№ 1 (4), 2021

UDC 681.51; 681.52

V. Moroz
Lviv Polytechnic National University,
Department of Electromechatronics and Computerized Electromechanical Systems
volodymyr.i.moroz@lpnu.ua

T.-M. Yanchak
Lviv Polytechnic National University,
Department of Electromechatronics and Computerized Electromechanical Systems
maximyanchak@ukr.net

THE INFLUENCE OF LIMITED BIT ON THE IMPLEMENTATION OF THE TRANSFER IN DIGITAL SYSTEM

http://doi.org/10.23939/sepes2021.01.074

The unexploring problem of digital control systems analyzes in this article – the impact on their behavior of the limited bit resolution of the hardware and, accordingly, the discrete transfer functions coefficients. The research conducted by the zeros/poles and transient characteristics methods using the mathematical application MATLAB with the package Control System Toolbox and confirmed the relevance of this problem.

The research purpose is to determine the minimum sampling period in digital systems, provided there stability and adequacy to the behavior of a continuous system (prototype), given the limited binary resolution and the data accuracy. This will make it possible to offer solutions to reduce the negative impact of the limited bit resolution in the developed digital systems and expand the range of rational sampling rate.

Boundary dependences for the minimum sampling period and inertial time of elementary first- and second-order transfer functions formulated. It is shown that in the case of limited precision arithmetic the order's growing of the transfer function polynomial increases its sensitivity to the its coefficients accuracy.

The study of the limited arithmetic precision impact carried out on the example of transfer functions corresponding to the fifth and seventh orders binomial forms. The influence of the accuracy of the transfer functions polynomials coefficients and the placement of zeros and poles of discrete transfer functions on the complex plane shown on depending on the sampling step. The transient characteristics of discrete systems with limited bit resolution compared to their continuous analogue.

Studies confirm that the method of a continuous transfer function decomposition for decomposition into elementary components of the first and second orders and their following sampling, allows expanding the bounds of allowable sampling steps in digital systems with limited precision arithmetic. The influence of errors shown on the example of polynomials from the second to the seventh order, where in particular with growing order the polynomial increases its sensitivity to the accuracy of its coefficients setting.

Key words: bit precision; decomposition; digital control systems; polynomial; sampling; transfer function; transient; zeros and poles of the transfer function.

1. Introduction

Nowadays, with the fast development of technology and the implementation of computer equipment, most technical solutions are made using digital control and data processing systems. Among the main reasons for this there are the wider capabilities of digital systems (microprocessors, controllers) compared to old and weighty analog technology in the case of complex control rules [1, 2, 3]. Digital control systems provide high accuracy and universality of the system and allow implementing complex algorithms and flexible control systems, which with minimal changes in the hardware can be used in various purposes systems.

As a result, it becomes appropriate to use digital systems to control electric drives in modern means of technological processes automation.

2. Problem definition

The use of analysis and synthesis methods of digital devices is necessary to study all the impacts that can cause them to work incorrectly. One of the little-studied and often ignored problems in digital control systems is the influence of the sampling time of the control algorithm implementation. Usually, it is believed that the decreasing of the sampling step in digital control system theoretically and according to traditional ideas came from applied mathematics, causes its behavior similar to a continuous prototype due to the reduction of errors from the sampling process by the zero-order hold [4, 5]. However, in this case the fact is not taken into account that all calculations in digital systems are performed with limited arithmetic precision (accuracy), while the traditional mathematical basis for such systems synthesis does not consider this fact, assuming the calculations accuracy unlimited.

This, quite unexpected, thesis can be confirmed by a simple example – consider the fifth order continuous transfer function. By the way, such an order has a simple linear model of an asynchronous motor in d-q coordinates, and in the description in phase coordinates the such model order increases to the seventh [6, 7]. Therefore, the analysis of discrete transfer functions of this order is relevant, given modern control systems based on the object model. To illustrate the effect of limited arithmetic accuracy, standard binomial forms in the form of fifth- and seventh-order transfer functions were used for testing:

$$W_5(s) = \frac{1}{s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + 1};$$

$$W_7(s) = \frac{1}{s^7 + 7s^6 + 21s^5 + 35s^4 + 35s^3 + 21s^2 + 7s + 1},$$

which are discretized by standard means from Control System Toolbox of the MATLAB program [8] using function **c2d** (the default method – with a zero order hold on an input is used). It is worth noting that all calculations in the MATLAB application performed with double accuracy, i.e. with an accuracy of 15–16 decimal digits. The built-in **step** function use to get transient characteristics of continuous and corresponding those discretized systems.

There are the transient characteristics of the above-mentioned continuous systems and their corresponding discretized with a step h = 0.1 s on Fig. 1. Note the good convergence of the transient characteristics between continuous and corresponding discrete systems for both test systems.

For completeness of the information the map of zeros and poles (Fig. 2) of the obtained fifth and seventh orders discrete systems for the step h = 0.1 s by means of the built-in function **pzmap** is constructed. Note again the practical convergence of the discrete poles of the both systems.

Next, we perform the following experiment: reduce ten times the sampling step, which will now be h = 0.01 s. According to traditional expectations, the convergence of transient characteristics for continuous and corresponding discrete systems should only increase. Let's check this assertion – plots of the step responses are shown in Fig. 3, and the first surprise appeared on them: if the convergence of both transient characteristics for the fifth-order system still continues, then for the seventh-order system we can see a significant difference in step responses between the continuous system and its discrete analogue.

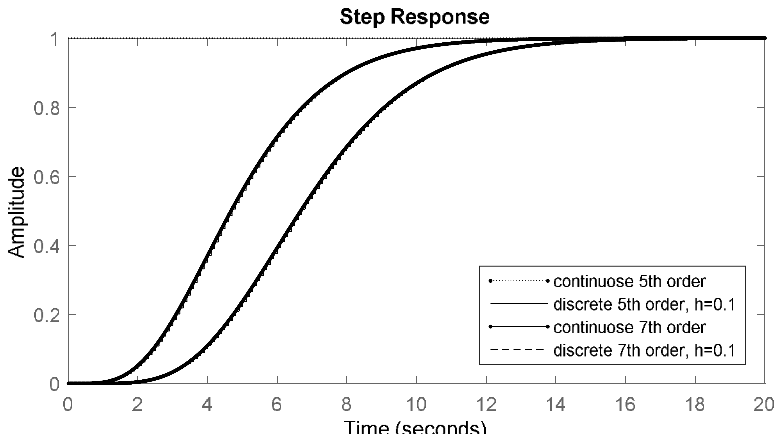

Fig. 1. Step response of the 5th and 7th orders analog systems and their discrete equivalents for step h = 0.1

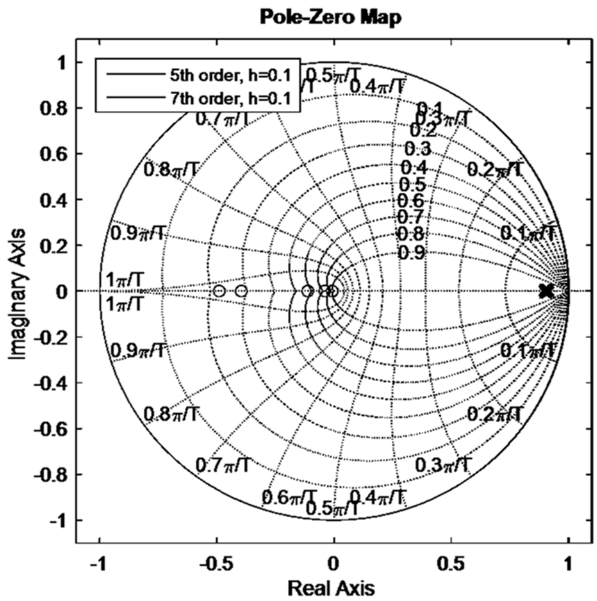

Fig. 2. Zeros and poles map of the 5th and 7th orders discrete systems for step h = 0.1

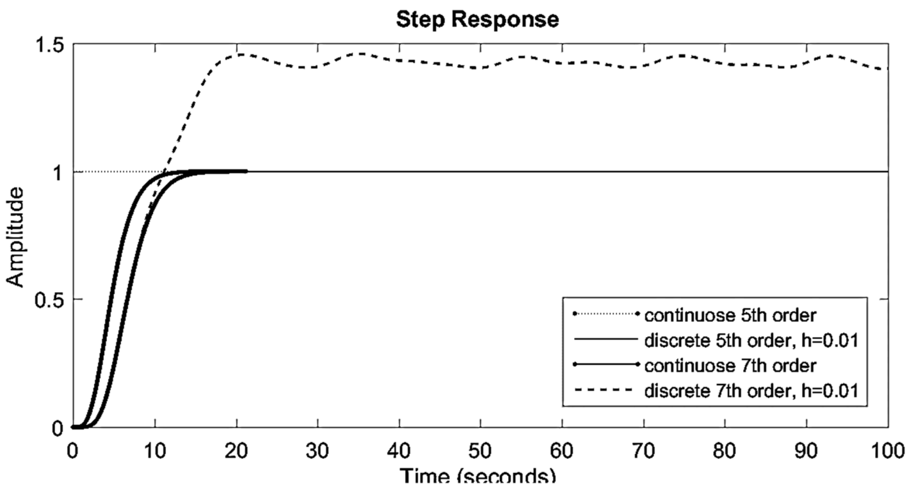

Fig. 3. Step response of the 5-th and 7-th orders analog systems and their discrete analog for step h = 0.01 s

The map of zeros and poles distribution on the complex plane for discrete systems helps to find out the reason of such phenomenon. It is known [1–4] that the region of discrete systems stability is contained within a unit circle. Let's look at Fig. 4 – it is seen that all discrete poles of both test systems have moved to the boundary of a unit circle, to a point with coordinates (1, 0). Consider this place in an enlarged form (Fig. 5).

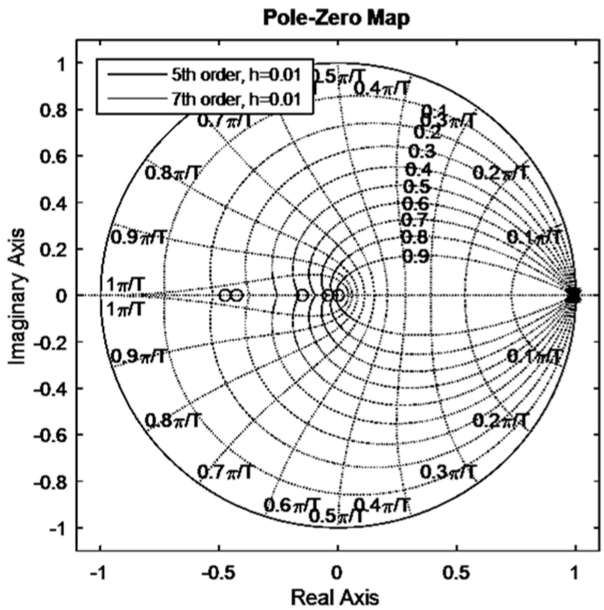

Fig. 4. Zeros and poles map of the 5th and 7th orders discrete systems for step h = 0.01 s

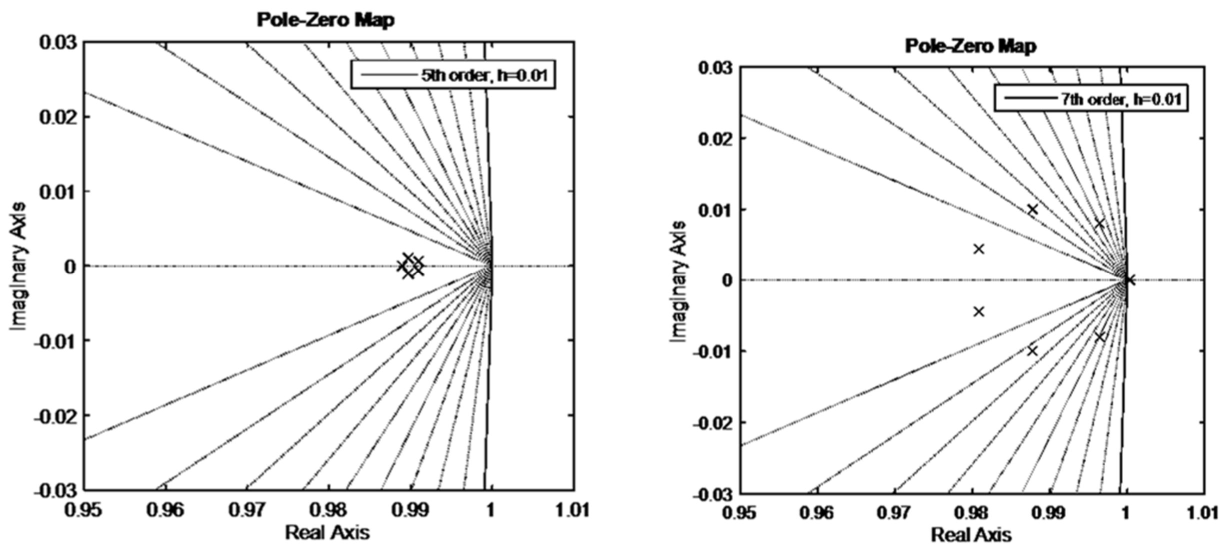

Fig. 5. Zeros and poles map of the 5-th and 7-th orders discrete systems for step h = 0.01 s (enlarged)

Even a brief analysis shows that for a fifth order discrete system all the poles placed in the unit circle, although slightly different from each other. However, for a seventh order discrete system everything immediately becomes clear – first, the poles are quite different (*a must be the equal for the binomial form polynomial*), and, secondly, one pole slightly beyond the unit circle.

Even more understandable is the traditional situation in engineering practice – setting the coefficients of the discrete transfer function in four decimal places: usually, during the development and fixing of digital systems, the discrete transfer functions coefficients entered into the control system with 3–4 decimal places. In this case, the difference in behavior between the continuous system and its discrete analogue becomes even clearer: for the 5-th order system, it already becomes visible for the step h = 0.2 s (Fig. 6), and for the 7-th order system the step h = 0.4 s becomes catastrophic (Fig. 7).

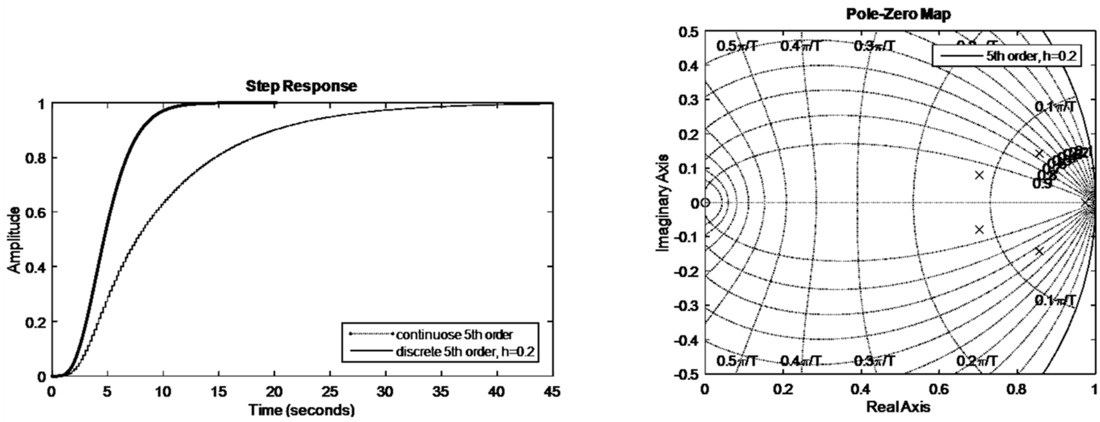

Fig. 6. Step response and zeros and poles map of the 5-th order discrete system for step h = 0.2 s in case of setting the coefficients of the discrete transfer function by four decimal digits

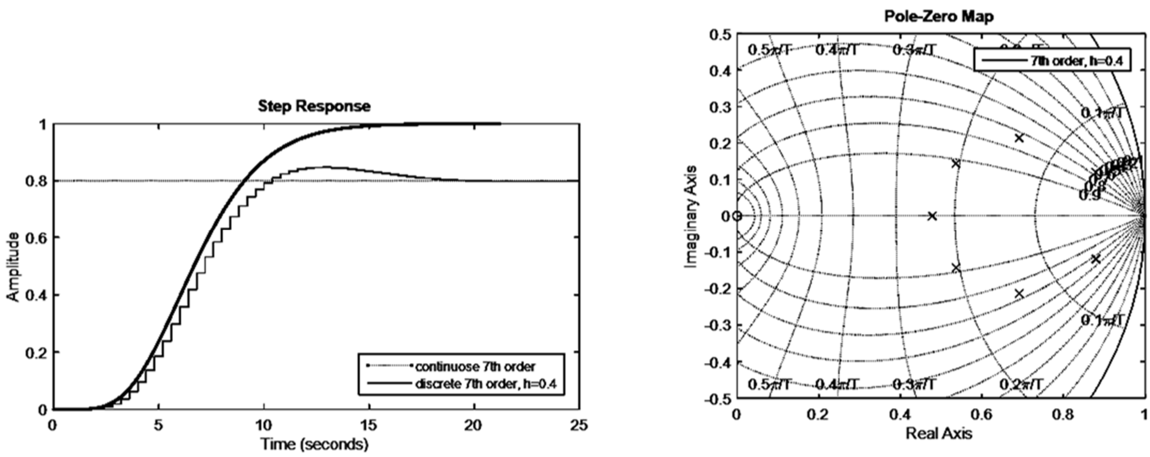

Fig. 7. Step response and zeros and poles map of the 7-th order discrete system for step h = 0.4 s in case of setting the coefficients of the discrete transfer function by four decimal digits

That is, in digital systems, the effect of the calculations accuracy and setting the coefficients of the discrete transfer function can significantly change the behavior of the synthesized discrete system compared to the expected one. Therefore, the answer to the following question is relevant: what is the reason for such effect?

Note that in the known literature, the effect of limited bit data on the digital control system is either ignored (for example, [5, 9]), or analyzed for two cases of effect [10, 11]:

- On the accuracy of maintaining controlled coordinates, which is determined by the bit resolution of the ADC or digital sensors (data accuracy);
- Location of zeros and poles of discrete transfer functions only in a limited number of discrete points on the complex plane.

Rolf Isermann [11] also proposes to consider the effect of limited bit resolution on the accuracy of calculations as a random variable (noise) with a uniform distribution. An extended formulation of the problem of the effect of limited arithmetic precision, the causes of which go beyond the generally accepted background, described in [12]. Analysis of possible causes of the effect of limited bit resolution on the functioning of digital systems initiated in [13, 14].

3. Goal of research

The purpose of the research is to determine the minimum sampling timed in digital systems, provided their stability and correspondence with the behavior of a continuous system (prototype), provided the limited bit count and the accuracy of data. This will offer solutions to reduce the impact of limited bit accuracye in digital systems and expand the range of rational sampling time.

4. Presentation of the main material

Let's return to the above-mentioned test transfer functions, which correspond to the binomial form with single poles in our case (*such a choice and the poles values are only for convenience*). Accordingly, in the sampling process we will have a mapping of the form $P_k^* = e^{P_k h}$ of discrete poles into a complex plane, where h is the sampling time; P_k – continuous k-th pole; P_k^* is a discrete k-th pole. It is clear that in the case of a stable discrete system all its poles must placed in a unit circle, i.e., the condition $|P_k^*| = \left|e^{P_k h}\right| < 1$ must be satisfied from the stability conditions, and taking into account that for the considered above the test systems, all continuous roots are unit ($P_k = -1$), this condition in our case will look like: $P_k^* = e^{-h} < 1$.

From the given stability condition, it is possible to understand that in case of a sampling step reduction **all** (*it is very important!*) discrete roots will move to a point with coordinates (1, 0) on unit circle border, that is, will appear on border of stability area: $\lim_{h \to 0} e^{-h} = 1$. For systems with limited arithmetic precision, this means that if the value of the sampling step h is outside the bit grid (lowest bit) of the device (*simply, equal to zero*), the discrete pole will move to the boundary of the stability region.

Let consider the generalized cases that are associated with two types of poles and their location on the complex plane:

- real pole;
- a pair of complex conjugate poles.

To start, consider the **real pole**, which characterizes the first order circuit. Such a circuit has a transfer function $\frac{1}{T \cdot s + 1}$, where T – time constant, and one real pole $P = -\frac{1}{T}$, which after sampling correspond to the discrete pole $P^* = e^{-\frac{h}{T}}$. In this case, in a discrete system with the binary resolution *D*, the ratio of the sampling step h to the time constant T is a significant value. In the case of limited bit resolution, this is necessary to ensure that the value of the exponent $e^{-\frac{h}{T}}$ is less than 1 from the condition of stability:

$$1 > \frac{2^D - 1}{2^D} \geq e^{-\frac{h}{T}}, \qquad \text{hence } \frac{h}{T} \geq -\ln\left(\frac{2^D - 1}{2^D}\right),$$

where the expression $\frac{2^D - 1}{2^D}$ for a given binary resolution *D* is a number that is less than one by the value of the lower binary digit (*i.e., from the condition of stability is less than one and, at the same time, for a given binary bit is closest to it*).

For a typical 32-bit system for unsigned numbers we get $\frac{h}{T} \geq 2.328 \cdot 10^{-10}$, and for unsigned numbers (one binary digit is allocated for sign) we get $\frac{h}{T} \geq 4.657 \cdot 10^{-10}$. For systems with other binary bit rate generalization for the ratio of the sampling step h to the time constant T is summarized in Table 1.

Table 1

The ratio of the allowable sampling step *h* to the time constant *T* for first-order systems

Binary resolution of the system	Boundary relations *h*/*T*	
	Without sign	With sign
8	3.9×10^{-3}	7.8×10^{-3}
10	9.8×10^{-4}	1.95×10^{-3}
12	2.9×10^{-4}	4.9×10^{-4}
16	1.53×10^{-5}	3.05×10^{-5}
32	2.33×10^{-10}	4.66×10^{-10}

The pair of complex-conjugate poles corresponds to the second-order circuit with the transfer function $\frac{1}{T^2s^2+2\xi Ts+1}$, where T is a time constant, ξ is the damping coefficient, and the two complex-conjugate poles have corresponding values and discrete mappings:

$$P_{1,2} = \frac{-\xi \pm \sqrt{\xi^2-1}}{T} \xRightarrow{Discretize} P_{1,2}^* = e^{-\xi \cdot \frac{h}{T}} \cdot e^{\pm j \cdot \sqrt{1-\xi^2} \cdot \frac{h}{T}}.$$

Discrete poles have two elements:

- the first element $e^{-\xi \cdot \frac{h}{T}}$ corresponds to the modulus of the complex number (*the length of the vector of the complex number*);
- the second element $e^{\pm j \cdot \sqrt{1-\xi^2} \cdot \frac{h}{T}}$ corresponds to the rotation angle of the complex number.

The analysis of the limiting ratio of the sampling step *h* to the time constant *T* for the second-order circuit (*a pair of complex-conjugate discrete poles*) will be performed in the same way as for the first-order circuit (*real discrete pole*)

$$\frac{2^D-1}{2^D} \geq e^{\pm j \cdot \sqrt{1-\xi^2} \cdot \frac{h}{T}} \xRightarrow{accordingly} \frac{h}{T} \geq -\frac{1}{\xi} \cdot \ln\left(\frac{2^D-1}{2^D}\right);$$

$$\frac{2^D-1}{2^D} \geq e^{-\xi \cdot \frac{h}{T}} \xRightarrow{accordingly} \frac{h}{T} \geq -\frac{1}{\xi} \cdot \ln\left(\frac{2^D-1}{2^D}\right).$$

For a typical 32-bit system and a damping factor $\xi = 0.3$ for unsigned numbers we get $\frac{h}{T} \geq 7.761 \cdot 10^{-11}$, and for unsigned numbers we have $\frac{h}{T} \geq 1.552 \cdot 10^{-9}$. The generalization of the ratio of the sampling step *h* to the time constant *T* for systems with different bits and the damping coefficient $\xi = 0.3$ is summarized in Table. 2.

Table 2

The ratio of the allowable sampling step *h* to the time constant *T* for a pair of complex-conjugate poles with a damping factor ξ = 0.3

Binary resolution of the system	Boundary relations *h*/*T*	
	Without sign	With sign
8	1.31×10^{-2}	2.61×10^{-2}
10	3.26×10^{-3}	6.51×10^{-3}
12	8.14×10^{-4}	1.63×10^{-3}
16	5.1×10^{-5}	1.02×10^{-4}
32	7.76×10^{-10}	1.55×10^{-9}

Another and, as it turned out, much more significant factor in the influence of limited arithmetic accuracy on the discrete system behavior is the influence of the accuracy of setting the coefficients of the numerator and denominator polynomials of the discrete transfer function. An illustration of this effect given at the beginning of the article.

To check the influence of the polynomial order on the error of finding the transfer function poles (polynomial roots) and their placement on the complex plane, polynomials (binomial form) of the second, third, fifth and seventh orders used for the test (see Table 3).

Table 3

Investigated polynomials

The polynomial order n	Polynomial
2	$s^2 + 2s + 1$
3	$s^3 + 3s^3 + 3s + 1$
5	$s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + 1$
7	$s^7 + 7s^6 + 21s^5 + 35s^4 + 35s^3 + 21s^2 + 7s + 1$

An experiment for introduction of a small error (0.1 %) in the polynomial coefficients is obvious for studying the effect of such errors, in particular, errors in setting the polynomials coefficients. For the experiment, we used the change only in the last coefficient of the above-mentioned polynomials (at the power of s^0) of the second, third, fifth, and seventh orders (see Table 4). In the case of errors in other polynomials coefficients, no significant differences were observed. The results were quite clear: the graphs of the roots location on the complex plane for both cases (precise setting of the coefficients and with small error) shown in Fig. 8–11.

Table 4

Investigated polynomials with introduced error

The polynomial order n	Polynomial
2	$s^2 + 2s + 1.001$
3	$s^3 + 3s^3 + 3s + 1.001$
5	$s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + 1.001$
7	$s^7 + 7s^6 + 21s^5 + 35s^4 + 35s^3 + 21s^2 + 7s + 1.001$

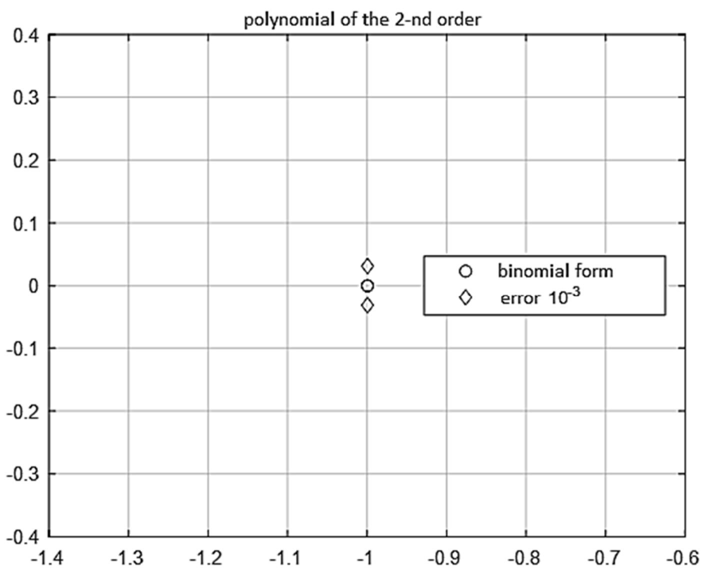

Fig. 8. Distribution of polynomial roots of the second order in the case of exact setting of coefficients (labeled by a circle) and error 0.1 % in the last coefficient (labeled by a diamond)

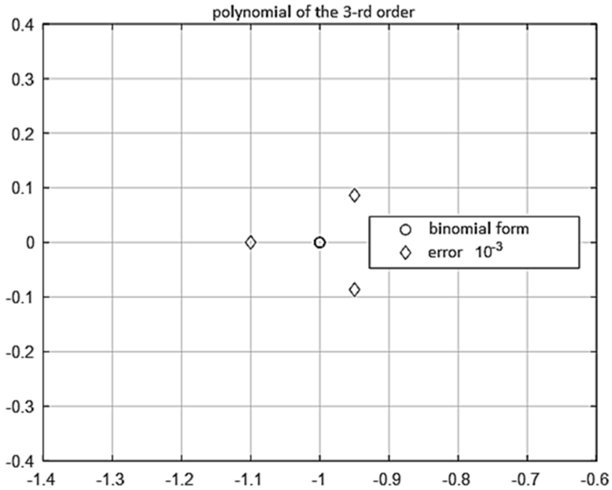

Fig. 9. Distribution of a polynomial roots of the third order in the case of exact setting of coefficients (labeled by a circle) and error 0.1 % in the last coefficient (labeled by a rhombus)

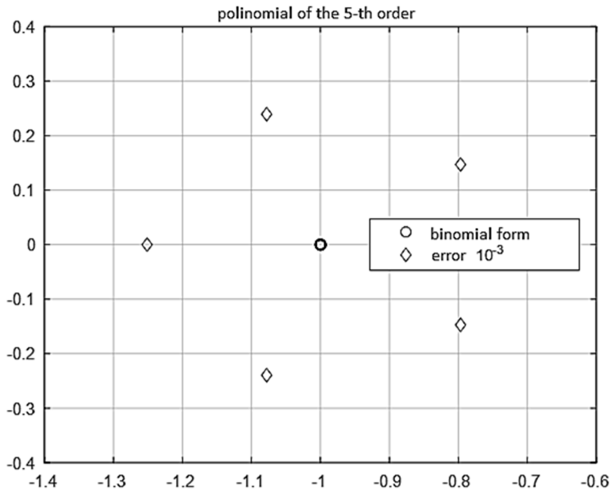

Fig. 10. Distribution of a polynomial roots of the fifth order in the case of exact setting of coefficients (labeled by a circle) and error 0.1 % in the last coefficient (labeled by a rhombus)

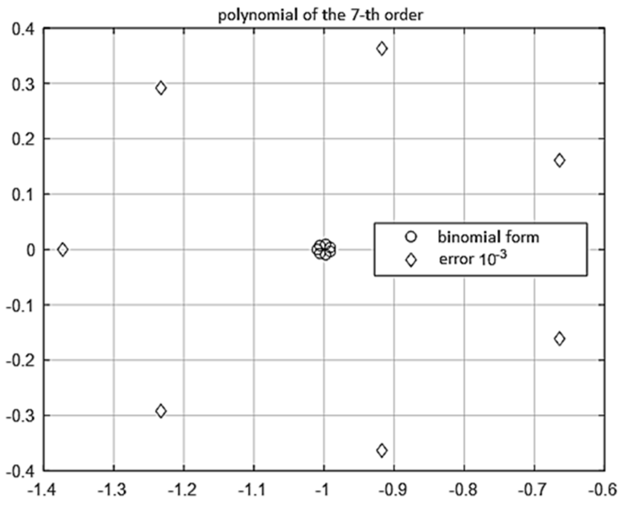

Fig. 11. Distribution of a polynomial roots of the seventh order in the case of exact setting of coefficients (labeled by a circle) and error 0.1 % in the last coefficient (labeled by a rhombus)

Let's notes that in the case of high-order polynomials (for example, a seventh-order polynomial in Fig. 11), even 15–16 exact digits to specify the coefficients can affect the placement of roots (this can be seen in the center of the last graph).

Thus, the **conclusion:** increasing the order of the polynomial increases its sensitivity to the coefficients accuracy, which confirmed by the experience of applied mathematics, especially in the situation of multiple roots [15, 16]. The occasion of multiple roots occurs in the case of a decrease in the sampling step – all discrete roots of the numerator and denominator polynomials of the discrete transfer function go to unit (see above).

There is only one way to reduce the influence of the accuracy of setting the polynomial coefficients - by reducing the order of the discrete transfer function. This can be realized by using the decomposition of a continuous transfer function (**prototype**) into elementary components – simple dynamic blocks of minimal order, which are then discretized by some method [17].

For electrical systems we will have a fractional-rational transfer function, i.e., the order of the numerator polynomial does not exceed the denominator polynomial order, which allows us to apply to it the Heaviside decomposition theorem (***i.e., to decompose the system***) [18]. As a result, instead of a continuous high order transfer function, we obtain the sum of continuous transfer functions of the first and second orders, which are much less sensitive to the accuracy of the coefficients setting. The next step is a simple procedure for sampling the obtained simple continuous transfer functions. An additional positive effect of these actions will be the presence of only one sampling point – the input for the sum of simple functions, which, accordingly, reduces sampling process errors.

Conclusion

In this article shows that the limited bit rate (arithmetic precision) of calculations in digital control systems significantly affects their implementation, and there are two main factors:

1) limited accuracy of the discrete pole reproduction of the transfer function;
2) bad conditionality of the denominator polynomial of the system's discrete transfer function in the case of reducing the sampling step.

Using decomposition of continuous transfer function for decomposition into elementary components not higher than second order with following sampling makes it possible to expand the boundaries of acceptable sampling frequency.

Direction of further research

Further studies planned: on the effect of decomposition and the used sampling method on the behavior of the synthesized discrete system.

References

1. William J. Dally, John W. Poulton, Digital Systems Engineering. Cambridge University Press, 2008. 704 p. [ISBN-13: 978-0521061759; ISBN-10: 052106175X].

2. Moudgalya Kannan M. Digital Control. John Wiley & Sons, Ltd.2007. 544 p. [ISBN-13: 978-0470031445; ISBN-10: 0470031441].

3. Goodwin, C. F. Grebe, M. Y. Salgado. Control systems project. Prentice Hall. 2000. ISBN 0-13-0958653-9. [https://csd.newcastle.edu.au/].

4. Raj Kamal, Digital Systems: Principles and Design. Pearson Education India, 2009. 524 p. [ISBN 8177585703, 9788177585704].

5. Dr. Farid Golnaraghi, Dr. Benjamin C. Kuo. Automatic Control Systems, 10th Edition. 2017, McGraw-Hill Education. ISBN: 9781259643835

6. LIVIU S. BOCÎI and VALENTIN MÜLLER. Mathematic model of three-phase induction machine connected to advanced inverter for traction system for electric trolley // Annals of the Brazilian Academy of Sciences. 2013. No. 85 (2). Pp. 849–858. Printed version ISSN 0001-3765 / Online version ISSN 1678-2690.

7. Alecsandru Simion, Leonard Livadaru and Adrian Munteanu. Mathematical Model of the Three-Phase Induction Machine for the Study of Steady-State and Transient Duty Under Balanced and Unbalanced States. Published: November 14 2012. DOI: 10.5772/49983. https://www.intech-open.com/books/induction-motors-modelling-and-control/mathematical-model-of-the-three-phase-induction-machine-for-the-study-of-steady-state-and-transient-.

8. Control System Toolbox: Design and analyze control systems. https://www.mathworks.com/help/control/?s_tid=srchbrcm.

9. Karl J Åström, Björn Wittenmark. Computer-Controlled Systems: Theory and Design, Third Edition. 2013, Published by Dover Books on Electrical Engineering. 578 p. [ISBN-13: 978-0486486130; ISBN-10: 0486486133].

10. B. Kuo. Digital Control Systems (The Oxford Series in Electrical and Computer Engineering), Second Edition. 1995, Oxford University Press. 784 p. [ISBN: 9780195120646].

11. R. Isermann. Digital Control Systems. Volume 1: Fundamentals, Deterministic Control [1989. 351 p. ISBN 13: 978-3-642-86417-9], Volume 2: Stochastic Control, Multivariable Control, Adaptive Control, Applications [1991. – 340 p. ISBN 978-3-642-86422-3] – Springer-Verlag Berlin Heidelberg.

12. Moroz V. Influence of finite-precision data on realization of digital control systems researches / V. Moroz, M. Solskyi // Computational Problems of Electrical Engineering. Vol. 3, No. 1, 2013. P. 69–74. [DOI: 10.23939/jcpee], [ISSN: 2224-0977 (Print) ; 2309-4834 (Online)].

13. Moroz V. Conditions of realization of digital regulators on digital systems with limited bit size / Moroz V., Solsky M., Holovach I. // Visnyk of the National Technical University “KhPI”. Collection of scientific works. Series: “Problems of automated electric drive. Theory and practice”. Kharkiv: NTU “KhPI”. 2013. No. 36 (1009). 600 p. [ISSN 2079-8024].

14. Moroz V. Analysis of existing methods for estimating the sensitivity of the roots of transfer functions to changes in their coefficients / Moroz V., Solsky M. // Visnyk of the Lviv Polytechnic National University “Electric power and electromechanical systems”. 2014. No. 785. P. 46–49. [DOI: 10.23939/sepes ; ISSN: 2706-9982].

15. Forsythe G. E., Malcolm M. A., Moler C. B. Computer Methods for Mathematical Computations. Englewood Cliffs, New Jersey 07632. Prentice Hall, Inc., 1977. 259 p. [ISBN-13: 978-0131653320 ; ISBN-10: 9780131653320].

16. Cleve Moler. Numerical Computing with MATLAB. [2004, Cleve Moler]. MathWorks, Inc. – http://www.mathworks.com/moler/chapters.html.

17. V. Moroz. High-Speed Precise Simulation Using Modified Z-Transform // Proc. of the XIII International Symposium on Theoretical Electrical Engineering ISTET’05. July 4–7, 2005, L’viv, Ukraine. Pp. 184–186. [https://www.researchgate.net/publication/288835403_High-Speed_Precise_Simulation_Using_Modified_Z-Transform].

18. William M. Siebert. Circuits, Signals, and Systems. 11th printing. 1998, The MIT Press. McGraw-Hill Book Company. 659 p. [ISBN-13: 978-0262690959; ISBN-10: 0262690950].

Список літератури

1. William J. Dally, John W. Poulton, Digital Systems Engineering. Cambridge University Press, 2008. 704 p. [ISBN-13: 978-0521061759; ISBN-10: 052106175X].

2. Moudgalya Kannan M. Digital Control. John Wiley & Sons, Ltd. 2007. 544 p. [ISBN-13: 978-0470031445; ISBN-10: 0470031441].

3. Goodwin, C. F. Grebe, M. Y. Salgado. Control systems project. Prentice Hall. 2000. ISBN 0-13-0958653-9. [https://csd.newcastle.edu.au/].

4. Raj Kamal, Digital Systems: Principles and Design. Pearson Education India, 2009. 524 p. [ISBN 8177585703, 9788177585704].

5. Dr. Farid Golnaraghi, Dr. Benjamin C. Kuo. Automatic Control Systems, 10th Edition. 2017, McGraw-Hill Education. ISBN: 9781259643835.

6. Liviu S. Bocîi and Valentin Müller. Mathematic model of three-phase induction machine connected to advanced inverter for traction system for electric trolley // Annals of the Brazilian Academy of Sciences. 2013. No. 85 (2). Pp. 849-858. Printed version ISSN 0001-3765 / Online version ISSN 1678-2690.

7. Alecsandru Simion, Leonard Livadaru and Adrian Munteanu. Mathematical Model of the Three-Phase Induction Machine for the Study of Steady-State and Transient Duty Under Balanced and Unbalanced States. Published: November 14 2012. DOI: 10.5772/49983. – https://www.intech-open.com/books/induction-motors-

modelling-and-control/mathematical-model-of-the-three-phase-induction-machine-for-the-study-of-steady-state-and-transient-.

8. *Control System Toolbox: Design and analyze control systems. https://www.mathworks.com/help/control/?s_tid=srchbrcm..*

9. *Karl J Åström, Björn Wittenmark. Computer-Controlled Systems: Theory and Design, Third Edition. 2013, Published by Dover Books on Electrical Engineering. 578 p. [ISBN-13: 978-0486486130; ISBN-10: 0486486133]*

10. *Kuo B. Digital Control Systems (The Oxford Series in Electrical and Computer Engineering), Second Edition. 1995, Oxford University Press. 784 p. [ISBN: 9780195120646].*

11. *Iserman. R. Digital Control Systems. Volume 1: Fundamentals, Deterministic Control [1989. 351 p. ISBN 13: 978-3-642-86417-9], Volume 2: Stochastic Control, Multivariable Control, Adaptive Control, Applications [1991. 340 p. ISBN 978-3-642-86422-3]/ Springer-Verlag Berlin Heidelberg.*

12. *Moroz V. Influence of finite-precision data on realization of digital control systems research-es / V. Moroz, M. Solskyi // Computational Problems of Electrical Engineering. Vol. 3, № 1, 2013. P. 69–74. [DOI: 10.23939/jcpee], [ISSN: 2224-0977 (Print) ; 2309-4834 (Online)].*

13. *Мороз В. І. Умови реалізації цифрових регуляторів на цифрових системах з обмеженою розрядністю / Мороз В. І., Сольський М. І., Головач І. Р. // Вісник Національного технічного університету “ХПІ”. Збірник наукових праць. Серія: “Проблеми автоматизованого електропривода. Теорія і практика”. Харків : НТУ “ХПІ”. 2013. № 36 (1009). 600 с. [ISSN 2079-8024].*

14. *Мороз В. І. Аналіз існуючих методів оцінки чутливості коренів передатних функцій до змін їх коефіцієнтів / Мороз В. І., Сольський М. І. // Вісник Національного університету “Львівська політехніка” “Електроенергетичні та електромеханічні системи”. 2014. № 785. С. 46–49. [DOI: 10.23939/sepes; ISSN: 2706-9982].*

15. *Forsythe G. E., Malcolm M. A., Moler C. B. Computer Methods for Mathematical Computations. Englewood Cliffs, New Jersey 07632. Prentice Hall, Inc., 1977. 259 p. [ISBN-13: 978-0131653320 ; ISBN-10: 9780131653320].*

16. *Cleve Moler. Numerical Computing with MATLAB. [©2004, Cleve Moler]. MathWorks, Inc. – http://www.mathworks.com/moler/chapters.html.*

17. *Moroz V. High-Speed Precise Simulation Using Modified Z-Transform // Proc. of the XIII International Symposium on Theoretical Electrical Engineering ISTET’05. July 4–7, 2005, L’viv, Ukraine. Pp. 184–186. [https://www.researchgate.net/publication/288835403_High-Speed_Precise_Simulation_Using_Modified_Z-Transform].*

18. *William M. Siebert. Circuits, Signals, and Systems. 11th printing. – 1998, The MIT Press. McGraw-Hill Book Company. – 659 p. [ISBN-13: 978-0262690959; ISBN-10: 0262690950].*

УДК 681.51; 681.52

В. І. Мороз
Національний університет “Львівська політехніка”,
кафедра електромехатроніки та комп’ютеризованих електромеханічних систем,
volodymyr.i.moroz@lpnu.ua

Т.-М. І. Янчак
Національний університет “Львівська політехніка”,
кафедра електромехатроніки та комп’ютеризованих електромеханічних систем,
maximyanchak@ukr.net

ВПЛИВ ОБМЕЖЕНОЇ РОЗРЯДНОСТІ НА РЕАЛІЗАЦІЮ ПЕРЕДАТНОЇ ФУНКЦІЇ В ЦИФРОВИХ СИСТЕМАХ

Проаналізовано малодосліджену проблему в цифрових системах керування – вплив на їхню поведінку обмеженої розрядності апаратної частини і, відповідно, коефіцієнтів дискретних передавальних функцій. Дослідження здійснено методом нулів і полюсів та перехідних ха-

рактеристик з використанням математичного застосунку MATLAB з бібліотекою Control System Toolbox і підтвердили актуальність цієї проблеми.

Мета досліджень – визначення мінімального періоду дискретизації в цифрових системах за умови забезпечення їхньої стійкості та відповідності поведінці неперервної системи (прототипу) за обмеженої розрядності обчислень і точності задавання даних. Це дасть змогу запропонувати рішення для зменшення негативного впливу обмеженої розрядності в розроблюваних цифрових системах і розширити діапазон раціональної частоти дискретизації.

Сформульовані граничні залежності для мінімального кроку дискретизації та сталої часу елементарних передавальних функцій першого і другого порядків. Показано, що у випадку обмеженої розрядності підвищення порядку полінома передавальної функції збільшує його чутливість до точності задавання його коефіцієнтів.

Дослідження впливу обмеженої розрядності обчислень виконано на прикладі передавальних функцій, що відповідають біноміальним формам п'ятого та сьомого порядків. Показано вплив точності задавання коефіцієнтів поліномів на перехідні функції та розміщення нулів і полюсів дискретних передавальних функцій на комплексній площині залежно від кроку дискретизації. Порівняно перехідні характеристики дискретних систем з обмеженою розрядністю з їхнім неперервним аналогом.

Дослідження підтверджують, що метод декомпозиції неперервної передавальної функції для розкладу на елементарні складові не вище ніж другого порядку з подальшою їх дискретизацією дає змогу розширити межі допустимих частот дискретизації у цифрових системах з обмеженою розрядністю даних. Показано вплив похибок на прикладі поліномів з другого по сьомий порядок, зокрема зі збільшенням порядку полінома зростає його чутливість до точності задавання його коефіцієнтів.

Ключові слова: двійкова розрядність; декомпозиція; дискретизація; нулі та полюси передавальної функції, передавальна функція, перехідний процес; поліном; цифрова система керування.