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Identifying the set of all admissible disturbances: discrete-time systems with perturbed gain matrix

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This paper focuses on linear controlled discrete-time systems which subject to the control input disturbances. A disturbance is said to be admissible if the associated output function verifies the output constraints. In this paper, we address the following problem: determine the set of all admissible disturbances from all disturbances susceptible to the deformation of control input. An algorithm for computing the maximum admissible disturbances set is described and the sufficient conditions for finite termination of this algorithm are given. Numerical examples are given. The case of discrete-time delayed systems is also considered.

Keywords: *discrete-time system, linear system, perturbed system, admissible disturbance, delayed system.*

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1. Introduction

Disturbed systems have also been extensively studied in the past decade. In most control systems, the existence of disturbances has a remarkable probability. The influence of the physical environment on the systems leads to the emergence of these undesirable parameters [1–5], and references therein. In this direction, and in the control literature, there are many works and techniques to avoid the effects of disturbances, [6–15] and references therein. These disturbances can be deterministic or stochastic and can affect different components of the system, for example, the system's dynamic, the control operator, the initial state ..., which can drive the system to unstable behavior, or constraints violations. In order to contribute to this thematic, we are interested in a class of systems described by

$$\begin{cases} x_{i+1} = Ax_i + Bu_i, \\ x_0 \in \mathbb{R}^n, \\ y_i = Cx_i, \quad i \geqslant 0, \end{cases}$$

where $A \in \mathbb{R}^{n\times n}$ is the dynamic matrix, $B \in \mathbb{R}^{n\times m}$ is the matrix governing the distribution of the control variable on the different components of the state x_i. $(u_i)_i \in \mathbb{R}^m$ is the control input and $y_i = Cx_i$ is the response output, where $C \in \mathbb{R}^{p\times n}$. We assume that the feed-back control $(u_i)_i$ is given by $u_i = Kx_i$, where $K \in \mathbb{R}^{m\times n}$. The gain matrix K is designed with the aim of forcing y_i to achieve the desired goal.

For reasons that are due to random phenomenon, or by the principle of action and reaction between a system and its surroundings or in the system simulations via a machine, task, which always generates approximations and various disturbances. We assume that when applying the gain K, one can not avoid disturbances $(K_i)_i$, in other words, it is rather the disturbed control $u_i = (K + K_i)x_i$ which acts on the system, where $K_i \colon \mathbb{R}^n \to \mathbb{R}^m$ is not necessarily linear.

Inspired by works of [16–21]. We develop in the present work a theoretical and algorithmic approach, for determining, among all disturbances K_i that may affect the system, those whose effect is relatively tolerable, i.e. given a set of constraints Ω, the aim is to determine all disturbances such that the corresponding output satisfies

$$y_i \in \Omega, \quad i \geqslant 0$$

A violation of these constraints can lead the closed-loop system to an unstable behavior, or serious damage may happen. Since time delay is encountered in various engineering systems, for examples, chemical processes, biological systems, economic systems and hydraulic/pneumatic systems. A class of delayed systems is also considered.

This paper is organized as follows. Some preliminaries are given in Section 2. Section 3 contains efficient algorithms for computing the set of all admissible disturbances that infect the control input. Section 4 provides numerical examples to show the efficiency of the proposed technique and contains some figures illustrating the performance of the algorithm defined in the previous section. A class of discrete-time delayed systems is also considered in Section 5 followed by conclusion in Section 6.

2. Preliminary results

The controlled linear discrete-time system considered is

$$\begin{cases} x_{i+1} = Ax_i + Bu_i, \\ x_0 \in \mathbb{R}^n. \end{cases} \tag{1}$$

The associated output function is

$$y_i = Cx_i, \quad i \geqslant 0 \tag{2}$$

and the infected control input is given by

$$u_i = (K + K_i)x_i \in \mathbb{R}^m, \tag{3}$$

where the state variable $x_i \in \mathbb{R}^n$ and A, B, C, K are respectively $(n \times n)$, $(n \times m)$, $(p \times n)$, $(m \times n)$ matrices, and $(K_i)_{i\geqslant 0}$ are maps which describe the disturbances that infect the control input. K_i represents all kinds of unwanted signals inputs which then affect the control-system's output. For instance, sensor noise signal, load disturbances, gusts of wind hitting the satellite dish of a tracking radar create unwanted large torques that affect the position of the antenna.

Replacing (3) in (1) we have

$$\begin{cases} x_{i+1} = Ax_i + B(K + K_i)x_i, \\ x_0 \in \mathbb{R}^n \end{cases}$$

by changing $A + BK \to A$ and $BK_i \to P_i$ we have

$$\begin{cases} x_{i+1} = (A + P_i)x_i, \\ x_0 \in \mathbb{R}^n. \end{cases} \tag{4}$$

For physical considerations, and without loss of generality, we assume that all disturbances susceptible of infecting the system (4) have a limited age, i.e $(P_i)_{i\geqslant 0}$ are persistent on a given time interval $\{0, \dots, I\}$ which means that

$$P_i \equiv 0, \quad \forall i > I,$$

I is called the age of the disturbances $(P_i)_{i\geqslant 0}$.

Motivated by practical considerations, the controlled output is required to satisfies

$$y_i \in \Omega, \quad i \geqslant 0, \tag{5}$$

where $\Omega \in \mathbb{R}^p$ is the set of constraints.

Definition 1. *We say that a disturbance $(P_i)_{0\leqslant i\leqslant I}$ is admissible, if the corresponding output satisfies*

$$y_i \in \Omega, \quad i \geqslant 0.$$

Otherwise $(P_i)_{0\leqslant i\leqslant I}$ is said inadmissible.

Then the principal goal in this paper, is to characterize the set Σ of all admissible disturbances which will be called the maximal admissible disturbances set described as follows

$$\Sigma = \{(P_i)_{0\leqslant i\leqslant I}/y_i \in \Omega, \, \forall i \geqslant 0\}. \tag{6}$$

We see that Σ can be written as follows

$$\Sigma = U \cap V, \tag{7}$$

where

$$\begin{aligned} U &= \{(P_i)_{0\leqslant i\leqslant I}/y_i \in \Omega, \, \forall i = 0, 1, \dots, I\}, \\ V &= \{(P_i)_{0\leqslant i\leqslant I}/y_i \in \Omega, \, \forall i \geqslant I+1\}. \end{aligned} \tag{8}$$

Note that U is determined by a finite number of inequalities but V is defined by an infinite number of inequalities. The idea of this decomposition will be useful for the algorithmic determinations of V so of Σ.

Proposition 1. The set V in (8) can be written as follows

$$V = \{(P_j)_{0\leqslant j\leqslant I}/CA^j\Gamma((P_j)_{0\leqslant j\leqslant I}) \in \Omega, \, \forall j \geqslant 0\}, \tag{9}$$

where $\Gamma((P_j)_{0\leqslant j\leqslant I}) = \prod_{j=0}^{I}(A + P_{I-j}).x_0 \in \mathbb{R}^n$.

Proof. For $i \geqslant I+1$, we have

$$\begin{aligned} x_i &= (A + P_{i-1})(A + P_{i-2}) \dots (A + P_0)x_0, \\ x_i &= \prod_{k=1}^{i}(A + P_{i-k}).x_0, \\ x_i &= \prod_{k=1}^{i-I-1}(A + P_{i-k}).x_0. \prod_{k=i-I}^{i}(A + P_{i-k}).x_0, \\ x_i &= A^{i-I-1}\prod_{k=i-I}^{i}(A + P_{i-k}).x_0, \\ x_i &= A^{i-I-1}\prod_{j=0}^{I}(A + P_{I-j}).x_0, \end{aligned}$$

therefore

$$x_i = A^{i-I-1}\Gamma\left((P_j)_{0\leqslant j\leqslant I}\right) \tag{10}$$

and

$$\Gamma((P_j)_{0\leqslant j\leqslant I}) = \prod_{j=0}^{I}(A + P_{I-j}).x_0.$$

Using (2), (8) and (10), V is written as follows

$$V = \left\{(P_i)_{0\leqslant i\leqslant I}/CA^{i-I-1}\Gamma((P_i)_{0\leqslant i\leqslant I}) \in \Omega,\, \forall i \geqslant I+1\right\}$$

or

$$V = \left\{(P_j)_{0\leqslant j\leqslant I}/CA^{j}\Gamma((P_j)_{0\leqslant j\leqslant I}) \in \Omega,\, \forall j \geqslant 0\right\}.$$

■

As $\Gamma((P_j)_{0\leqslant j\leqslant I})$ is a vector of $\mathbb{R}^n$ then we will introduce a set Λ defined by

$$\Lambda = \{x \in \mathbb{R}^n/CA^j x \in \Omega, \forall j \geqslant 0\} \quad \text{and} \tag{11}$$

$$\Lambda_k = \{x \in \mathbb{R}^n/CA^j x \in \Omega, \forall j = 0, 1, \ldots, k\}. \tag{12}$$

In [22], the set Λ is called the maximal output admissible set.

Remark 1. We note that for all $k \geqslant 0$: $\Lambda \subset \Lambda_{k+1} \subset \Lambda_k$.

For a complete determination of Λ we need the following results.

Proposition 2. If there is an integer k^* such that $\Lambda_{k^*} = \Lambda_{k^*+1}$ then $\Lambda_{k^*} = \Lambda$, and Λ is said to be finitely determined.

Proof. let $x \in \Lambda_{k^*}$ then $CA^j x \in \Omega$ $\forall j = 0, 1, \ldots, k^*$ since $\Lambda_{k^*} = \Lambda_{k*+1}$ then $CA^{k^*+1}x = CA^{k^*}(Ax) \in \Omega$ therefore $Ax \in \Lambda_{k^*}$, then we have $Ax \in \Lambda_{k^*}$ $\forall x \in \Lambda_{k^*}$. By iteration we have $A^j x \in \Lambda_{k^*}$ $\forall j \geqslant 0$, therefore $\Lambda_{k^*} \subset \Lambda$, and since $\Lambda \subset \Lambda_{k^*}$, we deduce that $\Lambda = \Lambda_{k^*}$. ■

Remark 2. We note that V is given by

$$V = \{(P_i)_{0\leqslant i\leqslant I}/\Gamma((P_i)_{0\leqslant i\leqslant I}) \in \Lambda\}. \tag{13}$$

Proposition 3. If $\Lambda_{k^*} = \Lambda_{k^*+1}$ for an integer k^* then the set of all admissible disturbances is given by

$$\Sigma = \{(P_i)_{0\leqslant i\leqslant I}/y_i \in \Omega \ \ \forall i = 0, 1, \ldots, I+1+k^*\}. \tag{14}$$

Proof. If $\Lambda_{k^*} = \Lambda_{k^*+1}$ then by proposition 2 and (13), the set V can be written as follows

$$\begin{aligned}
V &= \{(P_j)_{0\leqslant j\leqslant I}/\Gamma((P_j)_{0\leqslant j\leqslant I}) \in \Lambda_{k^*}\},\\
V &= \{(P_j)_{0\leqslant j\leqslant I}/CA^j\Gamma((P_j)_{0\leqslant j\leqslant I}) \in \Omega,\, \forall j = 0, 1, \ldots, k^*\},\\
V &= \{(P_i)_{0\leqslant i\leqslant I}/CA^{i-I-1}\Gamma((P_i)_{0\leqslant i\leqslant I}) \in \Omega,\, \forall i = I+1, \ldots, I+1+k^*\}
\end{aligned}$$

according to (2) and (10)

$$V = \{(P_i)_{0\leqslant i\leqslant I}/y_i \in \Omega,\, \forall i = I+1, \ldots, I+1+k^*\}$$

and by (8), we have

$$\Sigma = \{(P_i)_{0\leqslant i\leqslant I}/y_i \in \Omega,\, \forall i = 0, 1, \ldots, I+1+k^*\}.$$

■

Hereafter, we will give some sufficient conditions for existence of such integers k^*.

Theorem 1. *If we have:*

1) A is asymptotically stable (i.e., the eigenvalues λ_i of A satisfy the condition $|\lambda_i| < 1$ for all i).

2) The pair (A, C) is observable (i.e., the matrix $\left[C^T, A^TC^T, \cdots, (A^T)^{n-1}C^T\right]$ has rank n).

3) Ω bounded and contains the origin in its interior,

then there exists an integer k^* such that $\Lambda = \Lambda_{k^*}$.

Proof. The pair (A, C) is observable implies that the matrix $H = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$ has rank n, then the matrix $H^T H$ is invertible.

Therefore

$$\begin{gathered} \exists \alpha > 0, \forall x \in \mathbb{R}^n \quad \text{such that} \quad \alpha \|x\|^2 \leqslant \langle H^T H x, x \rangle, \\ \text{then} \quad \alpha \|x\|^2 \leqslant \left\|H^T\right\| \|Hx\| \|x\|, \forall x \in \mathbb{R}^n. \end{gathered} \tag{15}$$

It follows from the definition of Λ_{n-1} that $Hx \in \overbrace{\Omega \times \Omega \times \cdots \times \Omega}^{n\ fois} \forall x \in \Lambda_{n-1}$ and since Ω is bounded, we deduce from (15) that

$$\begin{gathered} \alpha \|x\|^2 \leqslant \beta \|x\| \ \forall x \in \Lambda_{n-1} \quad \text{for some} \quad \beta \in \mathbb{R}, \\ \text{hence} \quad \|x\| \leqslant \gamma, \forall x \in \Lambda_{n-1} \quad \text{for some} \quad \gamma \in \mathbb{R}. \end{gathered}$$

Since the origin belongs to the interior of Ω then there exists $\varepsilon > 0$ such that $B(0, \varepsilon) \subset \Omega$ (where $B(0, \varepsilon)$ is the open ball of radius ε).

From the asymptotic stability of A we deduce that

$$\exists k^* \geqslant n-1 \quad \text{such that} \quad \left\|CA^{k^*+1}\right\| \leqslant \frac{\varepsilon}{\gamma}.$$

On the other hand, $\Lambda_{k^*} \subset \Lambda_{n-1}$ implies that

$$\|x\| \leqslant \gamma, \forall x \in \Lambda_{k^*},$$

thus for all $\forall x \in \Lambda_{k^*}$ we have

$$\begin{aligned} \left\|CA^{k^*+1} x\right\| &\leqslant \left\|CA^{k^*+1}\right\| \|x\| \\ &\leqslant \frac{\varepsilon}{\gamma} \gamma = \varepsilon \end{aligned}$$

Hence $CA^{k^*+1} x \in B(0, \varepsilon) \subset \Omega$, $\forall x \in \Lambda_{k^*}$, this shows that $\Lambda_{k^*} \subset \Lambda_{k^*+1}$ or equivalently $\Lambda_{k^*} = \Lambda_{k^*+1}$ (since $\Lambda_{k^*+1} \subset \Lambda_{k^*}$). ■

3. Algorithmic determination

To determine the integer k^* defined above, the following algorithm is suggested.

Algorithm 1 Version 1

Step 1: set $k = 0$ and move to Step 2

Step 2: if $\Lambda_k = \Lambda_{k+1}$ then $k^* = k$
else continue

Step 3: $k = k + 1$ and return to Step 2

This algorithm is conceptually similar to what is done in [22]. We show how the test in Step 2 can be implemented, in the case where Ω is described as follows

$$\Omega = \{x \in \mathbb{R}^m / f_i(x) \leqslant 0, \forall i = 0, 1, \ldots, s\},$$

where $f_i : \mathbb{R}^m \to \mathbb{R}$ are a given functions.

Therefore Λ_k can be written as follows

$$\Lambda_k = \{x \in \mathbb{R}^n / f_i(CA^j x) \leqslant 0,\ \forall j = 0, 1, \ldots, k, \forall i = 0, 1, \ldots, s\ \},$$

we note that for every integer k we have $\Lambda_{k+1} \subset \Lambda_k$, then $\Lambda_k = \Lambda_{k+1}$ if and only if $\Lambda_k \subset \Lambda_{k+1}$ which is equivalent to

$$f_i\big(CA^{k+1}x\big) \leqslant 0,\ \forall i = 0, 1, \ldots, s,\ \forall x \in \Lambda_k,$$

or

$$\sup_{x \in \Lambda_k} f_i\big(CA^{k+1}x\big) \leqslant 0,\ \forall i = 0, 1, \ldots, s,$$

or

$$\sup f_i\big(CA^{k+1}x\big) \leqslant 0,\ \forall i = 0, 1, \ldots, s,$$
$$\begin{cases} f_l\big(CA^j x\big) \leqslant 0, \\ j \in \{0, \ldots, k\}, \\ l \in \{1, \ldots, s\}. \end{cases}$$

Then the algorithm 1 will be implemented as follows

Algorithm 2 Version 2

Step 1: set $k = 0$ and move to Step 2

Step 2:
For $i = 0, \ldots, s$ then
maximize $F_i(x) = f_i\big(CA^{k+1}x\big)$
$$\begin{cases} f_i(CA^j x) \leqslant 0 \\ j \in \{0, \ldots, k\} \\ i \in \{1, \ldots, s\} \end{cases}$$
Let F_i^* the maximum value calculated of F_i.
If $F_i^* \leqslant 0$ for For $i = 0, \cdots, s$ then
Set $k = k^*$ and Stop
else continue

Step 3: $k = k + 1$ and return to Step 2

Remark 3. i) This algorithm can never be useful, if there are no methods to solve rather large mathematical programming problems which arise in Step 2. The search for a global optimum will be more difficult. But in the case where Ω is a polyhedron (i.e., f_i are affine functions for all $i = 0, 1, \ldots, s$), the difficulty disappears as the programming problems are linear and an efficient algorithm for it still exists.

ii) Assumptions of theorems 1 are sufficient but not necessary. If these conditions are not verified, there is not guarantee that Algorithm 2 will stop. If the Algorithm 2 converge then the set Λ is finitely determined, else it is not.

4. Examples

Consider the following system

$$\begin{cases} x_{i+1} = Ax_i + Bu_i \\ x_0 = \begin{pmatrix} -0.9 \\ -0.2 \end{pmatrix} \in \mathbb{R}^2, \end{cases} \tag{16}$$

where A and B are described as follows

$$A = \begin{pmatrix} 2 & 0 \\ 0.9 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 2 \\ 1 & -3 \end{pmatrix}. \tag{17}$$

The infected control input is

$$u_i = K + K_i, \tag{18}$$

where

$$K = \begin{pmatrix} 2.675 & 0.55 \\ 1.225 & 0.55 \end{pmatrix} \tag{19}$$

is the desired gain matrix, and $(K_i)_{i\geqslant 0}$ are a maps which describe the disturbances that infect the control input. We assume that the disturbances are unknown and disappears at $I = 10$. The associated output

$$y_i = Cx_i, \tag{20}$$

where $C = \begin{pmatrix} -0.1 & -1 \end{pmatrix}$, is required to verify the constraints

$$y_i \in \Omega = [-0.7, 0.7]\,, \ \forall i \geqslant 0.$$

From (17) and (19), the change $\tilde{A} = A + BK$ give

$$\tilde{A} = \begin{pmatrix} -0.9 & 0 \\ -0.1 & -0.1 \end{pmatrix},$$

we see that $\tilde{A}$ is asymptotically stable, and by means of simple hand calculations it is possible to verify that the pair $(\tilde{A}, C)$ is observable, thus by theorem 1 it follows that algorithm 2 will converge.

By execution of the algorithm 2 with this data, we have $k^* = 2$. As a result, the only disturbances which did not affect our system are those which verify the following equations

$$|y_i| \leqslant 0.7, \quad \forall i \in \{0, \dots, 13\}$$

and the set Σ in (14) is given by

$$\Sigma = \{(K_i)_{0\leqslant i\leqslant 10} / \, |y_i| \leqslant 0.7, \ \forall i = 0, 1, \dots, 13\}, \tag{21}$$

while the set Λ in (11) is given by

$$\Lambda = \{x \in \mathbb{R}^2 / \left|C\tilde{A}^i x\right| \leqslant 0.7, \ \forall i = 0, 1, 2\}, \tag{22}$$

which is represented in Fig. 1 by the filled area.

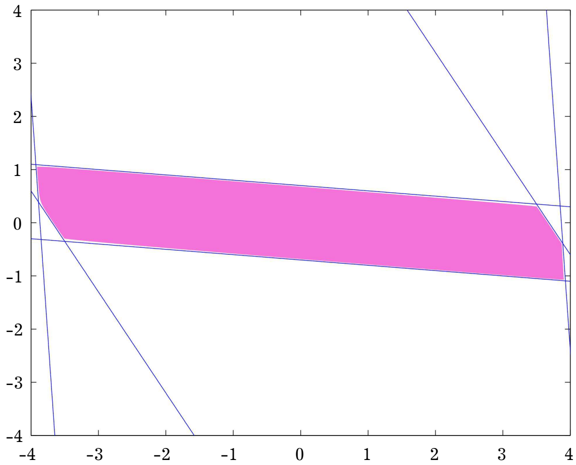


Fig. 1. The colored area represents the set Λ given by (22).

Let us define a set of indices $\mathcal{O}$ as follows

$$\mathcal{O} = \{0, 1, \dots, 10\} \tag{23}$$

and the indicator function of the set $\mathcal{O}$ given by

$$\mathbb{1}_{\mathcal{O}}(x) = \begin{cases} 1, & \text{if } x \in \mathcal{O}, \\ 0, & \text{if } x \notin \mathcal{O}. \end{cases} \tag{24}$$

Case 1. We consider an example of nonlinear disturbances $(K_i)_{i\geqslant 0}$ defined as follows

$$K_i \colon \begin{pmatrix} x \\ y \end{pmatrix} \to \left\langle \begin{pmatrix} x \\ y \end{pmatrix}, M_i \begin{pmatrix} x \\ y \end{pmatrix} \right\rangle v, \tag{25}$$

where

$$M_i = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \mathbb{1}_{\mathcal{O}}(i), \ a \in \mathbb{R} \ \text{ and } \ v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Where $\mathcal{O}$ and $\mathbb{1}_{\mathcal{O}}$ are given by (23) an (24), respectively. It is obvious that

$$K_i\begin{pmatrix} x \\ y \end{pmatrix} = \left\langle \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \right\rangle v$$

for $i = 0, 1, \ldots, 10$, and

$$K_i\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ \forall i > 10$$

and consequently the age of disturbances K_i is $I = 10$.

By (16), (18) and (25) we have

$$x_1 = \tilde{A}x_0 + 2ax_0^{(1)}x_0^{(2)}Bv, \tag{26}$$

$$x_i = 2a\left(\sum_{k=0}^{i-2} x_k^{(1)}x_k^{(2)}\tilde{A}Bv + x_{i-1}^{(1)}x_{i-1}^{(2)}Bv\right) + \tilde{A}^i x_0. \tag{27}$$

For $i \geqslant 2$, where $x_i = \begin{pmatrix} x_i^{(1)} \\ x_i^{(2)} \end{pmatrix} \in \mathbb{R}^2$. From (20), (21) and (27) we deduce that

$$\Sigma = \{a \in \mathbb{R} / \, |F_i(a)| \leqslant 0.7, \ \forall i = 0, 1, \ldots, 13\}, \tag{28}$$

where

$$F_0(a) = Cx_0,$$

$$F_1(a) = C\tilde{A}x_0 + 2ax_0^{(1)}x_0^{(2)}CBv,$$

$$F_i(a) = 2a\left(\sum_{k=0}^{i-2} x_k^{(1)}x_k^{(2)}C\tilde{A}Bv + x_{i-1}^{(1)}x_{i-1}^{(2)}CBv\right) + C\tilde{A}^i x_0, \ i \geqslant 2.$$

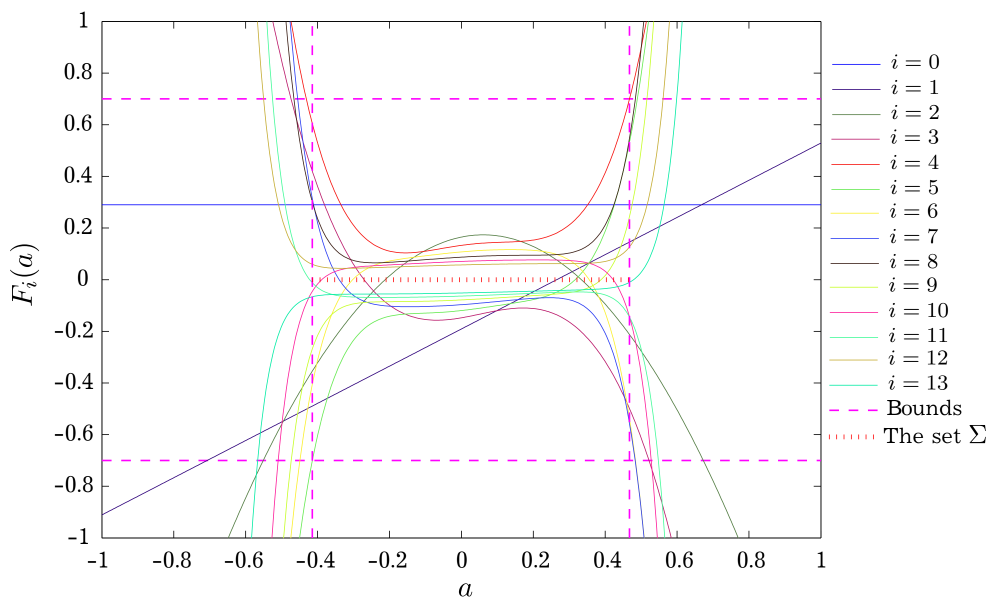


Fig. 2. Geometric representation of the set Σ given by (28).

In Fig. 2, we plot functions F_i, $i = 0, \ldots, 13$ with different colors, in order to simplify the determination of the set Σ given by (28). From the definition of Σ in (28), its clear that Σ in this case is a segment of $\mathbb{R}$, which is plotted in this figure with the dotted segment in red.

Case 2. We also consider here an example of non linear disturbances $(K_i)_{i\geqslant 0}$ defined as follows

$$K_i\colon \begin{pmatrix} x \\ y \end{pmatrix} \to \left\langle \begin{pmatrix} x \\ y \end{pmatrix}, M_i \begin{pmatrix} x \\ y \end{pmatrix} \right\rangle v, \tag{29}$$

where

$$M_i = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mathbb{1}_{\mathcal{O}}(i), \quad (a,b) \in \mathbb{R}^2 \quad \text{and} \quad v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

with $\mathcal{O}$ and $\mathbb{1}_{\mathcal{O}}$ are given by (23) an (24) respectively. It is obvious that

$$K_i \begin{pmatrix} x \\ y \end{pmatrix} = \left\langle \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\rangle v$$

for $i = 0, 1, \ldots, 10$, and

$$K_i \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \forall i > 10$$

and consequently the age of disturbances K_i is $I = 10$.

By (16), (18) and (29) we have

$$x_1 = \tilde{A}x_0 + \left(a(x_0^{(1)})^2 + b(x_0^{(2)})^2\right) Bv, \tag{30}$$

$$\begin{aligned} x_i = \sum_{k=0}^{i-2} \left(a(x_k^{(1)})^2 + b(x_k^{(2)})^2\right) \tilde{A}Bv \\ \left(a(x_{i-1}^{(1)})^2 + b(x_{i-1}^{(2)})^2\right) Bv + \tilde{A}^i x_0. \end{aligned} \tag{31}$$

For $i \geqslant 2$, where $x_i = \begin{pmatrix} x_i^{(1)} \\ x_i^{(2)} \end{pmatrix} \in \mathbb{R}^2$. From (20), (21) and (31) we deduce that

$$\Sigma = \left\{(a,b) \in \mathbb{R}^2 / \, |F_i(a,b)| \leqslant 0.7, \, \forall i = 0, 1, \ldots, 13\right\}, \tag{32}$$

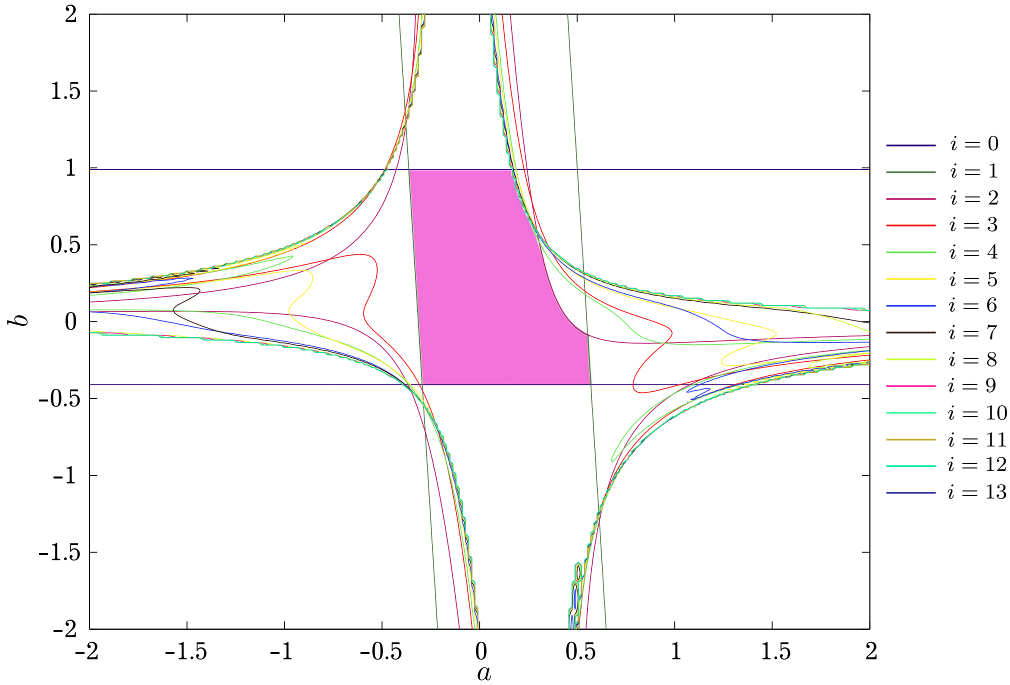
where

$$\begin{aligned} F_0(a,b) &= Cx_0, \\ F_1(a;b) &= C\tilde{A}x_0 + \left(a(x_0^{(1)})^2 + b(x_0^{(2)})^2\right) CBv, \\ F_i(a,b) &= \sum_{k=0}^{i-2} \left(a(x_k^{(1)})^2 + b(x_k^{(2)})^2\right) C\tilde{A}Bv \\ &\quad \left(a(x_{i-1}^{(1)})^2 + b(x_{i-1}^{(2)})^2\right) Bv + C\tilde{A}^i x_0 \end{aligned}$$

for $i \geqslant 2$. Let us define functions G_i as follows

$$\begin{cases} G_{2i}(a,b) = F_i(a,b) - 0.7, \\ G_{2i+1}(a,b) = F_i(a,b) + 0.7. \end{cases}$$

In Fig. 3, we plot equations $G_{2i} = 0$ and $G_{2i+1} = 0$, for each i in $\{0, 1, \ldots, 13\}$, with the same color, in order to simplify the appearance of iterations, then the set Σ in (32) is represented in this figure with the filled area.



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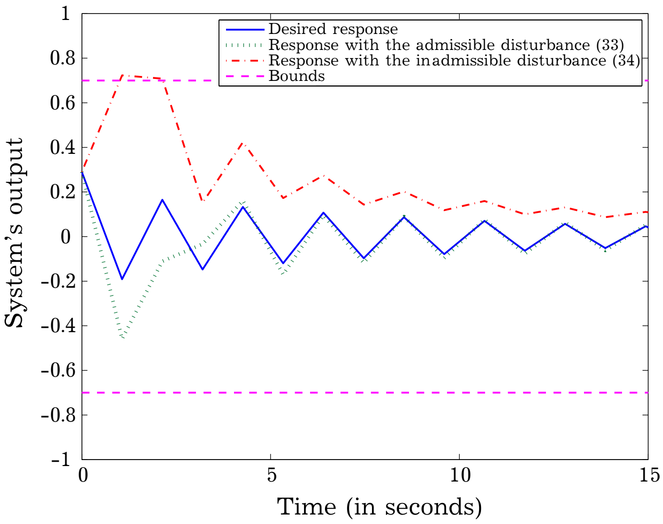

Fig. 4. System's response with an admissible and inadmissible disturbances.

Case 2. We give here an example corresponding to the two families of disturbances given by

$$K_i\colon \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} \frac{\sin(y^2)}{i+1} \\ \frac{-xy}{i+1} \end{pmatrix} \mathbb{1}_{\mathcal{O}}(i), \tag{33}$$

and

$$K_i\colon \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} \frac{y\sin(x)}{i+1} \\ \frac{\cos(y)}{2i+x^2} \end{pmatrix} \mathbb{1}_{\mathcal{O}}(i). \tag{34}$$

Where $\mathcal{O}$ and $\mathbb{1}_{\mathcal{O}}$ are given by (23) an (24) respectively. Its clear that the age of these disturbances is $I = 10$. Fig. 4 shows the impact of these disturbances on the system's output for $x_0 = \begin{pmatrix} -0.9 \\ -0.2 \end{pmatrix}$, where the impact of the disturbances (33) do not cause constraints violation, which means that these disturbances are admissible for the chosen x_0, while the system's output exceeds 0.7 when there are disturbances (34), which means that disturbance (34) could possibly cause serious damage, thus, are inadmissible.

Case 3. We consider here an example of linear disturbances $(K_i)_{0\leqslant i\leqslant 10}$ defined as follows

$$K_i\colon \begin{pmatrix} x \\ y \end{pmatrix} \to M_i \begin{pmatrix} x \\ y \end{pmatrix}, \ \forall i \in \{0,\dots,10\}, \tag{35}$$

where $M_i = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mathbb{1}_{\mathcal{O}}(i)$ with $(a,b) \in \mathbb{R}^2$, $\mathcal{O}$ and $\mathbb{1}_{\mathcal{O}}$ are given by (23) an (24) respectively. It is obvious that

$$K_i \begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} \begin{pmatrix} ax \\ by \end{pmatrix}, & \text{if } \ i \in \{0,\dots,10\}, \\ 0, & \text{elsewhere.} \end{cases}$$

Thus the system (16) is rewritten as follows

$$x_i = (\tilde{A} + BM_i)^i x_0 \tag{36}$$

for $i \in \mathcal{O}$. By (21) and (36), the maximal disturbances set corresponding to this example is given then by

$$\Sigma = \left\{(a,b) \in \mathbb{R}^2 / \, |F_i(a,b)| \leqslant 0.7, \; \forall i = 0,1,\ldots,13\right\}, \tag{37}$$

where

$$F_i(a,b) = C \begin{pmatrix} -2a-0.9 & 2b \\ a-0.1 & -3b-0.1 \end{pmatrix}^i x_0, \quad i \geqslant 1,$$
$$F_0(a,b) = Cx_0 = 0.29.$$

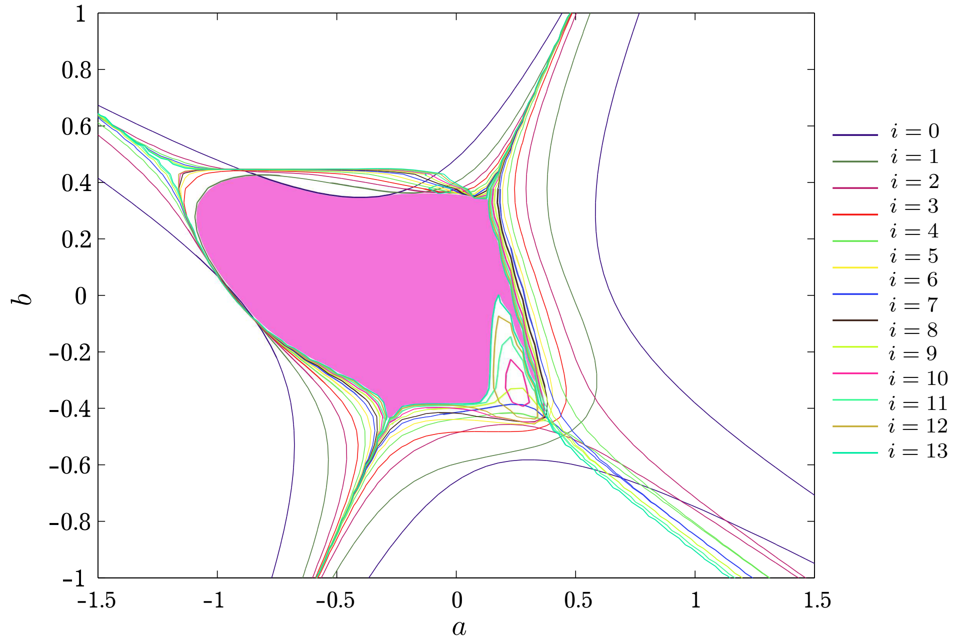

Fig. 5. The colored area represents the set Σ given by (37).

Fig. 5 depicts a geometric illustration of the set Σ given by (37), we have to note that $F_0(a,b)$ is not plotted in this figure because it does not depend neither on a nor on b, and its clear that $|F_0(a,b)| \leqslant 0.7$.

We give in Table 1 some examples of the execution of the algorithm 2, with different choice of matrices that define the system.

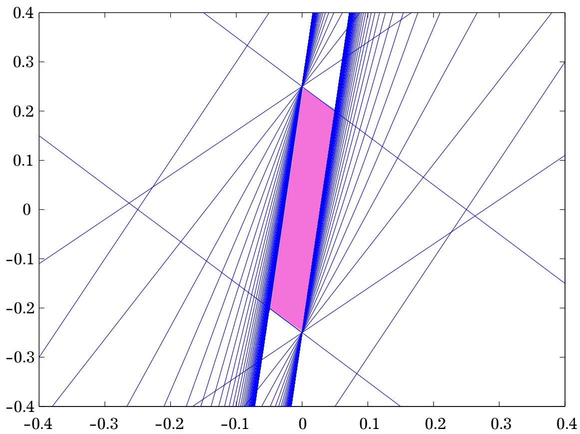

Fig. 6. The colored area represents the set Λ in (11) corresponding to example 2 with $k^* = 327$.

Remark 4. i) While the conditions in the theorem 1 are sufficient for the convergence of the algorithm 2, example 4 show that they are not necessary, we can see that the matrix $\tilde{A}$ is unstable. Furthermore, $\tilde{A}$ is just Lyapunov stable in examples 3 and 5.

ii) In example 2, the fact of writing $k^* = \infty$ does not mean that the algorithm is not convergent, but we explain that at the time of the analysis of this example on a computer, we obtained higher values of k^* ($k^* = 327$) in a somewhat larger time. Fig. 6 shows the set Λ defined by (11) with data of example 2.

5. Discrete-time delayed systems

Table 1. Data of examples 2–5.

Ex	A	B	C	K	$\tilde{A}$	Ω	k^*
2	$\begin{pmatrix} 1 & 0 \\ 0.1 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} -2 & 2 \end{pmatrix}$	$\begin{pmatrix} -0.3 & -4 \\ -1.1 & -2 \end{pmatrix}$	$\begin{pmatrix} -0.9 & 0 \\ 0.9 & -1 \end{pmatrix}$	$[-0.5, 0.5]$	∞
3	$\begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}$	$\begin{pmatrix} -2 & 0 \\ -1 & 2 \end{pmatrix}$	$\begin{pmatrix} -0.1 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 1.25 & -1.55 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0.5 & -0.1 \end{pmatrix}$	$[-0.1, 0.1]$	3
4	$\begin{pmatrix} 1 & -2 \\ 0.2 & 7 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}$	$\begin{pmatrix} -0.9 & 1.9 \end{pmatrix}$	$\begin{pmatrix} 4.4 & 2 \\ -2.4 & -4 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ -0.2 & 1 \end{pmatrix}$	$[-0.2, 0.2]$	1
5	$\begin{pmatrix} 1 & 3 & 0 \\ -2 & 1 & 0 \\ 0 & 2 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 & 0 \\ 1 & 2 & -2 \\ 1 & 2 & 0 \end{pmatrix}$	$\begin{pmatrix} -0.1 & -0.3 & 0.2 \end{pmatrix}$	$\begin{pmatrix} 1 & 8.4 & 0 \\ -1 & -5.4 & 0 \\ -1 & -0.5 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ -1 & -0.4 & 0 \\ -1 & -0.4 & 0 \end{pmatrix}$	$[-0.2, 0.2]$	4

The considered discrete-time delayed systems are described by

$$\begin{cases} x_{i+1} = A_0 x_i + \ldots + A_r x_{i-r} + B_0 u_i + \ldots + B_s u_{i-s}, \\ x_0 \in \mathbb{R}^n \quad \text{given} \\ x_j = \alpha_j, \quad -r \leqslant j \leqslant -1 \end{cases} \tag{38}$$

with delayed output function

$$y_i = C_0 x_i + \ldots + C_d x_{i-d} \in \mathbb{R}^p, \tag{39}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ are, respectively, the state, control input of the system (38). $(A_i)_{0 \leqslant i \leqslant r}$, $(B_i)_{0 \leqslant i \leqslant s}$ and $(C_i)_{0 \leqslant i \leqslant d}$ are real matrices of appropriate dimension.

The infected delayed control input is given by

$$u_{i-k} = (K + K_i) x_{i-k}, \quad 0 \leqslant k \leqslant s, \tag{40}$$

where K is a real matrix of compatible dimension, and $(K_i)_{i \geqslant 0}$ are a nonlinear maps which describe the disturbances that infect the control input. We assume that there exists an integer I for which $K_i \equiv 0$, $\forall i > I$, in this case I is called the age of disturbances.

We assume that the control (40) is introduced such that the corresponding output function (39) satisfies the constraints

$$y_i \in \Omega, \quad i \geqslant 0. \tag{41}$$

We assume hereafter that $r = s$ and using (40) in (38), we have

$$\begin{aligned} x_{i+1} &= A_0 x_i + \ldots + A_r x_{i-r} + B_0 (K + K_i) x_i + \ldots \\ &\quad + B_r (K + K_i) x_{i-r}, \\ x_{i+1} &= \sum_{k=0}^{r} (A_k + B_k K + B_k K_i) x_{i-k}. \end{aligned}$$

By changing

$$\tilde{A}_k = A_k + B_k K \quad \text{and} \quad P_{i,k} = B_k K_i,$$

the system (38) can be written as follows

$$\begin{cases} x_{i+1} = \sum_{k=0}^{r} (\tilde{A}_k + P_{i,k}) x_{i-k}, \\ x_0 \in \mathbb{R}^n \quad \text{given} \\ x_j = \alpha_j, \quad -r \leqslant j \leqslant -1. \end{cases} \tag{42}$$

We will investigate the admissible disturbances, i.e. the disturbances such that the corresponding output function satisfies also the constraints (41). As above, the set of all admissible disturbances Σ is given by

$$\Sigma = \{ (P_{i,k})_{0 \leqslant i \leqslant I, 0 \leqslant k \leqslant r} / y_i \in \Omega, \ \forall i \geqslant 0 \} \tag{43}$$

or

$$\Sigma = U \cap V, \tag{44}$$

where

$$\begin{gathered} U = \{(P_{i,k})_{0\leqslant i\leqslant I, 0\leqslant k\leqslant r} / y_i \in \Omega,\ \forall i = 0,1,\dots,I\}, \\ V = \{(P_{i,k})_{0\leqslant i\leqslant I, 0\leqslant k\leqslant r} / y_i \in \Omega,\ \forall i \geqslant I+1\}. \end{gathered} \tag{45}$$

First, we give the following result.

Proposition 4. The system (42) is equivalent to

$$\begin{cases} z_{i+1} = (\Delta + \Gamma_i) z_i, \\ z_0 \in \mathbb{R}^{(r+1)n}, \end{cases} \tag{46}$$

where

$$\begin{gathered} z_i = (x_i, x_{i-1}, \dots, x_{i-r})^T, \\ z_0 = (x_0, \alpha_{-1}, \dots, \alpha_{-r})^T, \end{gathered}$$

and

$$\Delta = \begin{bmatrix} \tilde{A}_0 & \tilde{A}_1 & \cdots & \tilde{A}_r \\ I & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & I & 0 \end{bmatrix},$$

and

$$\Gamma_i = \begin{bmatrix} P_{i,0} & P_{i,1} & \cdots & P_{i,r} \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Proof. From (42), we have

$$x_{i+1} = (\tilde{A}_0, \tilde{A}_1, \dots, \tilde{A}_r) \begin{pmatrix} x_i \\ x_{i-1} \\ \vdots \\ x_{i-r} \end{pmatrix} + (P_{i,0}, P_{i,1}, \dots, P_{i,r}) \begin{pmatrix} x_i \\ x_{i-1} \\ \vdots \\ x_{i-r} \end{pmatrix}$$

and

$$x_i = (I, 0, \cdots, 0) \begin{pmatrix} x_i \\ x_{i-1} \\ \vdots \\ x_{i-r} \end{pmatrix}, \quad \dots, \quad x_{i-r+1} = (0, 0, \cdots, I, 0) \begin{pmatrix} x_i \\ x_{i-1} \\ \vdots \\ x_{i-r} \end{pmatrix}.$$

Let $z_i = (x_i, x_{i-1}, \dots, x_{i-r})^T$, then we have $z_{i+1} = (\Delta + \Gamma_i) z_i$. ■

The output function y_i can be written in terms of the new state variables z_i as follows

$$y_i = \tilde{C} z_i, \quad i \geqslant 0,$$

where

$$\tilde{C} = (C_0, \dots, C_d, 0, \dots 0) \in \mathbb{R}^{p\times(r+1)n}.$$

Remark 5. A disturbance $(\Gamma_i)_{0\leqslant i\leqslant I}$ is admissible for system (46), if and only if the corresponding disturbance $(P_{i,k})_{0\leqslant i\leqslant I, 0\leqslant k\leqslant r}$ is also admissible for system (42). Since system (46) have the form of system (4) in Section 2, we can apply the above results to characterize the set of all admissible disturbances. Which means that

Theorem 2. *If the following assumptions hold:*

1) Δ is asymptotically stable,

2) the pair $(\triangle, \tilde{C})$ is observable,

3) Ω is bounded and contains the origin in its interior,

then there exists an integer k^*such that

$$\Sigma = \{(\Gamma_i)_{0 \leqslant i \leqslant I} / y_i \in \Omega, \ \forall i = 0, 1, \ldots, I + 1 + k^*\}.$$

Remark 6. i) In most case of delayed systems, one can find an equivalent system in the form (46), and we can use the above ideas to solve the problem of perturbed control input.

ii) To execute the algorithm 2, described above, the following change is made

$$A = \Delta \quad \text{and} \quad C = \tilde{C}.$$

Example. Without loss of generality, we consider the following discrete-time delayed model with $r = 2$:

$$\begin{cases} x_{i+1} = -1.3x_i + 2.4x_{i-1} - 3.1x_{i-2} + \frac{4}{15}u_i - \frac{34}{15}u_{i-1} + \frac{16}{15}u_{i-2}, \\ x_0 = 0.2, \\ x_{-1} = 0.1, \\ x_{-2} = 0.1. \end{cases} \tag{47}$$

With the delayed output

$$y_i = -0.1x_i + x_{i-1}.$$

The perturbed control function

$$u_{i-k} = (1.5 + K_i)\, x_{i-k}, \quad 0 \leqslant k \leqslant 2 \tag{48}$$

is introduced in this model to satisfy the output constraint

$$y_i \in \Omega = [-0.5, 0.5]\,, \quad \forall i \geqslant 0$$

We assume that the perturbation K_i is inevitable for all $i \in \{0, 1, \ldots, 10\}$. By substituting the control (48) in the model (47) we have

$$x_{i+1} = -1.3x_i + 2.4x_{i-1} - 3.1x_{i-2} + \frac{4}{15}(1.5 + K_i)\, x_i - \frac{34}{15}(1.5 + K_i)\, x_{i-1} + \frac{16}{15}(1.5 + K_i)\, x_{i-2},$$

after simplification we have

$$x_{i+1} = \left(-0.9 + \frac{4}{15}K_i\right) x_i + \left(-1 - \frac{34}{15}K_i\right) x_{i-1} + \left(-1.5 + \frac{16}{15}K_i\right) x_{i-2}.$$

Then, by using the change of variables as before, we note

$$z_i = \begin{pmatrix} x_i \\ x_{i-1} \\ x_{i-2} \end{pmatrix}.$$

Therefore, the new equivalent model is written as

$$z_{i+1} = (\triangle + P_i)\, z_i,$$

where

$$\Delta = \begin{pmatrix} -0.9 & -1 & -1.5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

and

$$P_i = \begin{pmatrix} \frac{4}{15}K_i & -\frac{34}{15}K_i & \frac{16}{15}K_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and

$$\tilde{C} = \begin{pmatrix} -0.1 & 1 & 0 \end{pmatrix}.$$

By a simple calculation we can verify that the matrix Δ is not stable and the pair $(\Delta, \tilde{C})$ is observable, nevertheless, the algorithm 2 converges, for the two matrices Δ and $\tilde{C}$ and the set Ω, and gives $k^* = 3$. As a result, the only disturbances P_i which did not affect this system are those which verify the following equations

$$|y_i| \leqslant 0.5, \quad \forall i \in \{0, \ldots, 14\}$$

and the set Σ in (14) is given by

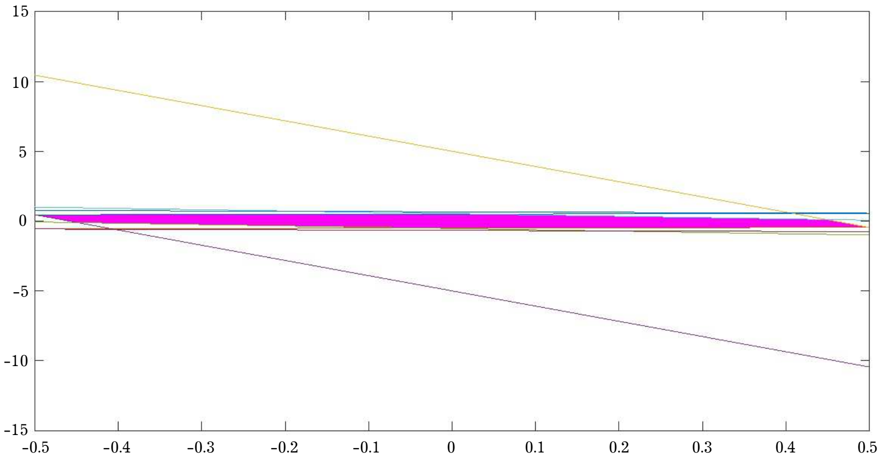

Fig. 7. The colored area represents the set Λ in (50) corresponding to example of delayed model, where $k^* = 3$.

$$\Sigma = \{(P_i)_{0 \leqslant i \leqslant 10} / \, |y_i| \leqslant 0.5, \ \forall i = 0, 1, \ldots, 14\}, \tag{49}$$

while the set Λ in (11) is given by

$$\Lambda = \{x \in \mathbb{R}^2 / |\tilde{C}\Delta^i x| \leqslant 0.5, \ \forall i = 0, 1, 2, 3\}, \tag{50}$$

which is illustrated by the filled area of the Fig. 7.

6. Conclusion

In this paper, we have developed a new technique that allows us to determine admissible disturbances susceptible to infecting the control input of a controlled linear discrete-time system. A disturbance is said to be admissible if the corresponding output satisfies specific constraints. In this paper, we restrict our interest in the determination of the set of all these admissible disturbances which is called the maximal admissible disturbances set. By assuming that disturbances have a limited age, we managed to develop an algorithmic method for computing this set, under some conditions. Numerical examples were used to demonstrate the effectiveness of the proposed technique. We have shown also, for a class of controlled discrete-time delayed systems, that the maximal admissible disturbances set can be computed with the same way of delay-free systems, after some changes.

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Ідентифікація набору всіх допустимих збурень: дискретно-часові системи зі збуреною матрицею підсилення

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Стаття присвячена лінійним керованим дискретно-часовим системам, на які діють вхідні збурення. Збурення вважаються допустимими, якщо функція виходу задовольняє вихідні обмеження. У цій статті вирішується наступна задача: визначити набір усіх допустимих збурень з усіх збурень, сприйнятливих до деформації керуючого входу. Описано алгоритм обчислення множини максимально допустимих збурень і наведено достатні умови для припинення цього алгоритму. Наведено числові приклади. Також розглядається випадок дискретно-часових систем із затримкою.

Ключові слова: *дискретно-часова система, лінійна система, збурена система, допустимі збурення, система із затримкою.*