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SH-WAVE SCATTERING FROM THE INTERFACE DEFECT

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Abstract**: The problem of the elastic SH-wave diffraction from the semi-infinite interface defect in the rigid junction of the elastic layer and the half-space is solved. The defect is modeled by the impedance surface. The solution is obtained by the Wiener-Hopf method. The dependences of the scattered field on the structure parameters are presented in analytical form. Verifica¬tion of the obtained solution is presented.**

Index Terms**: Elastic layer, impedance, rigid junction, defect, diffraction, normal wave, Wiener-Hopf technique.**

I. INTRODUCTION

Prediction of the reliable work of the engineering constructions using layered junctions leads to the developing of the diagnostic methods. Information signals formed by the interaction of the elastic fields with material's inhomogeneity to define interface defects are used. They have the complex dependencies on constructive, physical and mechanical parameters. It takes a lot of time and effort to carry out the natural experiments. In this case the mathematical modelling is an important stage of planning new technological means of diagnostic that elastic waves use. The theoretical basis of this modelling are the solutions of the diffraction problems of elastic waves from the defects in layers and its junctions. For simplifying, the materials layers are defined as elastic waveguides and the defects/cracks are modeled by the free stress surface [1–9]. General theory of wave propagation in the waveguide is given in [10–16]. It is based on using integral equation methods and direct numerical analysis of the boundary value problems [17–19]. But the analytical methods are crucial in this investigation as it allows to understand better the physical features of waves and defects interaction. One of the most important analytical approaches to this analysis is based on the usage of functional Wiener-Hopf equation [7, 11, 20]. It allows to obtain the solutions of the mixed boundary value problems in the wide frequency domain that are controlling to check the results given by the general approximate methods.

One of the modern key diagnostic problem is a developing of the methods of collecting and data analysis to define the features that prevent the possible defects. The aim of the problem is to define the changing properties of the material that lead to the defect. The solutions of the mixed boundary value problems with impedance boundary conditions are used to describe the damage [21–24]. There was little research into these models. Therefore, the developing of the analytical methods of their analysis that give reliable results are important [6, 25–28]. In this paper the diffraction problem of the elastic SH-wave from the semi-infinite impedance surface formed on the rigid junction of a layer with a half-space is solved exactly by the Wiener-Hopf technique.

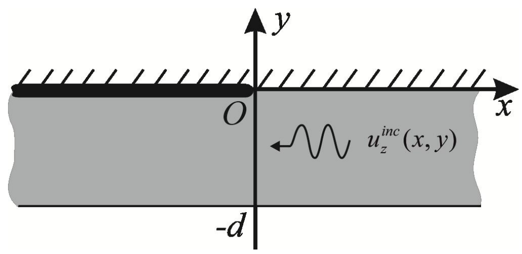

Fig. 1. Geometry of the problem

The solution can be used for the estimation of changed condition of junctions by its changed impedance.

II. STATEMENT OF THE PROBLEM

Let us consider the elastic layer in the Cartesian coordinate system xOy as

$$P:\{x\in(-\infty,\infty), y\in(-d,0),\ z\in(-\infty,\infty)\}$$

that is joined with a half-space $y>0$.

Impedance half-plane

$$\Gamma:\{x\in(-\infty,0), y=0, z\in(-\infty,\infty)\}$$

is a model of junction defect (Fig. 1).

Let this structure is irradiated by the normal SH-mode of layer P that propagates in the negative direction of the axis x. The time factor is assumed to have harmonic variation $e^{-i\omega t}$ and is suppressed through this paper. The problem is formulated in terms of the scalar-valued function $u=u(x,y)$ which is the nonzero component of the displacement vector $\mathbf{u}\equiv\vec{e}_z u(x,y)$, satisfies the Helmholtz equation

$$\partial_x^2 u(x,y)+\partial_y^2 u(x,y)+k^2 u(x,y)=0\ ,\ (x,y)\in P\ , \quad (1)$$

and boundary impedance condition type on Γ

$$\partial_y u^{tot}(x,y) + \eta u^{tot}(x,y) = 0,\ (x,y) \in \Gamma, \quad (2)$$

where η is an impedance ($\operatorname{Im}\eta \le 0$ [24]), and describes the connection on the plane Γ between the stress and displacement. The unknown function u has to satisfy the stress-free boundary condition on the surface layer

$$\tau_{zy} = \mu\partial_y u^{tot}(x,y) = 0, y = -d,\ x \in (-\infty,\infty) \quad (3)$$

and condition of the rigid junction with a half-space

$$u^{tot} = 0,\ y = 0;\ x \in (0,\infty). \quad (4)$$

Here $u^{tot} = u + u^{inc}$, u is the unknown diffracted field, u^{inc} is the incident wave,

$$u^{inc}(x,y) = e^{\gamma_j x}\sin(\beta_j y), \quad (5)$$

$\beta_j = \pi(2j-1)/2d$, $j = 1,2,3...$; $\gamma_j = \sqrt{\beta_j^2 - k^2}$, $\operatorname{Re}\gamma_j > 0$; $k = k' + ik''$ is the wave number ($k', k'' > 0$, $k' >> k''$).

It is necessary to find the solution of the diffraction problem (1)–(4) in the class of functions that satisfy the boundary absorption condition at infinity, if $|x| \mapsto \infty$ and the Meixner condition as

$$u \sim \rho^{\delta},\ \partial u/\partial y \sim \rho^{-(1-\delta)},$$

$$\text{when } \rho = [x^2 + y^2]^{1/2} \to 0, \quad (6)$$

where ρ is the distance to the edge of the defect in the local coordinate system; $0 < \delta < 1/2$.

Note, the impedance η in the boundary condition (2) is defined as a parameter, where the inverse value $q = 1/\eta$ (admittance) defines the level of damage; its limit value shows the absence of damaging when $q \to 0$ and, if $q \to \infty$ the crack is formed.

III. THE PROBLEM SOLUTION

Let introduce the unknown field by the Fourier integral

$$u(x,y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} U(\alpha,y)e^{-i\alpha x}d\alpha, \quad (7)$$

where $U(\alpha,y) = B(\alpha)e^{\gamma y} + C(\alpha)e^{-\gamma y}$, $\gamma = (\alpha^2 - k^2)^{1/2} = -i(k^2 - \alpha^2)^{1/2}$, $\operatorname{Re}\gamma \ge 0$; function $U(\alpha,y)$ is regular in the stripe $\alpha \in \Pi : \{-\tau_0 < \tau < \tau_0\}$, where $\tau_0 \le \min\{\operatorname{Im}k, \operatorname{Re}\gamma_1)\}$, $\operatorname{Re}\gamma_1 < \operatorname{Re}\gamma_j$, if $j > 1$; $B(\alpha)$, $C(\alpha)$ are unknown functions. Applying the Jones's method [11], we transform the boundary value problem (1)–(4) to the Wiener-Hopf equation as

$$M(\alpha)U'^{+}(\alpha,0) + \frac{i\beta_j M(\alpha)}{\sqrt{2\pi}(\alpha - i\gamma_j)} - \bar{U}^{-}(\alpha,0) = 0,$$

$$\alpha \in \Pi \quad (8)$$

Here,

$$\bar{U}^{-}(\alpha,0) = \frac{d^{-1}}{\sqrt{2\pi}}\int_{-\infty}^{0} u(x,0)e^{i\alpha x}dx \quad (9)$$

$$U'^{+}(\alpha,0) = \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} \partial_y u(x,0)e^{i\alpha x}dx. \quad (10)$$

The kernel function

$$M(\alpha) = \frac{\operatorname{ch}(\gamma d)}{\gamma d\operatorname{sh}(\gamma d) + \bar{\eta}\operatorname{ch}(\gamma d)} \quad (11)$$

is even and meromorphic; in the stripe Π the function $M(\alpha)$ is regular and outside Π has simple zeros and poles. If $|\alpha| \mapsto \infty$, the asymptotic evaluation $M(\alpha) = O(\alpha^{-1})$ is correct; $\bar{U}^{-}(\alpha,0)$ and $U'^{+}(\alpha,0)$ are unknown Fourier integrals of the displacement and stress fields on the surfaces Γ and $\{x > 0, y = 0,\ z \in (-\infty,\infty)\}$ respectively, $\bar{\eta} = \eta d$.

The asymptotic behavior of the unknown functions $U'^{+}(\alpha,0)$ and $\bar{U}^{-}(\alpha,0)$, if $|\alpha| \mapsto \infty$ in the regularity regions $\operatorname{Re}\alpha > -\tau_0$, $\operatorname{Re}\alpha < \tau_0$ respectively are defined according to the conditions (6) [20] as

$U'^{+}(\alpha,0) = O(|\alpha|^{-1/2})$, $|\alpha| \mapsto \infty$, when $\alpha > -\tau_0$;

$\bar{U}^{-}(\alpha,0) = O(|\alpha|^{-3/2})$, $|\alpha| \mapsto \infty$, when $\alpha < \tau_0$.

Let us represent the Fourier transform of the displacement field (7) as

$$U(\alpha,y) = \left[U'^{+}(\alpha,0) - \bar{\eta}\bar{U}^{-}(\alpha,0) + \frac{(2\pi)^{-1/2}i\beta_j}{(\alpha - i\gamma_j)}\right] \times$$

$$\times\frac{\operatorname{ch}(\gamma(y+d))}{\gamma\operatorname{sh}(\gamma d)}. \quad (12)$$

The even function (11) admits the factorization [11,20]:

$$M(\alpha) = M_{+}(\alpha)M_{-}(\alpha), \quad (13)$$

where the functions $M_{+}(\alpha)$ and $M_{-}(\alpha)$ are regular and do not have zeros and poles in the upper ($\tau > -\tau_0$) and lower ($\tau < \tau_0$) half-planes of the complex variable α respectively. Let us indicate $\pm i\gamma_{nc}$ and $\pm i\gamma_{ns}$ the zeros and poles of the function $M(\alpha)$ as

$$\gamma_{nc} = d^{-1}\sqrt{4^{-1}\pi^2(2n-1)^2 - k^2 d^2},\ n = 1,2,..., \quad (14)$$

$$\gamma_{ns} = \gamma_{ns}(\bar{\eta}) = d^{-1}\sqrt{z_n^2 - k^2 d^2},\ n = 1,2,.... \quad (15)$$

Here z_n are the roots of the transcendental equation

$$z\sin(z) - \bar{\eta}\cos(z) = 0. \quad (16)$$

Let us represent the numerator and denominator of the function (11) in the form of the infinite product [11] as

$$M_{\pm}(\alpha)=\frac{\prod_{n=1}^{\infty}\left[1\pm\frac{\alpha}{i\gamma_{nc}}\right]e^{\pm i\alpha\frac{d}{\pi n}}}{\sqrt{\bar{\eta}-kd\,\mathrm{tg}(kd)}\prod_{n=1}^{\infty}\left[1\pm\frac{\alpha}{i\gamma_{ns}}\right]e^{\pm i\alpha\frac{d}{\pi n}}}. \quad (17)$$

Taking into account the asymptotic behavior of zeroes and poles of the function (11), we arrive at the asymptotic estimates of the functions (17) in the regularity regions as $M_{\pm}(\alpha)=O(|\alpha|^{-1/2})$, if $|\alpha|\to\infty$.

Applying the procedures of factorization and decomposition [20] to the Wiener-Hopf equation (8), its solution is written as follows

$$U'^{+}(\alpha,0)=-\frac{i\beta_j}{\sqrt{2\pi}\,(\alpha-i\gamma_j)}\left(1-\frac{M_{+}(i\gamma_j)}{M_{+}(\alpha)}\right), \quad (18)$$

$$\bar{U}^{-}(\alpha,0)=\frac{i\beta_j M_{+}(i\gamma_j)M_{-}(\alpha)}{\sqrt{2\pi}\,(\alpha-i\gamma_j)}. \quad (19)$$

IV. FIELDS REPRESENTATION

Substituting the expressions (18) and (19) in (12), we obtain the Fourier transform of the displacement field. In order to transform into coordinate area, we apply the inverse Fourier transformation. For this purpose we close the integration path into the upper and lower complex half-planes for $x<0$ and for $x>0$ respectively, where Jordan's lemma is satisfied. The scattered field for each of the regions is written as follows:

$$u(x,y)=\begin{cases}u_1(x,y), & x>0,\\ u_2(x,y), & x<0.\end{cases} \quad (20)$$

Here,

$$u_1=u_1(x,y)=\sum_{q=1}^{\infty}R_{jq}e^{-\gamma_{qc}x}\sin\left(\frac{\pi(2q-1)}{2d}y\right), \quad (21)$$

$$u_2=u_2(x,y)=-u^{\mathrm{inc}}(x,y)+ +\sum_{q=1}^{\infty}T_{jq}e^{\gamma_{qs}x}\cos\left(\frac{\varphi_q(y+d)}{d}\right), \quad (22)$$

where R_{jq}, T_{jq} are coefficients of mode transformations on the defect tip in the domains $x>0$ and $x<0$ respectively;

$$R_{jq}=\frac{\pi\beta_j M_{+}(i\gamma_j)}{2d}\frac{(2q-1)M_{+}(i\gamma_{qc})}{(\gamma_{qc}+\gamma_j)\gamma_{qc}}, \quad q=1,2,\ldots,\ j=1,2,\ldots. \quad (23)$$

$$T_{jq}=\frac{\beta_j M_{+}(i\gamma_j)}{d}\times \times\frac{i\varphi_q M_{+}^{-1}(i\gamma_{qs})}{\gamma_{qs}\cos(\varphi_q)(\varphi_q+(1+\bar{\eta})\mathrm{tg}(\varphi_q))(\gamma_{qs}-\gamma_j)}, \quad q=1,2,\ldots,\ j=1,2,\ldots. \quad (24)$$

Then applying the asymptomatic analysis we obtain that $R_{jq},T_{jq}=O(q^{-3/2})$, if $q\to\infty$; therefore, if $y=0$ we arrive at $u=O(x^{1/2})$ and $\partial_y u=O(x^{-1/2})$, when $x\to 0$. These estimations guarantee the uniform convergence of the series (21), (22) and their derivatives in the domain $\{-\infty<x<0\bigcup 0<x<+\infty,\ -d\le y\le 0\}$. If $x\to 0$ our field representation formulas (21), (22) satisfy the Meixner condition (6) and the boundary conditions on the layer faces $y=-d$ and $y=0$.

The formulas (21), (22) give the exact solution of the problem that satisfies all the necessary conditions. They can be used to determinate the diffracted displacement field for the arbitrary values of geometrical parameters of our structure and frequency. If $\bar{\eta}\to 0$ the expressions (21), (22) are transformed into the previous results in [7].

V. EIGEN MODES IN THE SURFACE IMPEDANCE AREA

In order to analyse the wave propagation in our impedance waveguide let us determine the complex value roots of transcendental equation (16). There are several approaches to the solution of nonlinear transcendental equations. These methods have local convergences to the roots and the incorrect choice of the initial approximation leads to the divergences of the algorithm. In order to omit this limitation, we propose the algorithm which needs the additional information concerning the roots position on the complex area.

For further convenience, we will introduce new notations for functions and domains as

$$f(z)=z\sin(z),\ g(z)=-\bar{\eta}\cos(z),\ z=x+iy, \quad (25)$$

$$\Omega_n^K=z:\{(n-1/2)\pi\le x\le \le(n+1/2)\pi\wedge|y|\le K\},\quad n=1,2,\ldots,\infty. \quad (26)$$

The constant K to be defined. Using the notations (25), let us rewrite the equation (16) as follows

$$f(z)+g(z)=0. \quad (27)$$

Obviously, the function $f(z)$ has a single root in each domain Ω_n^K. As follows from the Rushe's theorem [29], the condition $|f(z)|>|g(z)|$ on the boundary $\partial\Omega_n^K$ is sufficient for existence of the single root of the equation (16) in Ω_n^K. The following example illustrates how it is satisfied.

On the segments $\{x=(n\pm 1/2)\pi\wedge|y|\le K\}$ is valid that

$$|f(z)|=2^{-1}\sqrt{(1\pm 2n)^2\pi^2+4y^2}\,\mathrm{ch}(y),$$

$$|g(z)|=|\bar{\eta}||\mathrm{sh}(y)|.$$

If $K\ge|\bar{\eta}|$ and $\mathrm{ch}(y)\ge|\mathrm{sh}(y)|$, then

$$|f(z)|\ge K\,\mathrm{ch}(y)\ge|\bar{\eta}|\,\mathrm{ch}(y)\ge|\bar{\eta}|\,\mathrm{sh}(y)=|g(z)|.$$

Let us consider the inequality as

$$a\,\mathrm{cth}(x)-x<0. \quad (28)$$

Let us show that the constant C_0 exists for which the inequality (28) is valid for any $a > 0$, if $x > C_0$.

Let us consider the function $v(x) = a\,\mathrm{cth}(x) - x$. Then we obtain that $\lim_{x\to 0} v(x) = +\infty$ and $\lim_{x\to+\infty} v(x) = -\infty$, thus there is at least one root of the equation $v(x) = 0$ on the interval $(0, +\infty)$. The function $v(x)$ is the straightly monotonically decreases on the interval $(0, +\infty)$ because $dv(x)/dx = -a\,\mathrm{sh}^{-2}(x) - 1 < 0$. This guarantees the existence of a single root $x = x_0$. If $x > x_0$ inequality (28) is satisfied. It is sufficient to use $C_0 = x_0$. In our segments (26) the following equalities are correct

$$|f(z)| = 2^{-1/2}\sqrt{(K^2 + x^2)}\sqrt{\mathrm{ch}(2K) - \cos(2x)},$$

$$|g(z)| = 2^{-1/2}\,|\bar{\eta}|\,\sqrt{\mathrm{ch}(2K) + \cos(2x)}$$

Let K' is the root of the equation $|\bar{\eta}|\,\mathrm{cth}(x) - x = 0$. Then, if $K > K_0 = \max(|\bar{\eta}|, K')$ the inequalities

$$|f(z)| = \sqrt{2^{-1}(K^2 + x^2)}\sqrt{\mathrm{ch}(2K) - \cos(2x)} \ge$$

$$\ge 2^{-1/2} K\sqrt{\mathrm{ch}(2K) - 1} \ge K\,\mathrm{sh}(K) > |\bar{\eta}|\,\mathrm{ch}(K) \ge |g(z)|,$$

are valid.

Thus, the equation (27) has a single root in any $\Omega_n^K \subset \Omega_n$, where $\Omega_n = \lim_{K\to\infty} \Omega_n^K$. Therefore, in the complex half-plane $\mathrm{Im}\, z > 0$ the equation (27) has an infinite number of roots $\{z_n\}_{n=1}^{\infty}$. The approximate expressions to determine the roots of the transcendental equation (16), if $|\bar{\eta}| << 1$ are as

$$z_1 \approx \sqrt{\bar{\eta}},\ z_n \approx \pi(n-1) + \bar{\eta}/\pi(n-1),\ n > 1. \quad (29)$$

We apply the Newton method for determine the roots of the equation (16) in the general case. The first 10 roots for different values of the parameter $\bar{\eta}$ are presented in Table 1.

VI. NUMERICAL ANALYSIS

To verify the obtained results we check the satisfying the continuity conditions of the displacement and stress fields at the surface $\{x = 0, -d < y < 0\}$ In Fig. 2 dependencies of $|\Delta u| = |u_1(0+, y) - u_2(0-, y)|$ and $|\Delta\partial_x u| = |\partial_x u_1(0+, y) - \partial_x u_2(0-, y)|$ on dimensionless parameter y/d are given.

These dependencies for different impedance parameters $\bar{\eta}$ and the dimensionless thickness/ frequencies kd of the layers are shown in Fig. 2. The curves in Fig. 2*a,b* and in Fig. 2*c,d* correspond to the different impedance values $\bar{\eta} = 1.5$ and $\bar{\eta} = 5$ respectively with $kd = 2$ (curves *1*) and $kd = 4$ (curves *2*). The five hundred terms of the series (21), (22) were used for calculation.

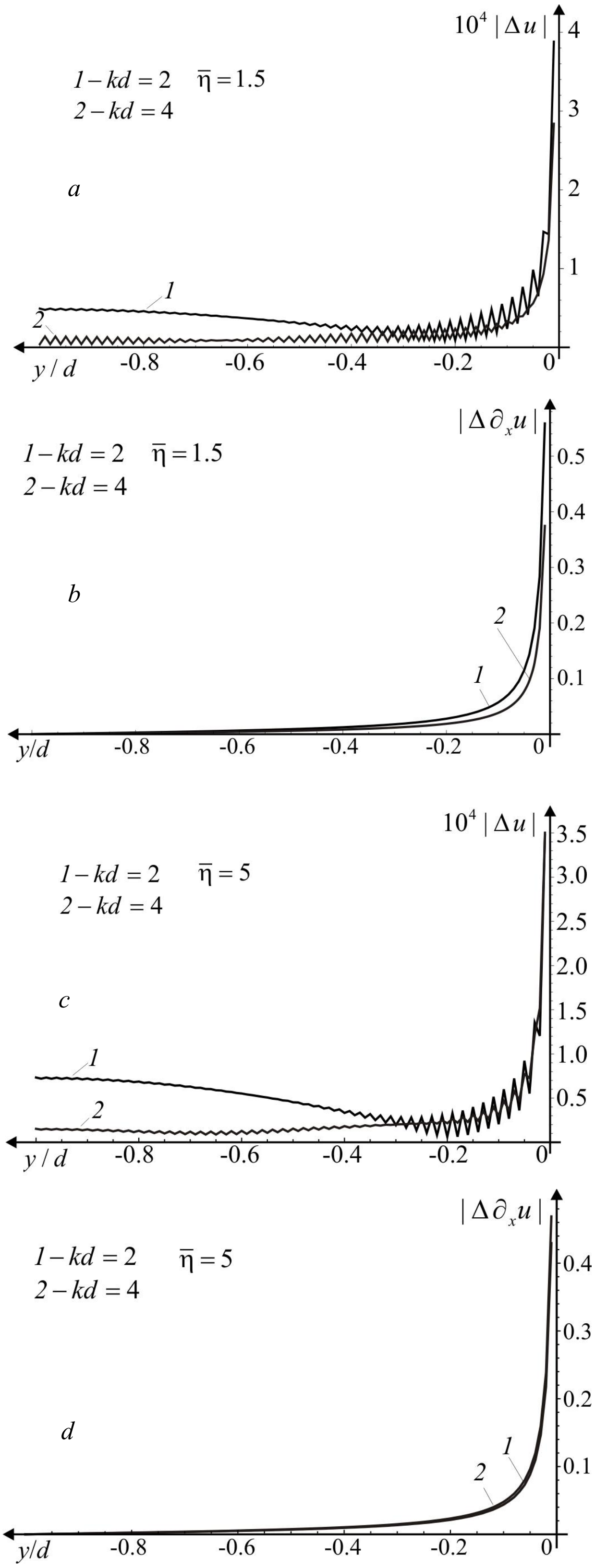

Fig. 2. Verification of the continuity conditions

The behavior of the curves in the figure show the excellent satisfaction of conditions of continuity of the displacement and stress fields in the internal domain $\{x = 0, -d < y < 0\}$, except of the tip of the defect $(x = 0, y = 0)$, where the stress has the singularity (6).

Table 1

Values of the first ten roots of the characteristic equation

$\overline{\eta}=0$	$\overline{\eta}=1.5$	$\overline{\eta}=5$	$\overline{\eta}=-0.1$
0.000	0.988	1.313	-0.322i
3.141	3.542	4.033	3.109
6.283	6.509	6.909	6.267
9.424	9.580	9.892	9.414
12.566	12.684	12.935	12.558
15.707	15.802	16.010	15.702
18.849	18.929	19.105	18.844
21.991	22.059	22.212	21.987
25.132	25.192	25.327	25.129
28.274	28.327	28.448	28.271

$\overline{\eta}=-1.5$	$\overline{\eta}=-5$	$\overline{\eta}=-0.15i$	$\overline{\eta}=3-0.15i$
-1.622i	-5.000i	0.281-0.267i	1.193-0.013i
2.622	1.941	3.142-0.048i	3.809-0.022i
6.0409	5.550	6.283-0.024i	6.704-0.018i
9.264	8.914	9.425-0.016i	9.724-0.014i
12.446	12.177	12.566-0.012i	12.797-0.011i
15.612	15.394	15.708- 0.010i	15.895-0.009i
18.770	18.587	18.850-0.010i	19.006-0.008i
21.923	21.765	21.991-0.007i	22.126-0.007i
25.073	24.935	25.133-0.006i	25.251-0.006i
28.221	28.098	28.274-0.005i	28.380-0.005i

VII. CONCLUSION

The solution of the diffraction problem of the elastic normal SH-wave from the tip of the defect in the rigid junction of a layer and a half-space is obtained. The elastic waveguide mode spectrum for domains with ideal (21) and impedance (22) boundaries is formed. Verification of the obtained solution for different values of the frequency and impedance parameter is made.

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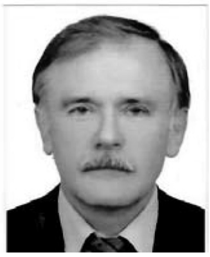

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