UDC 528.9

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https://doi.org/10.23939/istcgcap2022.95.094

CARTOMETRIC INVESTIGATION OF THE ACCURACY PLAN OF LVIV IN 1894

Ancient maps and plans are important sources of information for multifaceted knowledge of the past. In many studies, the accuracy parameters of spatial data are in demand. The purpose of our work is to study the geometric accuracy of the Lviv plan 1894 by Józef Khovanec. The methodology for studying the accuracy assessment is based on the transformation and geometric analysis of sets of identical points in the ancient plan and the modern reference one. For such a transformation, the Helmert transformation with four parameters and multiquadratic interpolation methods are used. The obtained results make it possible to graphically visualize the inaccuracies of the old plan in the form of displacement vectors, scale and rotation isolines, which clearly territorially diversify the distortions of the cartographic image. Using the method of least squares, a value was obtained that characterizes the positional accuracy of the ancient plan. All calculations and illustrations were made in the MapAnalyst software package, which specializes in the cartometric analysis of old maps. The results of cartometric analysis are influenced by a number of different factors, the decisive ones for the study were the following: the quality of the original; selection of a set of identical items; interpolation technique. When choosing identical points, the main attention is paid to their uniform distribution over the entire area of the plan at a constant position in time. The results obtained represent only one of the possible mathematical models built on the basis of the input data. However, we consider the achieved results to be valid. The processed technique significantly speeds up and simplifies the study of the accuracy of old plans and can be used for similar studies of other cartographic works, and the obtained numerical results and graphic visualizations can be used to compare old plans with each other.

Key words: ancient plans of Lviv, estimation of map accuracy, distortion on the map, Helmert transformation, displacement vectors, scale and rotation isolines, standart deviation.

Introduction

Maps and other cartographic works are an important part of the cultural heritage. Ancient maps are original sources of information about the past, as they reflect the geographical features of the territory and its socio-economic development over a period of time. Ancient maps and plans are indispensable in the study of the history of settlements, especially large cities, which are better provided with relevant cartographic materials.

The process of mapping in different epochs was determined by the level of development of science, technology and engineering, which largely determined the qualitative characteristics of cartographic reproduction of objective reality, in particular its spatial component. Self-evident is the trend of progressive movement from less accurate ancient maps to modern accurate maps. However, the quality of the cartographic image depends on many factors, so in each case it is different.

Ancient maps and city plans are an important source for conducting research on the development of Lviv. In many cases, you need to know the real accuracy of the mathematical basis of the map. Analysis of the accuracy of Lviv ancient maps and plans allows us to assess their features of geometric construction and take into account the information obtained in conducting relevant research.

The development of modern information technologies has made it possible to carry out cartometric analysis of ancient maps using GIS tools, making it faster and more accessible. This determines the relevance of our study of the accuracy of Lviv ancient plans.

Analysis of recent research and publications

Scientists from Switzerland, the Czech Republic, Poland and other countries have studied the accuracy of ancient maps, mainly in their countries [Jenny, 2008], [Petkiewicz, 1995], [Szeliga, 1993], [Zimová, 2006]. The legacy of Polish scientists whose research was founded by Henryk Merchyng in 1913 is of particular note [Ostrowski, 2014]. When studying the accuracy of ancient maps, scientists are beginning to widely use GIS tools [Manzano-Agugliaro et al., 2012].

Cartographic bibliography and cartographic aspects of Lviv's plans are currently quite fully studied

[Gavrilova, 1956], [Czerner, 1997], [Linda et al., 2004], [Shyshka, 2011, 2012], [Capraral, 2014], [Sossa et al., 2016], [Sossa, 2020].

The first major publication on the systematic study of ancient maps of Lviv was the work of Gavrilova E. "Map of Lviv and its development", which will be a presentation of her dissertation research [Gavrilova, 1956]. The author, in particular, thoroughly studied the scientific and technical features of topographic surveys of the city, geodetic and mathematical basis of plans, including a partial study of their geometric accuracy.

Research on the accuracy of ancient plans of Lviv is conducted at the Department of Cartography and Geospatial Modeling of the Lviv Polytechnic National University. Scientists of the department determined the geometric accuracy of the Lviv plans in 1844 and 1931 on the basis of the application of quantitative methods of such analysis. Direct measurements of lengths of lines and angles between two directions on ancient and modern plans were used and the analysis of results of measurements on the basis of the statistical theory of errors was carried out. [Holubinka et al., 2018].

Purpose

The main purpose of this work is to study the geometric accuracy of the Lviv plan in 1894 using graphical methods of visualization of distortions, which will assess and illustrate the spatial variations of errors in the plan.

Methods and results

The object of research is the "Plan of the Royal Capital City of Lviv, published and completed by the Society for the Development and Decoration of the City" (Plan król. Stoł. Miasta Lwowa wydany i uzupełniony przez Towarystwo dla rozwoju i upiększenia miasta) of 1894. The plan at scale 1: 7 200, published on a sheet measuring 47 × 68 cm, is known in the history of Lviv mapping as "Small Chowaniec" (Figs. 1, 2).

This plan is a fragment of the central part of the "Plan of the Royal Capital City of Lviv with the enclave Yalivets..." (Plan królewskiego stołecznego miasta Lwowa (z enkławą Jałowiec)...), which was concluded in 1890 by the engineer of the city Construction Administration Jozef Chowaniec on a scale of 1: 7,200 (known as the "Big Chowaniec"). The plan contains a large amount of geographical and statistical information about the city, an index of street and square names and a list of administrative and public buildings shown in the plan.

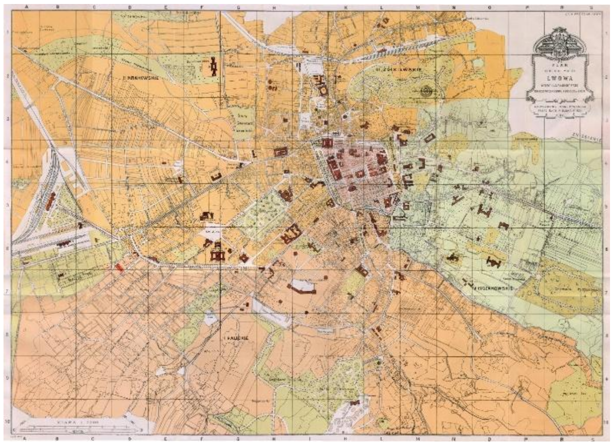

Fig. 1. Plan of Lviv by Jozef Chowaniec in 1894, scale 1: 7 200 (reduced image).

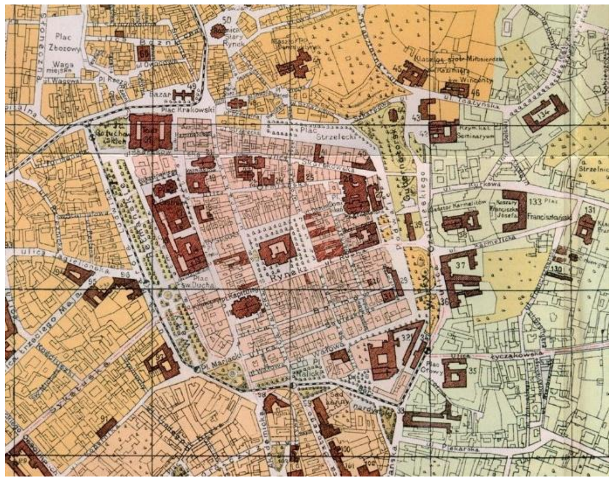

Fig. 2. Fragment of the central part of the plan of Lviv by Jozef Chowaniec in 1894

The plan of Lviv in 1894, given its compactness, was quite widely used. City services used it for various planning works. The plan has been repeatedly issued for tourism purposes.

The general procedure for cartometric analysis of the ancient plan of Lviv is to compare it with a modern map with a known geodetic coordinate system. We use two sets of corresponding control points. One set of points is contained on a modern map and is considered quite accurate in terms of spatial characteristics (reference), while the points on the ancient plan are considered inaccurate. Two sets of control points are used to bring two cartographic products into a common coordinate system.

The technological scheme of carrying out the study of the accuracy of the plan is shown in Fig. 3.

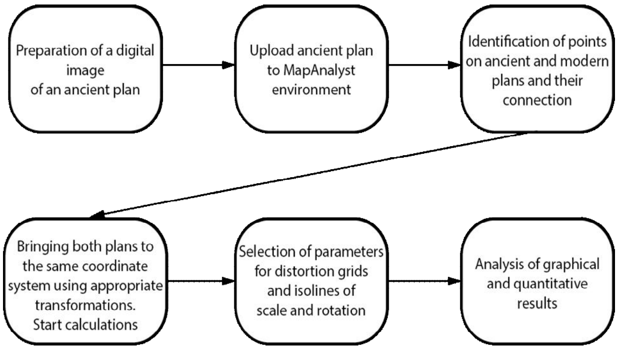

Fig. 3. Technological scheme of research of accuracy of the plan of Lviv in 1894

So, the first step of our study was to scan the plan using a GRAPHTEC CX530-09 wide-format scanner with a resolution of 500 dpi in TIFF format. The resulting raster image is imported into the MapAnalyst software.

By default, the application uses OpenStreetMap as a modern map, which is well suited for analyzing maps of large and medium scales. As we may know, OpenStreetMap is a common project to create a free editable map of the world. It covers a large part of Europe, North America and other parts of the world. Maps are created using data from portable GPS devices, aerial photography and other free sources. MapAnalyst downloads the map from the OpenStreetMap server. So, an Internet connection is enough to use OpenStreetMap.

The next step was to identify the system of points on the map of Lviv in 1894 and on the modern OpenStreetMap. The choice of a set of identical points used to calculate the transformation key plays an important role. Identical points should be distributed, ideally, throughout the mapping area evenly, so that the resulting transformation key is global, i.e takes into account the geometric and cartographic characteristics of the entire cartographic image. After receiving the transformation key, errors are visualized on the old map or the new map, which we accept as accurate. Graphical methods are used to visualize errors: displacement vectors, distortion grids and isofermats, i.e lines connecting points with the same errors. Special software products, one of which is MapAnalyst, are best for building graphic illustrations of the inaccuracies of ancient plans. MapAnalyst is developed and maintained by Bernhard Jenny, Monash University, Melbourne [*Jenny, 2006*]. Some parts were contributed by Adrian Weber, then at ETH Zurich. Most programming was carried at the Institute of Cartography and Geoinformation of ETH Zurich [*Jenny et al., 2008*].

To calculate the transformation key, we chose 98 points. This number, at first glance, may seem relatively small. However, the items were chosen carefully, taking into account the above principles, and they appear to be a relatively representative sample (Fig. 4). Identity points represented temples, cathedrals, castles, historical buildings, street intersections. The set of points is almost uniform, except for the northeastern part.

To bring the coordinate systems of the modern and ancient plans closer together, a four-parameter affine transformation (the Helmert transform with four parameters) with one scale factor, one angle of rotation, and two offsets was chosen. Further, the transformation ratios were corrected in accordance with the least squares method $V^TV = \min$.

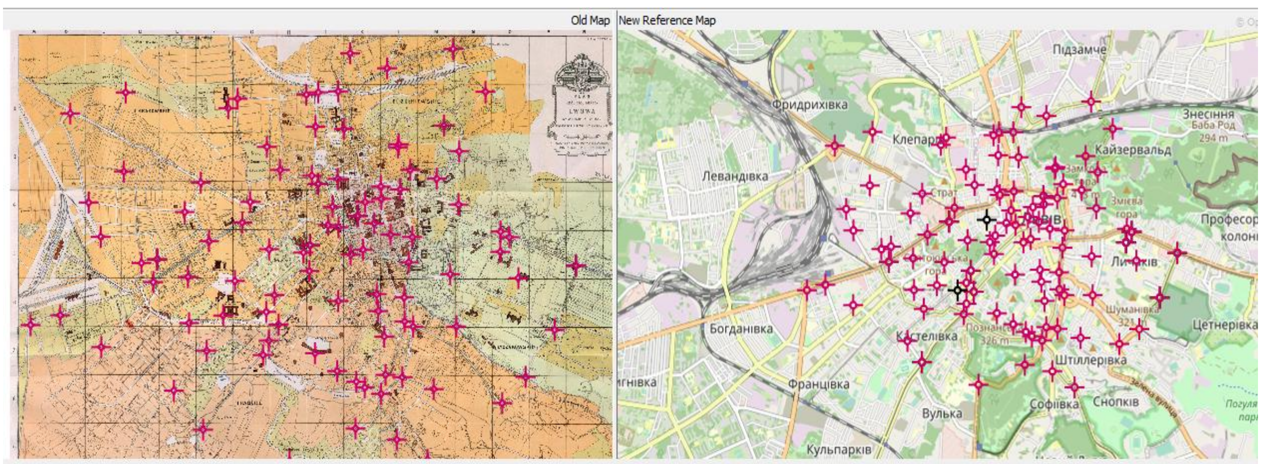

Fig. 4. Distribution of 98 identical points on Lviv's plans

The affine transformation equation can be written in the general form:

$$X_i = x_0 + m\cos\alpha\, x_i - m\sin\alpha\, y_i,$$
$$Y_i = y_0 + m\sin\alpha\, x_i + m\cos\alpha\, y_i,$$

where x_i and y_i – point coordinates on the modern plan; X_i and Y_i – coordinates of the corresponding point on the ancient plan; m – scale coefficient; α – rotation angle; x_0, y_0 – displacement along the coordinate axes respectively.

Parameters determined from a set of 98 identical points using the MapAnalyst program and supplemented by the characteristics of their determination (standard deviation) are shown in Table 1.

As we can see, the plan of Lviv of 1894 is generally turned counterclockwise in comparison with the reference data set by about 0.2 ± 0.07°. The global scale factor differs from the scale of the plan by 512±10 units, ie by 7.1±0.14 %.

Table 1

The values of the coefficients of affine transformation and their standard deviations

Coefficient	Value	Standart deviation
m	7 712	10.2
α	0.2°	0.07
x_0	2.9 м	0.0002
y_0	718 м	0.0002

The purpose of cartometric analysis is to check the mathematical and spatial parameters of the plan. To achieve the ideal result, both plans should be vectorized. Since a cartographic product is a large set of point, line and area cartographic signs, this process is time-consuming. Therefore, it is advisable to choose only a subset of such elements, representing a sample set in which the highest degree of positional accuracy can be assumed.

After completing the affine transformation, we can perform a cartometric analysis of Lviv's plan. The results of the analysis serve as an approximate evaluation criterion, illustrating the accuracy of the plan. Presentation of the results is possible in the form of tables or in the form of graphic images in plan. It should be emphasized that the following results of the cartometric analysis depend on the choice of a set of identical points that serve to calculate the transformation key. If the number or distribution of identical points were changed, the results would be slightly different, but not fundamentally different.

Affine transformation belongs to the group of residual transformations, the same points in both coordinate systems do not completely overlap. Correction values of identical points can be used to assess the accuracy of the image elements in the plan (Fig. 5). Such values are called displacement vectors. Each vector starts at a point previously identified on the old plan and ends where it would be if that plan was as accurate as the reference. Using the specified transformation key, the image of the central part of the city (Rynok Square) was rated as the most accurate, and the northern and eastern ones were rated as the least accurate.

Based on the selected set of n identical points and displacement vectors, the standard deviation $\sigma_0 = \sqrt{\dfrac{V^T V}{2n-4}}$ is calculated, which describes the root mean square error of the coordinate and the value $\sigma = \sigma_0\sqrt{2}$ that describes the positional accuracy of the Lviv plan. In our case σ_0 is 16.28 m, and $\sigma = 23.03$ m, is 0.3 %.

The analysis of the scale value and the angle of rotation is an important factor illustrating the accuracy of constructing a cartographic product. In our study, we used the MapAnalyst program, which specializes in the cartometric analysis of ancient maps [Jenny et all., 2008]. To do this, the application contains a complex geometric-analytical apparatus that uses multiquadratic interpolation of a set of identical points. Detailed information on the analysis methodology using multiquadratic interpolation is presented in [Beineke, 2000].

The process of cartometric analysis is fully automated, except for the identification of a set of points. This fact contributes to the speed and overall reliability of the analytical process. Using a set of 98 points and affine transformation parameters, MapAnalyst generates scale and rotation isolines.

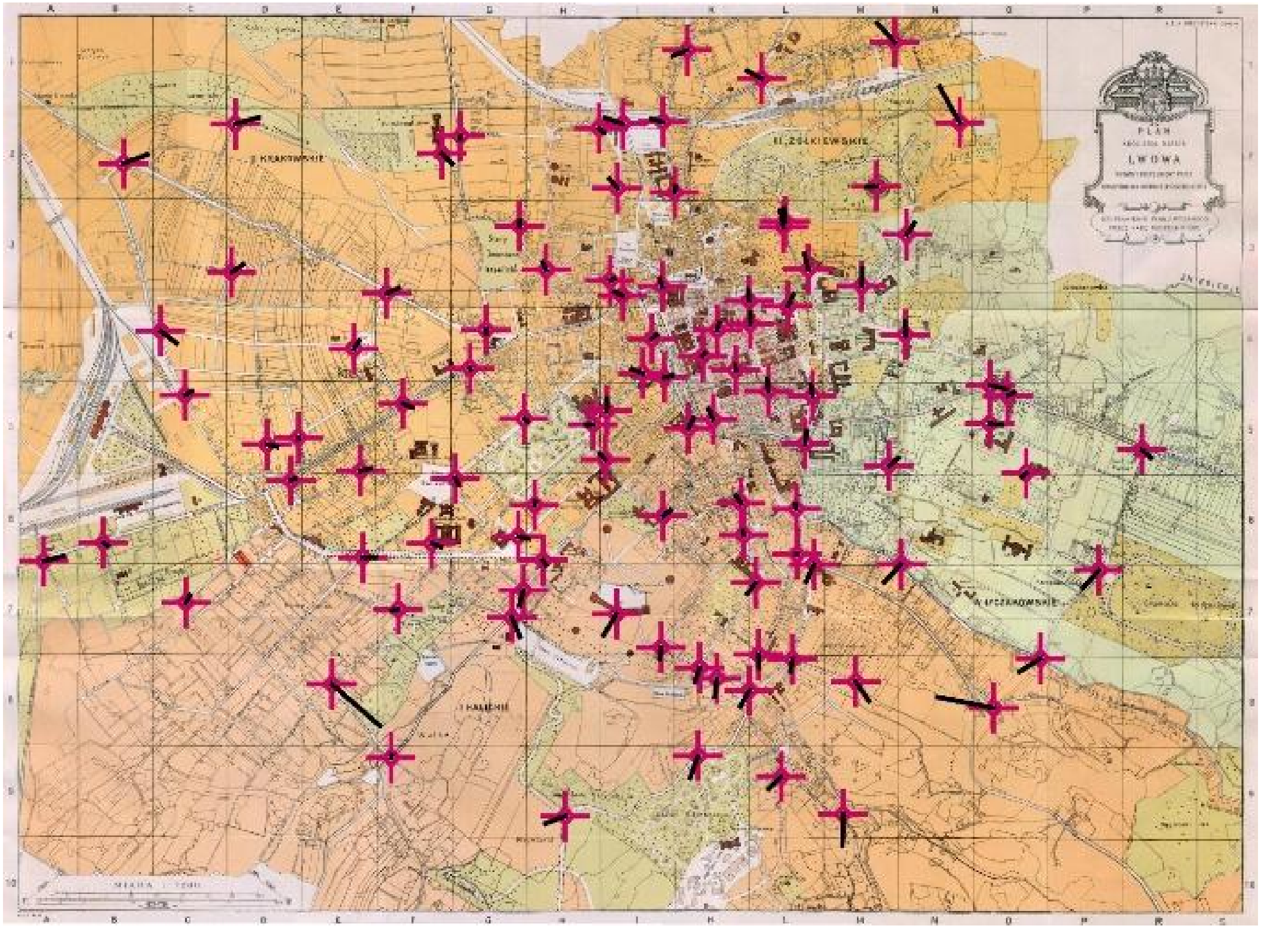

Fig. 5. Vectors of displacements on the plan of Lviv in 1894

Isolines are a way to show local variations in scales and rotations. They are curves which are connecting points of uniform scale and rotation. To reproduce them, two transparent raster grids are required, containing a stable scale location and a rotation scale value. The algorithm for constructing isolines consists of three stages:

1. Creating two raster grids that will contain the scale and rotation data.
2. Calculation of the scale and parameters of rotation to each cell of the raster.
3. Finally, we build isolines using the parameters of the cells of the raster grid.

When calculating the parameters of a certain cell, instead of using all valid point pairs, only points located within the circle of the current radius describing the cell are used (Fig. 6).

Realizing that points more distant from the center of the circle have less effect on processing than points closer to the center. Each point is assigned an individual weighting factor due to its location towards the center of the circle. Thus, the number and shape of the resulting isolines depend on the radius of the circle. Contours with a small radius of the impact circle show local variations in magnitude and rotation scales, a larger radius of the impact circle will result in more global values and smoother contours. The correct choice of radii is the main point when using this method. It is recommended to try different values for the radius of influence until you find the most descriptive isolines. A good value to start with is approximately a quarter of the scale of the map.

To create isolines of local scale variations on the studied plan, we apply the impact radius of 800 m with a step of 200 meters (Fig. 7) and 1000 m with a step of 300 (Fig. 8).

On Fig. 7, the largest scale value of 1:6 000 and less than 1:8 000 is observed in the southern part of the plan. At the same time, the central part of Lviv does not contain such strong distortions. The amplitude of fluctuations of scale values in the plan is 2 000 units. This can be caused both by the choice of identical points, and by errors in the plan itself.

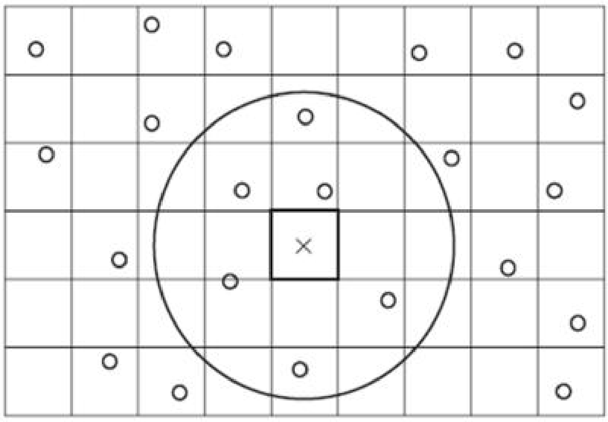

Fig. 6. Six points inside a circle, calculating the scale or rotation angle of the cell in the center

In a case with an impact radius of 1000 (Fig. 8), slight differences in the scale values are observed. The largest value of 1:7.500 is located in the central and northwestern parts, and the smallest value of 1:7,800 is located in the southwestern and southeastern parts of the plan. Amplitude is 300 units.

As can be seen from Figs. 7–8, an increase in the impact radius from 800 m to 1000 m led to a significant decrease in the number of isolines. However, the rest give a more global indicator, more stable deviations of the scale in different parts of the plan from the scale of 1:7.712 obtained by affine transformation and the scale of 1:7.200 declared on the ancient plan.

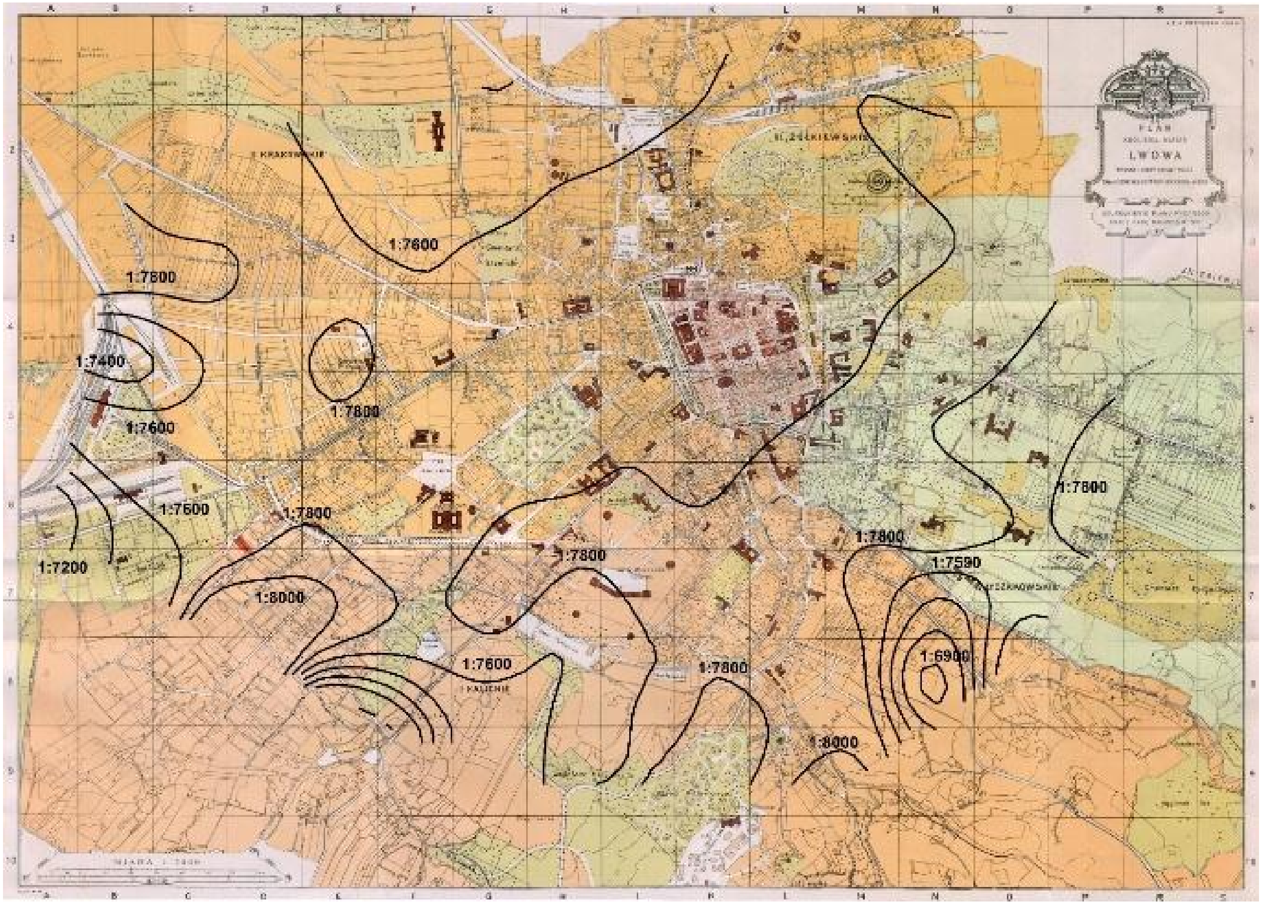

Fig. 7. Isolines of scale variations with an impact radius of 800 m, built with a step of 200 m

The isolines of rotation (Fig. 9) show the following: the largest rotations of 3° are observed in the south-western and north-eastern parts of the plan. There are no rotations in the central part.

If we summarize all the graphical constructions and the obtained numerical characteristics, it can be argued that the plan demonstrates high accuracy with respect to angular values. With respect to linear measurements and positional errors, the accuracy is worse. This is evidenced by rather high values of the root mean square error of coordinates σ_0 =16.28 m, and the root mean square position error σ = 23.03 m. High accuracy of linear measurements was recorded in the central part of the plan, where there are minimal positional and angular distortions. Inaccuracies in the vicinity are partly caused by the insufficient number of identified points in the northeast, southwest and west parts of the city. In addition, the global scale indicator of 1:7 700 differs by 500 units (nearly 7 %) from the value indicated on the work itself – 1:7 200.

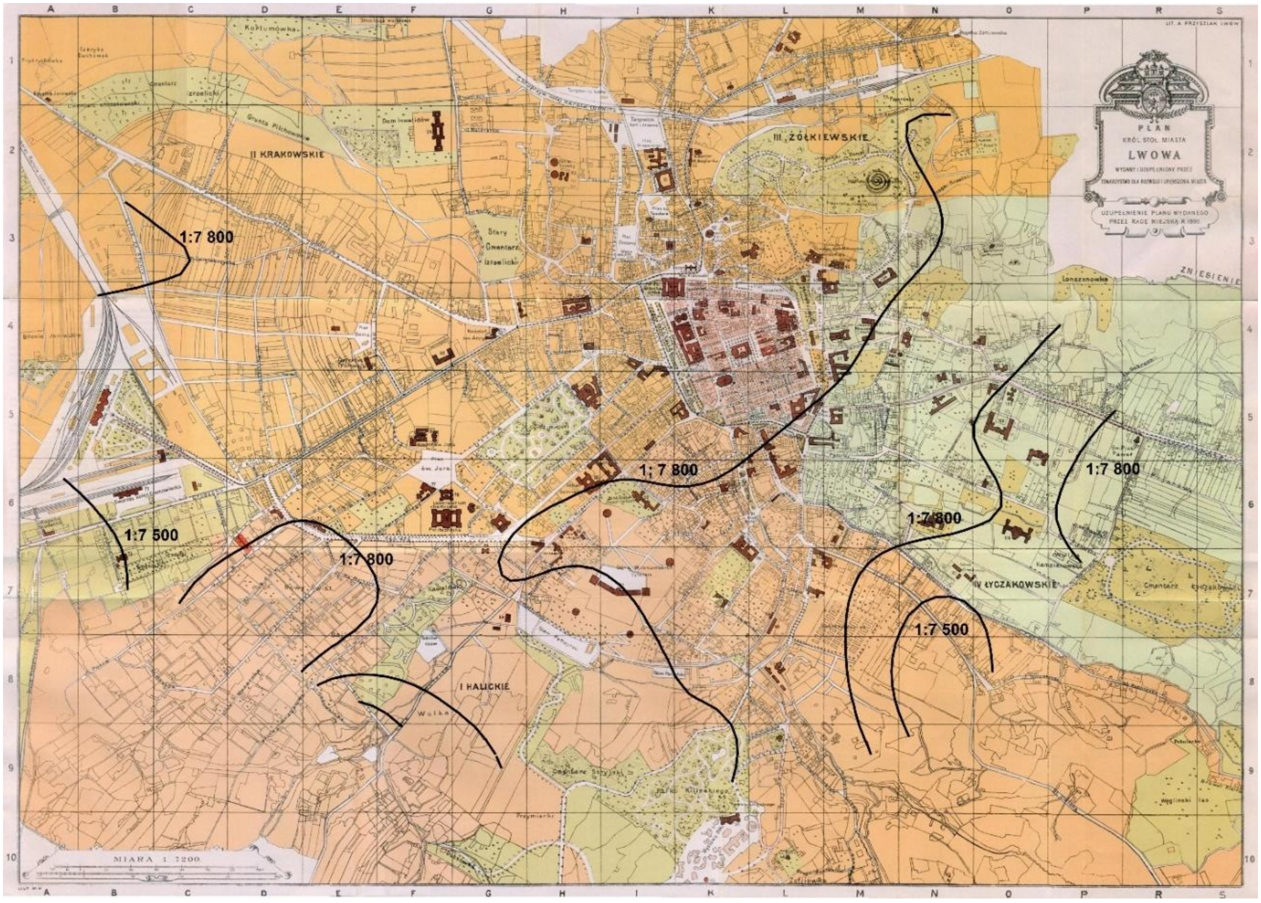

Fig. 8. Isolines of scale variations with an impact radius of 1000 m, built with a step of 300 m

Conclusions

The accuracy of the mathematical basis of the map largely describes the accuracy of the cartographic representation of objects on the map. To identify the location of features that were shown on an ancient map but no longer exist, it is important to know the accuracy of the map's mathematical basis. It is also necessary in the study of the general development of mapping.

A cartometric study of the accuracy of the old plan of Lviv in 1894 was carried out on the basis of comparison of sets of identical points on the old and modern plans of the city. A well-developed methodology allows to investigate distortions on ancient plans or maps using modern mathematical and geometric solutions, which contributes to the speed and reliability of obtaining results. The calculated distortion parameters were visualized using the MapAnalyst software product, which clearly showed information about the geometric accuracy of building the Lviv plan of 1894. It has been established that the results of cartometric analysis are influenced by a number of different factors. In our case, we consider three decisive factors: the quality of the original; selection of set of identical items; interpolation technique. When choosing identical points, the main attention should be paid to their uniform distribution over the entire area of the plan or map. Another condition for the choice is the invariance of the position of these points from the time of the creation of an ancient cartographic work to the present. The applied methods based on multi-square interpolation are suitable for processing a dataset with approximately the same density of points. Unevenly distributed clusters of points or places that do not contain points negatively affect the result achieved. The used set of 98 identical points can be considered sufficient, although it was not always possible to maintain the same density. There were not the required number of identical points in the vicinity of the city, so we can assume that they will have more weight when calculating some parameters of the transformation key. The technique of multiquadratic interpolation also played an important role and was used to reconstruct a continuous surface from discrete data. The results obtained represent only one of the possible mathematical models built on the basis of the input data. However, despite the above facts, we consider the achieved results to be valid. None of the factors played such an important role that would significantly affect the results of our cartometric study.

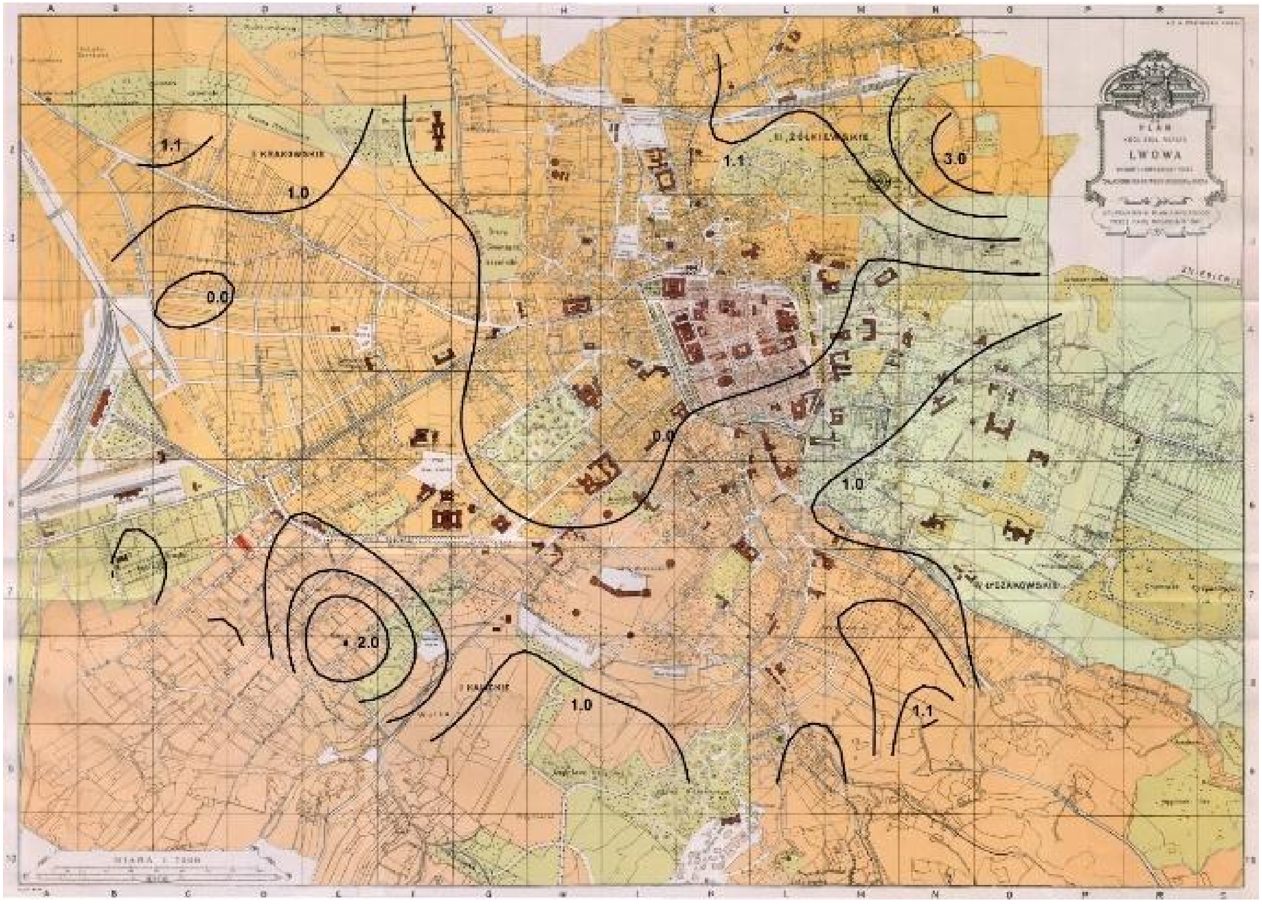

Fig. 9. Rotation isolines on the Lviv plan in 1894 built with a step of 0.1°.

The developed technique significantly simplifies the study of the accuracy of old plans and can be used for similar studies of other cartographic products, and the obtained numerical results and graphical visualizations can be used to compare old plans with each other, subject to careful consideration of the above factors affecting the results of cartometric analysis using MapAnalyst.

REFERENCES

Beineke, D. (2000). Verfahren zur analysis Genauigkeitsanalyse fur AltKarten, Munchen.

Capral, M. (2014). *Atlas of Ukrainian historical cities. Lviv. (V.1).* For science. ed. M. Capral. Kyiv: Cartography Research Institute, 95p. + 25 original maps, 11 reconstruction maps, 6 types of the city (*in Ukrainian*).

Gavrilova, E. Map of the city of Lviv and its development. L.: LPI, 1956. 80 p. (*in Russian*).

Czerner, O. (1997). Lwów na dawnej rycinie i planie. Wrocław, Warszawa, Kraków.

Holubinka, Yu., Nikulishyn, V., Sossa, R., & Yurkiv, M. (2018). Investigation of the accuracy of plans of Lviv in 1844 and 1932. *Geodesy, cartography and aerial photography*, 88, 33-46. https://doi.org/10.23939/istcgcap2018.02.033.

Jenny, B. MapAnalyst – A digital tool for the analysis of the planimetric accuracy of historical maps. e-Perimetron, 1–3, 2006, p. 239–245. http://www.e-perimetron.org/Vol_1_Table_of_Contents.pdf.

Jenny, B., Weber, A., & Hurni, L. (2007). Visualizing the planimetric accuracy of historical maps with MapAnalyst. Cartographica: The International Journal for Geographic Information and Geovisualization, 42(1), 89–94. https://doi.org/10.3138/carto-v42-1-089.

Linda, S. M., Oleshko, O. P., & Ivanochko, U.I. (2004). Study of the territorial development of Lviv during the Austrian period (1772–1918) based on cartographic material. *Bulletin of the Lviv Polytechnic National University*, 2004. No. 505: Architecture, 127–141. (*in Ukrainian*).

Manzano-Agugliaro, F., Martinez-Garcia, J., & San-Antonio-Gomez, C. (2012). GIS analysis of the accuracy of Tomas Lopez's historical cartography in the Canary Islands (1742–1746). *Scientific Research and Essays,* 7(1). https://doi.org/10.5897/SRE11.1271.

Ostrowski, J. Praca Henryka Merczynga z 1913 roku o radziwiłłowskiej mapie Litwy i jej wpływ na polskie badania dokładności dawnych map (w setną rocznicę publikacji). *Dawne mapy jako źródła w badaniach geograficznych i historycznych*. (Z Dziejów Kartografii. V 17, pp.35–52). Warszawa. 2014.

Petkiewicz, S. (1995). Analiza dokładności kilku map z XVII, XVIII I XIX wieku obejmującyh Polske w dawnich granicach. *Prace i studia geograficzne*, 17, 103–109.

Shyshka, O. V. (2011). Mapping of Lviv (1582–1770). *Bulletin of geodesy and cartography,* 4, 42–47. (*in Ukrainian*).

Shyshka, O. V. (2011). Mapping of Lviv in the pre-Austrian period (1772–1918). *Bulletin of geodesy and cartography,* 6, 42–46. (*in Ukrainian*).

Shyshka, O. V. (2012). Mapping of Lviv (1918 – 1944). *Bulletin of geodesy and cartography,* 1, 44–47. (*in Ukrainian*).

Sossa, R., Kryshtalovych, U., & Shyshka, O. (2016). *Cartographic editions of Lviv:* Exhibition catalog. Lviv, December 6–8, 2016. According to the general ed. R. Sossa. L: Lviv Polytechnic Publishing House, 28 p. (*in Ukrainian*).

Sossa, R. (2020). Lviv mapping in the Soviet-period. *Geodesy, Cartography and Aerial Photograph*. 91, 59–69. https://doi.org/10.23939/istcgcap2020.91.059

Szeliga, J. (1993). Metody i stan dokładnościowych badań dawnych map z obszaru Polskie. *Dorobek polskiej historii kartografii,* 6, 51–67.

Zimová, R., Pešt'ák, Ja., & Veverka, B. (2006). Positional Accuracy of Old Maps. Historical Military Mapping of Czech Lands. *GIM International*. Retrieved from: https://www.gim-international.com/content/ article/positional-accuracy-of-old-maps.

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КАРТОМЕТРИЧНЕ ДОСЛІДЖЕННЯ ТОЧНОСТІ ПЛАНУ ЛЬВОВА 1894 РОКУ

Стародавні карти та плани є важливими джерелами інформації для різноаспектного пізнання минулого. У багатьох дослідженнях затребуваними є точностні параметри просторових даних. Метою нашої роботи є дослідження геометричної точності плану Львова 1894 року Юзефа Хованця. Методика дослідження оцінки точності базується на перетворенні та геометричному аналізі наборів ідентичних точок на стародавньому плані та сучасному еталонному. Для такого перетворення використовується трансформація Гельмерта з чотирма параметрами та техніка мультиквадратичної інтерполяції. Отримані результати дають можливість графічно візуалізувати неточності старого плану у вигляді векторів зміщення, ізоліній масштабу та обертання, що унаочнюють територіальну диверсифікацію спотворень картографічного зображення. За допомогою методу найменших квадратів отримано значення, що характеризують позиційну точність давнього плану. Всі розрахунки та ілюстрації виконані у програмному пакеті MapAnalyst, який спеціалізується на картометричному аналізі стародавніх карт. На результати картометричного аналізу впливає низка різних факторів, вирішальними з яких для нашого дослідження були: якість оригіналу; вибір набору ідентичних пунктів; техніка інтерполяції. При виборі ідентичних точок основну увагу приділено їх рівномірному розподілу по всій площі плану при незмінному положенні у часі. Отримані результати представляють лише одну з можливих математичних моделей, побудованих на основі вхідних даних. Однак, досягнуті результати вважаємо дійсними. Опрацьована методика значно прискорює та спрощує вивчення точності старих планів і може бути використана для аналогічних досліджень інших картографічних творів, а отримані числові результати та графічні візуалізації – для порівняння старих планів між собою.

Ключові слова: стародавні плани Львова, оцінка точності карти, спотворення на карті, трансформація Гельмерта, вектори зміщень, ізолінії масштабу та обертання, стандартне відхилення.

Received 15.04.2022