{2\pi} \Delta g \cdot Q(\psi) \begin{Bmatrix} \cos A \\ \sin A \end{Bmatrix} d\psi dA \quad (2)$$

where Δg – gravimetric anomaly of free fall acceleration, $Q(\psi)$ – Vening Meinesz function, A – azimuth of direction.

In the presence of gravimetric data on the entire earth's surface, gravimetric components of the vertical line deviations, which are assigned to the level ellipsoid, whose center coincides with the center of mass of the Earth, can be calculated according to gravimetric anomalies. The problem of determining the gravimetric component of the deviations of the vertical lines on the physical surface of the Earth according to the anomalies measured was solved by M. S. Molodenskyi. Molodenskyi's formulas for the calculation of these variables, consisting of the main and correctional elements, takes into account the complex nature of the earth's surface. The main elements of the formulas coincide with the corresponding Vening Meinesz's formulas, which have been practically used for the flat areas. Thus, according to the formulas of Vening Meinesz, gravimetric components of the vertical lines' deviations with a mean square error (0.3–0.5)$''$ for flat areas and with (1–1.4)$''$ for mountains, can be calculated.

The Molodenskyi's method of the study of the figure and the external gravitational field of the Earth allows to accurately compute the gravimetric components of the vertical line deviations on the physical surface of the Earth using the gravimetric and topographic maps [Molodenskij, Eremeev & Jurkina, 1960].

There is a dependence between the astronomical-geodesic and gravimetric components of vertical line deviations:

$$\xi_{аг} = \xi_{гр} + \delta B,$$
$$\eta_{аг} = \eta_{гр}, \qquad (3)$$
$$\delta B = 0{,}171'' \cdot H \cdot \sin 2B$$

In these formulas:

δB – correction for curvature of the power line of the normal field; B – geodetic latitude of the point; H – geodetic height, denoted in km.

Note that the constituents of the deviations of plumb lines $\xi_{аг}$ and $\eta_{аг}$ are calculated in the system of the accepted reference-ellipsoid, and the components of the deviations of the vertical lines $\xi_{гр}$ and $\eta_{гр}$ – in the system of the level ellipsoid..

The magnitude of the deviations of the sloping lines in the flat terrain is in average 3–5$''$, sometimes they can reach 10–15$''$ in complex anomalous zones (Moscow gravimetric attraction). There exists a particularly noticeable change in the deviation of straight lines in mountainous terrain. Thus, in the western part of the Caucasus, the component of the vertical line deviation in the plane of the meridian $\xi_{аг}$ at a distance of about 300 km varies from -27$''$ to 20$''$. In the Lake Baikal area, this change reaches 0.67$''$/km with a maximum deviation of 30$''$. In Ukraine, the maximum deviation of vertical lines is fixed at the Crimean Astrophysical Observatory with the value of 35.7$''$. The maximum value of deviations of vertical lines on Earth is found in the Hawaiian Islands (≈97$''$) [Jakovlev, 1989].

Determining vertical line deviations using the two different described methods on the physical surface of the Earth have their advantages and disadvantages. In the first method, the components of deviations of vertical lines (astronomical-geodesic) are calculated from the known astronomical and geodetic coordinates of the points on the physical surface of the Earth.

Such astronomical definitions of φ and λ were performed in the so-called Laplace points, the distance between which was 70–100 km. The duration of such astronomical observations in the middle latitudes lasted up to one month, and the accuracy of calculations of $\xi_{аг}$ and $\eta_{аг}$ was low. The components of $\xi_{аг}$ and $\eta_{аг}$ deviations in the intermediate points between the astronomical points were determined by interpolation, and the accuracy of the determination of astronomical latitudes and longitudes was respectively. $m_{\varphi} = 0{,}3''$, $m_{\lambda} = 0{,}5''$.

Instead, the gravimetric method of determining the components of the vertical line deviations has significant advantages compared to the first method: 1) the use of gravimetric surveying, which does not depend on weather conditions; 2) an error of interpolation according to research data was 0.3–0.5$''$ for flat areas and 1.0–1.4$''$ for mountains [Dvulit, 1998; Dvulit, 2008; Heiskanen & Moritz, 1981].

Other methods of determining the astronomical-geodesic deviations of the vertical lines also deserve attention. These methods include the transfer of astronomical and geodetic deviations of the vertical lines using trigonometric (geodetic) levelling and GNSS-measurements, which will be considered in detail in the next section.

Analysis of recent research

The question of the effect of deviations of straight lines on the results of measured zenith distances is considered in many publications [Ogorodova, 2006; Litynskyi, & Perii, 2006; Jordan, Eggert & Kneissl, 1969; Czarnecki, 2010; Ceylan, 2009]. In addition, the measured zenith distance differs from the geodetic for the effect of the deviations of the vertical lines and the effect of refraction [Jakovlev, 1989; Torge, 1989; Biro, 1983]. The accuracy of trigonometric levelling is influenced by various factors: the measurement errors of zenith distances, device heights and sighting targets, and the error of determining the vertical refraction coefficient. The accuracy of trigonometric levelling essentially depends on the stability of the state of the atmosphere throughout the entire observation process. Therefore, in practice, in order to increase the accuracy of the measurement, two-level alignment is attempted, measuring the zenith distances and distances in the forward and reverse directions. Based on these measurements of zenith distances, the components of the vertical line deviations can be determined. There are, however, some difficulties in determining the effect of refraction. This method was used to determine the deviations of the vertical lines in the Himalayas, the Alps and the Tatras [Brovar, 1983; Hradilek, 1963; Eremeev & Jurkina, 1972]. The accuracy of determining the deviations of vertical lines is estimated by the magnitude of the error with the value of 1–2$^{//}$.

The main research material

Consider the possibility of determining the deviations of vertical lines with the correct precision for the case of a simplified approach to trigonometric alignment with short distances (in small areas) up to 500–1000 meters with two-way synchronous measurements of zenith distances. To do this, turn to Fig. 1, which shows the connection between elements of trigonometric levelling and the field of acceleration of free fall.

Figure 1 introduces the following notation:

– z_{12} and z_{21} – measured zenith distances in points P_1 and P_2,

– S' – the refractive curve,

– u_1 and u_2 – the vertical line deviation of P_1 and P_2,

– δ_1 and δ_2 – refraction angles in points P_1 and P_2,

– g_1 and g_2 – vectors of real acceleration of free fall in corresponding points P_1 and P_2,

– n_1 and n_2 – vectors of normal acceleration of free fall in corresponding points P_1 and P_2,

– $S_{12} = S_{21}$ – a straight line connecting P_1 and P_2 points,

– $\overline{S_{12}}$ – projection of measured distance between points P_1 and P_2 on the surface of the ellipsoid,

– γ – the central angle.

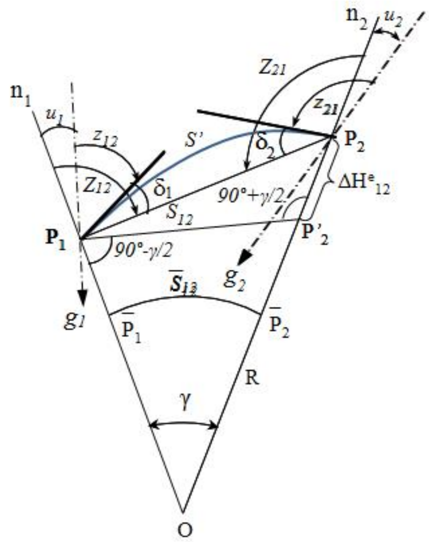

Fig. 1. The connection of trigonometric levelling with the field of free fall acceleration

From Fig. 1 the dependence arises:

$$Z_{12} = z_{12} + \delta_1 + u_1, \quad Z_{21} = z_{21} + \delta_2 - u_2 \qquad (4)$$

where Z_{12} and Z_{21} – geodetic azimuthal distances at observation points P_1 and P_2 to the ellipsoid normal.

The difference in geodetic heights ΔH^e_{12} between P_1 and P_2 points is obtained from the triangle $P_1 P_2 P_2^{'}$ using the famous Jordan's formula [Jordan, Eggert & Kneissl, 1969] which is most often used for one-way trigonometric levelling:

$$\Delta H^{e}_{12} = \overline{S_{12}}\left(1+\frac{H_1+H_2}{2R}\right)ctgZ_{12} + \frac{\overline{S_{12}}}{2R\sin^2 Z_{12}} \quad (5)$$

or for two-way trigonometric levelling:

$$\Delta H^{e}_{12} = \overline{S_{12}}\left(1+\frac{\overline{S^2}_{12}}{12R^2}+\frac{H_1+H_2}{2R}\right)tg\frac{Z_{21}-Z_{12}}{2}+\ldots \quad (6)$$

R means the average radius of curvature of normal section $\overline{P_1}\,\overline{P_2}$. Then for two-way synchronous measurements of zenith distances z_{12} and z_{21} at trigonometric levelling for short distances of 500–1000 meters one can assume that the effect of refraction on two points of observation is the same, that is $\delta_1 \approx \delta_2$.

In addition, the central angle γ is very small at short distances of trigonometric levelling.

$$\cos\frac{\gamma}{2} \approx 1$$

Then the difference of geodetic heights ΔH^{e}_{12} of two points P_1 i P_2 will be:

$$\Delta H^{e}_{12} = \overline{S_{12}}\left(1+\frac{\overline{S^2}_{12}}{12R^2}+\frac{H_1+H_2}{2R}\right)tg\frac{Z_{21}-Z_{12}}{2} - u^{A}_{m}\ldots \quad (7)$$

where $u^{A}_{m} = \frac{u_1+u_2}{2}$ – the average value of the vertical line deviation on the area.

From the GNSS measurements, the geodetic elevation points of observation P_1 and P_2, the length of normal section $\overline{S_{12}}$ and the average radius of the curvature of a normal section R can be calculated. Performing a two-way trigonometric levelling, we obtain zenith distances in two observation points P_1 and P_2. The following is denoted by:

$$F_{GNSS} = \frac{\Delta H^{e}_{12}}{\overline{S_{12}}}\left(1+\frac{\overline{S_{12}}^{2}}{12R^2}+\frac{H_1+H_2}{2R}\right)^{-1} \quad (8)$$

$$\frac{1}{2}\Delta z = \frac{z_{21}-z_{12}}{2} \quad (9)$$

Consequently, the final average value of the deviations of the vertical lines on the area will be:

$$tgu^{A}_{m} = \frac{tg\frac{1}{2}\Delta z - F_{GNSS}}{1+F_{GNSS}\cdot tg\frac{1}{2}\Delta z} \quad (10)$$

Using the formula (10), it is possible to determine the mean value of the deviation of vertical lines between two points P_1 and P_2 with trigonometric levelling and GNSS measurement of a given geodetic azimuth direction. This average deviation of the vertical lines can be calculated by another method, if the components of the deviations of the vertical lines for each point in the specified direction are separately determined. For this purpose, the Fai's field of gravimetric anomalies of a local area is most often used to calculate gravimetric constituents of deviations of straight lines using known formulas. [Brovar, Jurkina, and oth., 2010;. Dvulit, 2008; Dvulit & Holubinka, 2005; Dvulit & Holubinka, 2008].

The full average value u_m of vertical line deviations is calculated by formula:

$$u_m = \xi_{a\text{г}}\cos A + \eta_{a\text{г}}\sin A, \quad (11)$$

And by formulas (3) the transition from gravimetric to astronomical-geodesic components of the deviations of the vertical lines can be carried out.

Consequently, formula 11 gives the opportunity to control the calculations independently.

Results

To determine the deviations of the straight lines for trigonometric levelling and GNNS measurements, we used observations at the points of the high-precision geodynamic network of the Dniester hydroaccumulating power plant (DHPP). The reference geodetic network of the Dniester HPP can be considered as a local geodynamic landfill. The network points are tubular signs of long-term storage, which enable the compulsory centering of geodetic instruments. Figure 2 shows a general scheme for measuring the vectors of the reference geodetic network of the DHPP.

According to the program, at each point of the reference geodetic network, three independent sessions of GNNS observations for 6 hour durations by different receivers of satellite signals were conducted. The total observation time at each point was at least 18 hours.

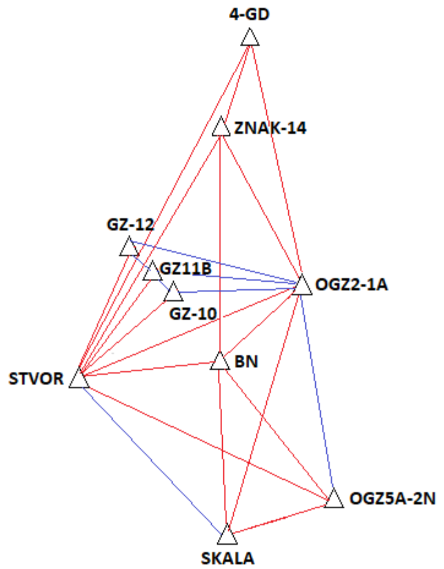

Fig. 2. The scheme of measuring the vectors of the reference geodetic network of the DHPP

The processing of the measured vectors was performed in the Leica Geo Office Combined ("LGO") software environment by calculating corrections for the effects of the ionosphere according to observations and taking into account the Hopfield tropospheric refraction. According to the results of the balancing of GNSS observations, the precision of determining the coordinates of the points does not exceed ±2 mm. For the entire period of observations, the vectors of horizontal displacements of points of the reference geodetic network are in the range from 4 to 16 mm with a mean squared error of their determination ±3 mm [Tretiak & Sidorov, 2005; Tretyak, Periy, Sidorov & Babiy, 2015].

Linear-angular measurements at the points of the geodynamic landfill were also performed on the territory of the DHPP. In 2018 high-precision linear-angular measurements of vectors were performed at the points of the reference geodetic network. Ground-based linear observations were performed by the Total Station Positioning System TSSPS TCRR-1201 from Leica. The mean squared error of measurement of horizontal and vertical angles with one reception is 1″, and the measurement of the lines ±(1 + 1.5 · ppm) mm. Measurement of the electronic tacheometer is carried out in the forward and reverse directions.

Angles and lines were measured in automated mode with 8 receivings recording the results in memory of the electronic tacheometer. The entire process of measuring the spatial network (10 observation points) was performed for 4 hours at a stable established atmospheric stratification (during the daytime period).

The values of deviations of the vertical lines have been calculated by the formula (10) and the gravimetric components were defined with the Fay's field of gravimetric anomalies at the points of the reference geodetic network of the DHPP. In addition, the average differences between the geodetic heights of vectors obtained from the results of trigonometric levelling and GNSS measurements are calculated. The results of these calculations are presented in the table, which shows:

– the name of the measured vector;

– $\overline{S}_{12}$ – projection of measured distance between points $\overline{P}_1$ and $\overline{P}_2$ on the surface of the ellipsoid;

– ΔH_{GNSS} – the difference in geodetic heights of the points P_1 and P_2 from GNSS-observations;

– $\Delta H_{trig\,liv.}$ – the average difference between the geodesic heights of the points P_1 and P_2 calculated from the results of the trigonometric levelling taking into account the correction for refraction;

– $\delta\Delta H$ – the difference between the geodesic heights obtained from GNSS measurements and trigonometric levelling;

– $u^{//}$ – the average value of the deviation of vertical lines calculated by the formula (10);

– $u^{//}_{grav.}$ – the average value of the deviation of vertical lines calculated by the Fay's field of gravimetric anomalies;

– $\delta u^{//}$ – the difference between the mean values of the vertical line deviations calculated by the formula (10) and gravimetric data;

– A – geodesic azimuths of the measured vectors.

Table 1

Comparative characteristics of the vertical line deviations and the differences in geodetic heights

The name of the measured vector	$\overline{S}_{12}$, m	ΔH_{GNSS}, m	$\Delta H_{trig.lev.}$, m	$\delta\Delta H$	$u^{//}$	$u^{//}_{грав.}$	$\delta u^{//}$	A°
SKALA-BN	954.914	127.018	127.035	-0.017	-4.5	-4.9	0.4	0
SKALA-OGZ5A-2N	621.112	121.281	121.255	0.026	7.9	4.9	3	61
BN-OGZ2-1A	481.019	-13.413	-13.424	0.011	5.5	3.7	1.8	62
BN-OGZ5A-2N	848.859	-5.738	-5.747	0.009	2	-0.5	2.5	140
GZ10-STVOR	508.479	-128.813	-128.793	-0.02	-5.4	-6.7	1.3	231
GZ11B-STVOR	504.119	-128.839	-128.823	-0.016	-4	-6.5	2.5	221
GZ12-STVOR	508.467	-128.819	-128.797	-0.022	-5.4	-6.3	0.9	215
STVOR-OGZ2-1A	1002.521	117.414	117.375	0.039	8	5.4	2.6	68
ZNAK14-OGZ2-1A	702.58	130.354	130.315	0.039	7	1.4	5.6	154
OGZ2-1A-ZNAK14	702.58	12.939	12.936	0.003	-1.9	-0.5	-1.4	334
4GD-ZNAK14	403.394	104.358	104.326	0.032	4.6	1	3.6	65
ZNAK14-BN	865.187	0.473	0.478	-0.005	-0.9	-3.2	2.3	187

Note that the differences in geodetic heights $\delta\Delta H$, calculated from trigonometric levelling and GNSS measurements, do not exceed ±3–4 cm depending on the excess between the points of the vectors, as well as from the distances between them. In Table 1, the distance of the measured vectors is from 400 to 1000 m. Such differences can be explained by the accuracy of measurements of zenith distances, distance between points, the error of determining the vertical refraction coefficient, and other factors. It should also be noted that the geodetic zenith distances were calculated taking into account the influence of refraction. But, on the results of calculations of excess geodetic heights with trigonometric leveling, this effect was not identified. The differences $\delta u^{//}$ do not exceed $(5–6)^{//}$. This is explained primarily by the fact that the gravimetric components of the vertical line deviations were calculated on the basis of gravimetric Fay's anomalies for the Krasovskii's ellipsoid, and for the points of the reference geodetic network of the DHPP, an ellipsoid WGS-84 was used. In addition, we took into account the field of Fay's anomalies for the influence of the central and near zones on the gravimetric components of the vertical line deviations. It is known that the influence of the distant zones of the field of gravimetric anomalies is 10–15 % of the total influence of anomalies on the globe according to experimental research [Dvulit & Holubinka, 2005; Dvulit & Holubinka, 2008]. However, in our studies, we did not take into account the influence of the gravitational field of the Earth on the accuracy of gravimetric levelling and the determination of vertical line deviations.

Scientific novelty and the practical importance

The proposed approach for determining the deviations of the vertical lines allows us to calculate the effect of the refraction and the gravitational field of the Earth on the results of trigonometric levelling with increased accuracy. Such technique makes it possible to evaluate the effect of the deviations of vertical lines on the results of two-way synchronous trigonometric levelling and on the study of geodynamic processes in zones of man-made load.

Conclusion

1. On the basis of theoretical studies and joint processing of the results of trigonometric levelling and GNSS measurements, an attempt was made to determine the astronomical-geodesic deviations of the vertical lines for the points in the territory of the DHPP.

2. The accuracy of determining the mean values of the astronomical-geodesic deviations of the slip lines can be estimated $(2–3)^{//}$ y using the proposed method, which depends on the method of

taking into account the influence of vertical refraction and the gravitational field of the Earth when performing two-way synchronous measurements of the zenith distances of the measured vectors.

3. In order to increase the accuracy of calculating the difference between geodetic heights with trigonometric levelling, it is necessary to take into account the effects of vertical refraction and gravimetric components of the vertical line deviations for the calculation of geodetic zenith distances.

4. In future studies, the local field of gravimetric anomalies should be used, which can be assigned to the level ellipsoid WGS-84 or to the reference ellipsoid, taking into account the formula for the normal value of the acceleration of free fall. The influence of the distant zones should also be taken into account, using the models of the gravitational field of the Earth.

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ВИЗНАЧЕННЯ ВІДХИЛЕНЬ ПРЯМОВИСНИХ ЛІНІЙ З ВИКОРИСТАННЯМ ТРИГОНОМЕТРИЧНОГО НІВЕЛЮВАННЯ ТА GNSS-ВИМІРЮВАНЬ

Метою дослідження є спроба визначити відхилення прямовисних ліній з використанням тригонометричного нівелювання та GNNS-вимірювань. За останні десятиліття з появою високоточних електронних теодолітів і тахеометрів, тригонометричне нівелювання стає конкурентом геометричного нівелювання II і III класів точності. Це насамперед визначення перевищень на відстанях до 1–2 км за топографічних знімань та досліджень геодинамічних процесів у зонах техногенного навантаження. Сьогодні з'явилися високоточні роботизовані електронні тахеометри, які дають змогу істотно підвищити точність вимірювання зенітних відстаней за рахунок автоматичного наведення на візирну ціль за максимум відбитого сигналу. Такі роботизовані тахеометри виконують вимірювання зенітних відстаней і віддалей без безпосередньої участі спостерігача. **Методику** досягнення цієї мети забезпечено теоретичними та експериментальними дослідженнями щодо підвищення точності тригонометричного нівелювання і використання високоточних GNSS-вимірювань. Тут також важливо перейти від просторових геодезичних координат B, L, H до локальних топоцентричних координат з метою забезпечення контролю за деформацією гідротехнічних споруд та території техногенного навантаження в зоні основних споруд Дністровської гідроакумулювальної електростанції (ДГАЕС). **Основний результат** дослідження – можливість враховувати вплив рефракції та гравітаційного поля Землі на точність тригонометричного нівелювання і визначити відхилення прямовисних ліній із двостороннього тригонометричного нівелювання з короткими відстанями від 500 до 1000 м. **Наукова новизна:** запропонований підхід дає змогу розрахувати вплив рефракції та гравітаційного поля Землі на результати тригонометричного нівелювання підвищеної точності. Крім цього, використовуючи тригонометричне нівелювання і GNSS-вимірювання, можна незалежно визначати відхилення прямовисних ліній. **Практична значущість:** запропонована методика дає можливість оцінювати вплив прямовисних ліній на результати двостороннього синхронного тригонометричного нівелювання.

Ключові слова: тригонометричне нівелювання, GNSS-вимірювання, вертикальна рефракція, відхилення прямовисних ліній, гравітаційне поле Землі.

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