MATHEMATICAL MODELING AND COMPUTING, Vol. 10, No. 1, pp. 195–203 (2023)

MHD Nanofluid boundary layer flow over a stretching sheet with viscous, ohmic dissipation

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(Received 8 June 2022; Revised 25 September 2022; Accepted 12 October 2022)

The objective of this research is to examine the steady incompressible two-dimensional hydromagnetic boundary layer flow of nanofluid passing through a stretched sheet in the influence of viscous and ohmic dissipations. The present problem is obtained with the help of an analytical technique called DTM-Pade Approximation. The mathematical modeling of the flow is considered in the form of the partial differential equation and is transformed into a differential equation through suitable similarity transformation. The force of fixed parameters like thermophoresis number Nt, Brownian motion number Nb, Prandtl number Pr, Lewis number Le, Magnetic field M, suction/injection S and Eckart number Ec are displayed with the aid of Figures. Our outcomes showed a greater trend in the velocity profile for the parameters of magnetics M, suction S, and nonlinear stretching parameter n. While the reverse trend is found against the temperature profile when the Prandtl number increases. Lewis number and other parameters have shown increasing behavior in the concentration profile.

Keywords: *magnetic field; Eckart number; thermophoresis number; Brownian motion; heat transfer; mass transfer.*

2010 MSC: 76D05, 76D10, 76A02, 80A20 **DOI:** 10.23939/mmc2023.01.195

1. Introduction

Heat transfer enhancement is a fascinating subject to explore in the present day because of its numerous applications in engineering fields. Conventional fluids such as water, engine oil, and ethylene glycol are being used for heat transfer, but they have limited conduction of heat. To complete the heat energy for industrial and engineering places, an alternate way to enhance the heat transfer is by adding nanoparticles to an ordinary fluid called nanofluid. Choi [1] introduced a new fluid in 1995 known as nanofluid. It is a colloidal suspension of nanoparticles, whose size is between $(1-100)$ nm. Nanoliquids are used in a variety of human endeavors, electrical and chemical engineering such as power systems, heat pipes, heat sinks,and thermal management of power electronics: semiconductors, air conditioning, refrigeration, fermentation, protein/cell separation, drug delivery, catalysts, lubricants, rotary seals, cancer therapy, antibacterial agents, NMR imaging, magnetic cell sorting, energy saving cooling medium, automobiles, solar energy, solar air collectors, fuel cells, refrigeration, electronic devices cooling, micro-electro-mechanical systems. The biochemical applications include gas sensing, protein and pathogen detection, DNA translocation and sequencing. In addition, utilizing magnetized nanofluid systems is a significant step toward achieving targeted medication administration and differential diagnostics.

Mustafa et al. [2] studied the flow through a stretching sheet is a significant issue in many engineering processes with industry sectors such as melt spinning, plastic and rubber sheets, production of paper, chemical engineering plant, food processing. Khan and Pop [3] deal with the boundary layer flow of nanofluid over a stretching sheet. Hassni et al. [4] depicted an analytical solution of the boundary layer flow of nanofluid by the Homotopy analysis method. Recently, Jabeeb et al. [5] analyzed the MHD flow over a nonlinear stretching sheet and solved the steady, in compressible problems using computational software with various standard techniques. Vennila et al. [6] investigated an outcome of a magnetic field on heat transfer carbon nanotubes suspended nanofluids by shooting type of Laplace Adomian decomposition method. Mabood et al. [7] introduced a notion of heat transfer of nanofluid flow over a stretching sheet in an occurrence of a magnetic field by the Runge–Kutta–Fehlberg fourth–fifth order method. Mustafa and Junaid [8] deal with Casson nanofluid magneto hydrodynamic flow and heat transfer across a non-linearly stretched sheet. Ellah et al. [9] investigated the combined impact of MHD and slip across a flat moving plate. Rashid et al. [10] discussed an entropy formation analysis of the hydrodynamic boundary layer flow of nanofuid past over a porous shrinking wall. Japil et al. [11] discussed the magnetic stagnation point flow past a stretching or shrinking sheet in a porous medium with momentum slip. Alias et al. [12] explained an impact of suction and magnetic-induced Navier slip flow due to nonlinear shrinking sheet.

Malik et al. [13] described an effect of viscous dissipation on the magnetic boundary layer flow of Sisko fluid. Prakash et al. [14] proposed an impact of viscosity and joule dissipation on magnetic field mixed convection nanofluid flow via a nonlinear stretched inclined plate exposed to varied suction embedded in a porous medium. Ganga et al. [15] discussed the magnetic boundary layer flow of nanofluid past a vertical plate in an influence of heat generation/absorption, viscous and Joule dissipation. Mohsen et al. [16] examined the concept of numerical simulation of MHD flow of nanofluid, heat transfer and effect of viscous dissipation. Nayak [17] expressed the hydrodynamic three-dimensional flow and heat transfer investigation of nanofluid through the shrinking sheet inspired by thermal radiation and viscous dissipation. Seyyedi Mehdi et al. [18] delivered the consequences of viscous dissipation and internal heat produced in the porous circular cavity on free convection of Cu-water nanofluid using the CVFEM.

The DTM has been explored and applied to a variety of linear and nonlinear issues in recent years. Rashidi et al. [19] explored a DTM method for solving both linear and nonlinear differential equations. Mujammad et al. [20] illustrated the DTM technique for unsteady nanoliquid flow and heat transfer. Dibyendu and Sanjib [21] approached the DTM and Pade approximation to derive the problems of magnetic field and mass transfer nanofluid flow through a stretching sheet with the influence of Soret and Dufour phenomena. Sheob Rashid et al. [22] solved a problem of free convection and MHD slip flow and heat transfer over a radially stretching sheet with thermal radiation using the differential transformation method.

A study of the previously mentioned literature motivated us to present this work with the effects of magnetic, ohmic and viscous dissipation in nanofluid boundary layer flow over a stretching sheet. Khashi et al. [23] discussed the effect of suction on the MHD flow in a doubly stratified micropolar fluid over a shrinking sheet. Majeed et al. [24] predicted the steady thermal boundary layer nanofluid flow was solved by the DTM-Pade approximation technique. Based on his motivation, we have expanded the work of nanoliquid motion with the effects of magnetic, ohmic and viscous dissipation in this problem.

2. Mathematical formulation

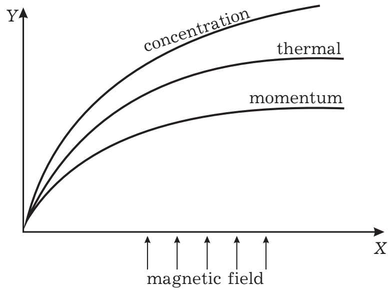

Fig. 1. Physical model.

The two-dimensional, laminar, steady nonlinear hydromagnetic flow of an incompressible, viscous, electrically conductive nanofluid has been addressed as Fig. 1. The flow takes place across a nonlinear stretching sheet. A variable magnetic field of strength $B(x) = B_0\, x^{\frac{n-1}{2}}$ is the applied normal force towards the sheet and parallel to the y-axis. The sheet corresponds to the plane $y = 0$, and the flow is confined $y > 0$. The x-axis is positioned in the same direction as the stretching sheet, the y-axis is angled in the opposite direction. Let $u = U_w(x) = ax^n$ symbolized as a stretching sheet velocity, where a is a positive constant and n is a nonlinear stretching parameter. An energy equation takes into account both viscous and ohmic dissipation. The ambient temperature and concentration are indicated as T_∞ and C_∞, respectively. We made an assumption that the constant temperature was $T_W > T_\infty$ and that the constant temperature was $C_W > C_\infty$. The governing equations for the boundary layer, which include momentum, energy, and concentration equations with dissipation effects are as follows [19, 24]

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial \upsilon}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2(x)u}{\varrho_{nf}}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\left(\frac{D_T}{D_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2u^2}{(\varrho c_p)_{nf}}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+\upsilon\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

where u and v are the velocity components along the x and y axis, ν is the nanofluid kinematic viscosity, α is the nanofluid thermal diffusivity, $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the nanoparticles and the heat capacity of the base fluid, ϱ_{nf} is the density of the nanoparticle. D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, and C is the volumetric expansion coefficient.

The boundary conditions are as follows:

$$\upsilon=V_w(x),\quad u=ax^n,\quad T=T_w,\quad C=C_w\quad \text{at}\quad y=0,$$
$$u\to 0,\quad T\to T_\infty,\quad C\to C_\infty\quad \text{at}\quad y\to\infty.$$

Let us introduce a similarity transformations

$$\eta=y\left(a\left(\frac{n+1}{2}\right)^{\frac{1}{2}}\right)x^{\frac{n-1}{2}},$$
$$u=ax^nf'(\eta),\quad \upsilon=-a\nu\left(\frac{n+1}{2}\right)x^{\frac{n-1}{2}}\left(f+\left(\frac{n-1}{n+1}\right)\eta f'\right),$$
$$T_\infty+\theta(\eta)(T_w-T_\infty)=T,$$
$$C_\infty+\phi(\eta)(C_w-C_\infty)=C.$$

Incorporating the above mentioned transformation into the governing equations (1) to (4) reduces to,

$$f'''+ff''-\frac{2n}{n+1}f'^2-Mf'=0, \tag{5}$$

$$\frac{1}{\mathrm{Pr}}\theta''+f\theta'+\mathrm{Nb}\,\phi'\theta'+\mathrm{Nt}\,\theta'^2\,\mathrm{Ec}\,f''^2+Mf'^2=0, \tag{6}$$

$$\phi''+\mathrm{Le}\,f\phi'+\frac{\mathrm{Nt}}{\mathrm{Nb}}\theta''=0, \tag{7}$$

and the corresponding boundary conditions are

$$f(0)=S,\quad f'(0)=1,\quad \theta(0)=1,\quad \phi(0)=1\quad \text{at}\quad \eta=0, \tag{8}$$

$$f'(\eta)\to 0,\quad \theta(\eta)\to 0,\quad \phi(\eta)\to 0\quad \text{at}\quad \eta\to\infty, \tag{9}$$

primes stands for differentiating with to η. M is the magnetic parameter, Pr is the Prandtl number, Nb is the Brownian motion parameter, Nt is the thermophoresis parameter, Le is the Lewis number, Ec is the viscous dissipation parameter (Eckart number), $S>0$ is the suction parameter and $S<0$ is the injection parameter

$$\mathrm{Pr}=\frac{\nu_f}{\alpha_f},\quad M=\frac{\sigma B_0^2}{a\varrho_f},\quad \mathrm{Ec}=\frac{U_w^2}{(C_P)_f(T_w-T_\infty)},$$
$$\mathrm{Nt}=\frac{(\rho C)_pD_T(T_w-T_\infty)}{(\rho C)_fT_\infty\upsilon},\quad \mathrm{Nb}=\frac{(\rho C)_pD_B(\phi_w-\phi_\infty)}{(\rho C)_p\upsilon}.$$

The Nusselt number Nu_x and Sherwood number Sh_x, which are defined as the parameters of practical significance

$$\mathrm{Nu}_x = \frac{\chi q_w}{k(T_w - T_\infty)}, \quad \mathrm{Sh}_x = \frac{\chi q_m}{D_B(C_w - C_\infty)},$$

where q_w and q_m are the wall heat flux and mass flux, respectivelty; the reduced Nusselt number Nu_r and Sherwood Sh_r can be written as

$$\mathrm{Nu}_r = \mathrm{Re}_x^{-0.5}\mathrm{Nu}_x = -\theta'(0), \quad \mathrm{Sh}_r = \mathrm{Re}_x^{-0.5}\mathrm{Sh}_x = -\phi'(0).$$

3. Differential transformation method with Pade approximation employment

The DTM is the one of the famous semi analytical tools to solve the nonlinear differential equation. The boundary condition can be considered from equations (8)–(9) as,

$$\begin{aligned} &F[0] = f(0) = S, && F[1] = f'(0) = 1, && F[2] = f''(0) = a,\\ &\theta[0] = \theta(0) = 1, && \theta[1] = \theta'(0) = b, &&\\ &\phi[0] = \phi(0) = 1, && \phi[1] = \phi'(0) = c. && \end{aligned}$$

Proceeding $F[3]$ for series of momentum profile, we would either predict the value of $F[2]$ or find it. For that purpose, the Pade approximation can be utilised to calculate the unknowns. We have to calculate $\theta[1]$ and $\phi[1]$ for thermal and concentration profiles.

DTM. The following are the basic definitions and operations of differential transformation.

Differential transformation of the function $f(\eta)$ is defined as follows

$$F(j) = \frac{1}{j!}\left[\frac{d^j f(\eta)}{d\eta^j}\right]_{\eta=\eta_0},$$

where $F(j)$, $\theta(j)$ and $\varphi(j)$ are the transformed functions of $f(j)$, $\theta(j)$ and $\varphi(j)$, respectively; they are given by

$$f(\eta) = \sum_{j=0}^{\infty} F(j)\eta^j, \quad \theta(\eta) = \sum_{j=0}^{\infty} \theta(j)\eta^j, \quad \varphi(\eta) = \sum_{j=0}^{\infty} \varphi(j)\eta^j.$$

Pade approximation. If a power series is used to represent a function $f(\eta)$, then

$$f(\eta) = \sum_{i=0}^{\infty} C_i\eta^i,$$

where C_i, $i = 0, 1, 2, \ldots$ is reserved for the given set of coefficients. The Pade approximant is a rational function and it is given by

$$P[L, M] = \frac{a_0 + a_1\eta + a_2\eta^2 + \ldots + a_L\eta^L}{b_0 + b_1\eta + b_2\eta^2 + \ldots + b_M\eta^M}.$$

Momentum

$$\begin{aligned} \frac{(j+1)(j+2)(j+3)}{j!}F[j+3] + \sum_{t=0}^{j}(j+1)(j+2)F[j-t+2]F[t]\\ - \left(\frac{2n}{n+1}\right)\sum_{t=0}^{j}(j-t+1)F[j-t+1](t+1)F[t+1] - M^2(j+1)F[j+1] = 0, \end{aligned}$$

$$\begin{aligned} f(\eta) = S + \eta + a\eta^2 + \frac{1}{6}\left(\frac{2n}{n+1} + M^2 - 2Sa\right)\eta^3\\ + \frac{1}{24}\left(\frac{8an}{n+1} + 2M^2a - 2a - S\left(\frac{2n}{n+1} + M^2 - 2Sa\right)\right)\eta^4 + \ldots. \end{aligned}$$

Temperature

$$\frac{1}{\Pr}\frac{(j+1)(j+2)}{j!}\theta[j+2]+\sum_{t=0}^{j}F[t](j-t+1)\theta(j-t+1)$$
$$+\mathrm{Nb}\sum_{t=0}^{j}(j-t+1)\phi(j-t+1)(t+1)\theta[t+1]+\mathrm{Nt}\sum_{t=0}^{j}(j-t+1)\theta(j-t+1)(t+1)\theta[t+1]$$
$$+\mathrm{Ec}\sum_{t=0}^{j}(t+1)(t+2)F[t+2](j-t+1)(j-t+2)F(j-t+2)$$
$$+M^2\sum_{t=0}^{j}(t+1)F[t+1](j-t+1)F(j-t+1)=0,$$

$$\theta(\eta)=1+b\eta+\frac{1}{2}(\Pr-b\,S-b\,c\,\mathrm{Nb}-b^2\,\mathrm{Nt}-a^2\,\mathrm{Ec}-M^2)\eta^2$$
$$+\frac{\Pr}{6}\Big(-\Pr S\,B-b-3\mathrm{Nt}\,b\Pr B-\frac{1}{2}b\,\mathrm{Nb}\,S\,\mathrm{Le}\,c-\mathrm{Nb}\,c\,B\Pr$$
$$+4a\,\mathrm{Ec}+\left(\frac{2n}{n+1}+M^2-2S\,a\right)+2a\,M^2\Big)\eta^3+\dots.$$

Concentration

$$\frac{(j+1)(j+2)}{j!}\phi[j+2]+\frac{1}{2}\mathrm{Le}\sum_{t=0}^{j}F[j-t](t+1)\phi[t+1]+\frac{\mathrm{Nt}}{\mathrm{Nb}}(j+1)(j+2)\theta[j+2]=0,$$

$$\phi(\eta)=1+c\eta+\frac{1}{2}\left(-\frac{1}{2}c\,S\,\mathrm{Le}-\frac{Nt}{Nb}\,B\Pr\right)\eta^2-\frac{1}{6}\Big(\frac{1}{2}\mathrm{Le}\left(2S\left(-\frac{1}{2}c\,S\,\mathrm{Le}-\frac{Nt}{Nb}B\Pr\right)+c\right)$$
$$+\Pr\frac{Nb}{Nt}\Big(-\Pr S\,B-b-3\mathrm{Nt}\,b\Pr B-\frac{1}{2}b\,\mathrm{Nb}\,S\,\mathrm{Le}\,c-\mathrm{Nb}\,c\,B\Pr$$
$$+4a\,\mathrm{Ec}\left(\frac{2n}{n+1}+M^2-2S\,a\right)+2a\,M^2\Big)\Big)\eta^3.$$

According to the theoretical works data, $\mathrm{Ec}=0.5$, $\mathrm{Nb}=\mathrm{Nt}=0.1$, $\mathrm{Le}=10$, $M=0.5$ so on.

Pade approximation of temperature and concentration

$$\mathrm{Pade}[5,5](\theta(\eta))=\frac{1-0.025461\eta+1.04764\eta^2-0.45441\eta^3-0.004535\eta^4+0.03423\eta^5}{0.9999-0.01622\eta+0.01376\eta^3-0.00147\eta^4+0.00679\eta^5},$$
$$\mathrm{Pade}[5,5](\varphi(\eta))=\frac{1-0.01194\eta+0.27415\eta^2-0.004207\eta^3-0.25106\eta^4+0.03393\eta^5}{0.9999-0.009763\eta+0.004563\eta^3-0.010054\eta^4+0.004023\eta^5}.$$

4. Results and discussion

We have used the analytical approach to solve the system of nonlinear ordinary differential equations (5)–(7) along with the boundary conditions (8) and (9). The obtained results demonstrated the effects of the governing parameters that are nondimensional, namely, the suction parameter S, Lewis number Le, thermophoresis parameter Nt, Brownian motion Nb, Eckart number Ec, magnetic parameter M on the velocity, thermal and nanoparticle concentration. Figure 2 examines the suction parameter S on the velocity profile $f(\eta)$. It is making a thinner momentum boundary layer. The increasing trend of velocity profile for different values of magnetic parameter M is studied in Figure 3 and shows that it accelerates the momentum flow. Therefore, the boundary layer thickness increases. Figure 4 illustrates that the velocity gradient increases when the stretching parameter increases due to the impact of volume fraction and thermal exponent. From Figure 5, we can see the diminution of thermal gradient, while increasing the Prandtl number. It means that Prandtl number controls the relative thickening of the momentum and thermal boundary layer. Figure 6 examining the Nb and Nt demonstrates that the temperature decreases when Nb and Nt increase. From Figure 7, it is clear that the temperature profiles become lower, which implies loss in the thickness of the thermal boundary layer. From Figures 8 and 9, it is seen that decreasing Ec number and increasing magnetized number cause the $\theta(\eta)$ profile decreases. In Figure 10 we observe that if the suction parameter is increasing then $\phi(\eta)$ profile also is increasing. In Figure 11, the same situation with the Pr number: with increasing Pr number the $\phi(\eta)$ profile is increasing. Figure 12: Nb and Nt parameters do not affect the concentration distribution. Figures 13 and 14 show that the effects of Lewis and Eckart numbers: their increase will enhance the concentration profile of nanofluid. The volume proportion of nanoparticles and the thickness of the concentration layer raised exponentially as the Le number increases.

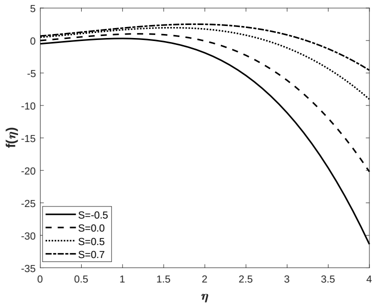

Fig. 2. The effect of S on velocity profile.

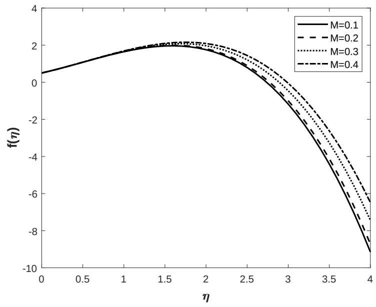

Fig. 3. The effect of M on velocity profile.

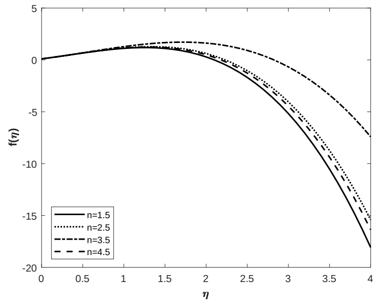

Fig. 4. The effect of n on velocity profile.

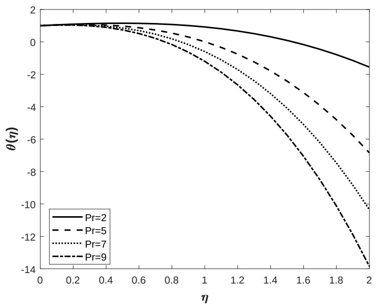

Fig. 5. The effect of Pr on thermal profile.

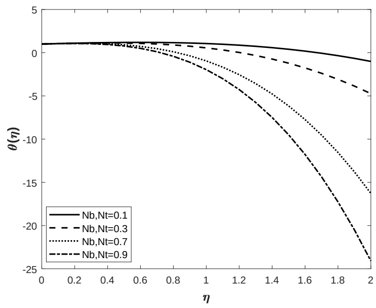

Fig. 6. The effect of Nb, Nt on thermal profile.

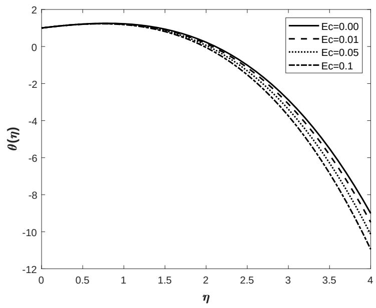

Fig. 7. The effect of Ec on thermal profile.

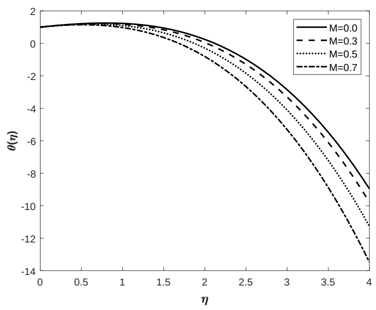

Fig. 8. The effect of M on thermal profile.

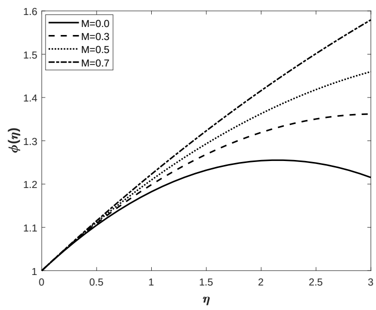

Fig. 9. The effect of M on concentration profile.

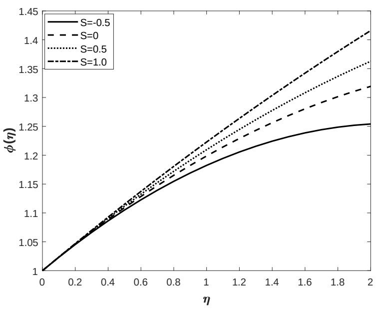

Fig. 10. The effect of S on concentration profile.

Both Figures 15 and 16 indicate that there is a drop in the rate of mass transfer while there is an increase in Prandtl number, Brownian motion, and thermophoresis. So that the reduced Sherwood number is a decreasing function of dimensionless parameters Pr, Nb and Nt. The effects of Pr and Nb parameters on heat transfer rates are shown in Figures 17–19, respectively, for a variety of Le values. It is obvious that the heat transfer rates will increase when the non-dimensional parameters are enlarged. So that the reduced Nusselt number is an increasing function of each dimensionless number.

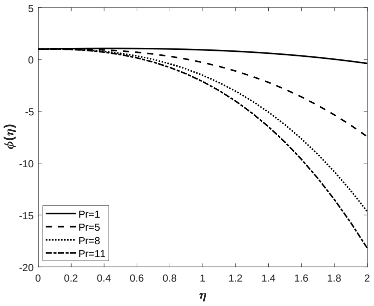

Fig. 11. The effect of Pr on concentration profile.

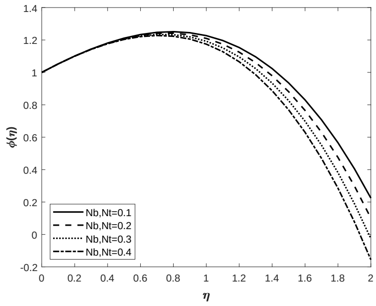

Fig. 12. The effect of Nb, Nt on concentration profile.

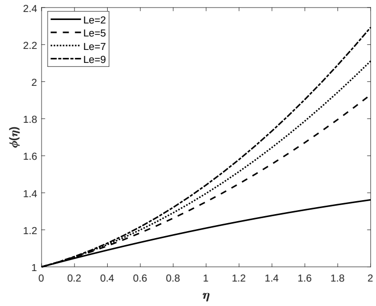

Fig. 13. The effect of Le on concentration profile.

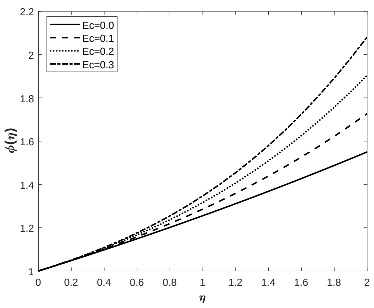

Fig. 14. The effect of Ec on concentration profile.

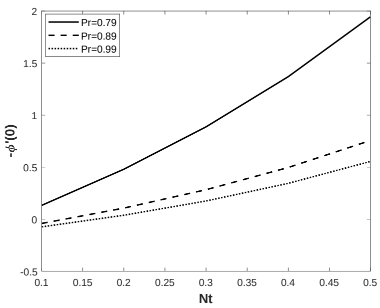

Fig. 15. The effect of Pr on mass transfer.

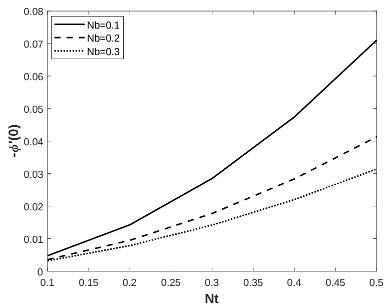

Fig. 16. The effect of Nb on mass transfer.

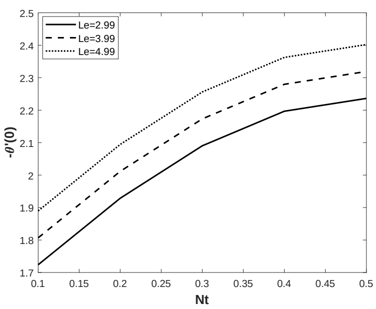

Fig. 17. The effect of Le on heat transfer.

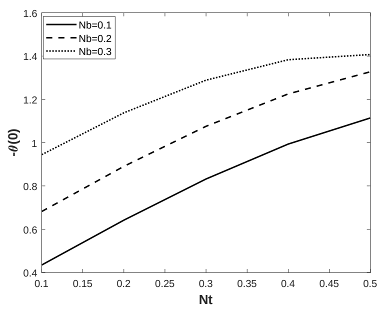

Fig. 18. The effect of Nb on heat transfer.

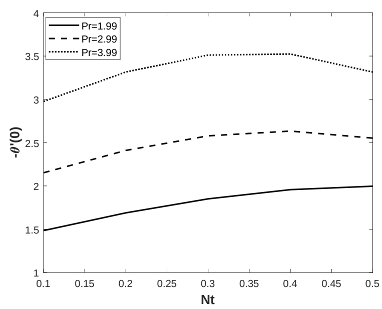

Fig. 19. The effect of Pr on heat transfer.

5. Conclusion

The primary goal of this work is to investigate a two-phase modeling solution for the movement of a boundary layer's heat and mass through a stretched sheet. The current issue is resolved using the analytical method DTM with the Pade approximation method. The Pade approximation is used to solve the issue in the $[5, 5]$ order of precision since the governing equations are nonlinear and there are some boundary conditions. Additionally, it looked for any connections between Pr, Le, suction/injection, and other parameters. In contrast to the reduced Nusselt number, which is a rising function of each dimensionless number, the reduced Sherwood number is a decreasing function of each dimensionless number, as we have learned.

[1] Choi S. U. S., Eastman J. A. Enhancing thermal conductivity of fluids with nanoparticles. ASME international mechanical engineering congress and exposition. San Francisco, 12–17 Nov 1995 (1995).

[2] Mustafa M., Khan J. A., Hayat T., Alsedi A. Boundary layer flow of nanofluid over a nonlinearly stretching sheet with convective boundary condition. IEEE Transactions on nanotechnology. **14** (1), 159–168 (2015).

[3] Khan W. A., Pop I. Boundary layer flow of a nanofluid past a stretching sheet. International Journal of Heat and Mass Transfer. **53** (11–12), 2477–2483 (2010).

[4] Hassani M., Mohammad Tabar M., Nemati H., Domairry G., Noori F. An analytical solution for boundary layer flow of a nanofluid past stretching sheet. International Journal of Thermal Sciences. **50** (11), 2256–2263 (2011).

[5] Jabeen K., Mushtaq M., Akram R. M. Analysis of the MHd boundary layer flow over a non linear stretching seet in a porous medium using semianalytical approaches. Mathematical Problem in Engineering. **2020**, 3012854 (2020).

[6] Vennila B., Nithya N., Kabilan M. Outcome of a magnetic field on heat transfer of carbon nanotubes (CNTs)-suspended nanofluids by shooting type Laplace–Adomian Decompostion method (LADM). In: Kondraivendhan B., Modhera C. D., Matsagar V. (eds) Sustainable Building Materials and Construction. Lecture Notes in Civil Engineering. Vol. 222, 153–160 (2022).

[7] Mabood F., Khan W. A., Ismail A. I. M. MHD boundary layer flow and heat transfer of nanofluid over a non linear stretching sheet: A numerical study. Journal of Magnetism and Magnetic Materials. **374**, 569–576 (2015).

[8] Mustafa M., Khan J. A. Model for flow of cassonnanofluid past a non linearly stretching sheet considering magnetic field effects. AIP Advances. **5** (7), 077148 (2015).

[9] Ellahi R., Alamri S. Z., Basit A., Majeed A. Effects of MHD and slip on heat transfer boundary layer flow over a moving plate based on specific entropy generation. Journal of Taibah University of Science. **12** (4), 476–482 (2018).

[10] Rashid I., Sagheer M., Hussain S. Entropy formation analysis of MHD boundary layer flow of nanofluid over a porous shrinking wall. Physica A: Statistical Mechanics and its Applications. **536**, 122608 (2019).

[11] Japili N., Rosali H., Bachok N. MHD stagnation point flow over a stretching sheet in a porous medium with velocity slip. Mathematical Modeling and Computing. **9** (4), 825–832 (2022).

[12] Alias N., Hafidzuddin M. E. H. Effect of suction and MHD induced Navier slip flow due ot non linear stretching/shrinking sheet. Mathematical Modeling and Computing. **9** (1), 83–91 (2022).

[13] Malik M. Y., Hussain A., Salahuddin T., Awais M. Effect of viscous dissipation on MHD boundary layer flow of sisko fluid over a stretching cylinder. AIP Advances. **6**, 035009 (2016).

[14] Prakash D., Narsu Sivakumar, Suriyakumar P., Rushikumar B. Influence of viscous and ohmic heating on MHD flow of nanofluid over an inclined nonlinear stretching sheet embedded in a porous medium. International Journal of Mechanical Engineering and Technology. **9** (8), 992–1001 (2018).

[15] Ganga B., Ansari M. Y. S., Vishnu Ganesh N., Abdul Hakeem A. K. MHD radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects. Journal of the Nigerian Mathematical Society. **34** (2), 181–194 (2015).

[16] Sheikholeslami M., Abelman S., Ganji D. D. Numerical simulation of MHD nanofluid flow and heat transfer considering viscous dissipation. International Journal of Heat and Mass Transfer. **79**, 212–222 (2014).

[17] Nayak M. K. MHD 3D flow and heat transfer analysis of nanofluid by shrinking surface inspired by thermal radiation and viscous dissipation. International Journal of Mechanical Sciences. **124–125**, 185–193 (2017).

[18] Dogonchi A. S., Chamkha A. J., Seyyedi S. M., Hashemi–Tilehnoee M., Ganji D. D. Viscous dissipation impact on free convection flow of Cu–water nanofluid in a circular enclosure with porosity considering internal heat sources. Journal of Applied and Computational Mechanics. **5** (4), 717–726 (2019).

[19] Rashidi M. M., Rabiel F., Naseri Nila S., Abbasbandy S. A review: Differential transform method for semi-analytical solution of differential equation. International journal of Applied Mechanics and Engineering. **25** (2), 122–129 (2020).

[20] Usman M., Hamid M., Khan U., Mohyud Din S. T., Iqbal M. A., Wang W. Differential transform method for unsteady nanofluid flow and heat transfer. Alexandria Engineering Journal. **57** (3), 1867–1875 (2018).

[21] Saha D., Sengupta S. Dual DTM–Pade approximations on for convection MHD mass transfer flow of nanofluid through a stretching sheet in presence of Soret and Dufour phenomena. WSEAS Transactions on Fluid Mechanics. **15**, 23–40 (2020).

[22] Sayyed S. R., Singh B. B., Makinde O. D., Bano N. DTM–Pade approach to MHD slip flow and heat transfer over a radially stretching sheet with thermal radiation. Latin American Applied Research. **50** (3), 175–184 (2020).

[23] Khashi'ie N. S., Wahi N., Arifin N. M., Ghani A. A., Hamzah K. B. Effect of suction on the MHD flow in double-stratified micropolar fluid over a shrinking sheet. Mathematical Modeling and Computing. **9** (1), 92–100 (2022).

[24] Yousif M. A., Hatamai M., Mahmod B. A., Rashidi M. M. Thermal boundary layer ananlysis of nanofluid flow past over a stretching flat plate in differential transpiration conditions by using DTM–Pade method. Journal of Mathematics and Computer Science. **17** (1), 84–95 (2017).

МГД-потік пограничного шару нанорідини на листі, який розтягується, з в'язкою омічною дисипацією

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Метою цього дослідження є вивчення усталеного нестисливого двовимірного гідромагнітного пограничного шару потоку нанофлюїду, що проходить через розтягнутий лист під впливом в'язкої та омічної дисипації. Ця задача вирішується за допомогою аналітичного методу під назвою DTM апроксимацією Паде. Математичне моделювання потоку розглядається у формі диференціального рівняння в частинних похідних і перетворюється на диференціальне рівняння за допомогою відповідного перетворення подібності. Впливи фіксованих параметрів, таких як число термофорезу Nt, число броунівського руху Nb, число Прандтля Pr, число Льюїса Le, магнітне поле M, всмоктування/впорскування S та число Екарта Ec показано на рисунках. Наші результати показали більшу тенденцію в профілі швидкості для параметрів магнітного поля M, всмоктування S та параметра нелінійного розтягування n. Тоді як на профілі температури виявляється обернена залежність зі збільшенням числа Прандтля, Число Льюїса та інші параметри збільшують профіль концентрації.

Ключові слова: *магнітний; Екарт; термофорез; броунівський; теплопередача; масообмін.*