ІНФОКОМУНІКАЦІЙНІ ТЕХНОЛОГІЇ ТА ЕЛЕКТРОННА ІНЖЕНЕРІЯ
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PROTECTION OF DOCUMENTS WITH THE HELP OF FRACTAL IMAGES FORMED BY A RANDOMIZED SYSTEM OF ITERATING FUNCTIONS

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This article examines the results of the development of the document protection algorithm with the help of fractal images formed by a randomized system of iterative functions of RSIF. This algorithm consists in building a fractal image which is applied to the document and built on the basis of the document number. Each digit of the document number will add two iterative functions to the RSIF, except for the digit 0. The fractal image, constructed using the algorithm, depends on the number of digits, the order of the location of the digit and its value. The algorithm for constructing a fractal image in calculations does not require large computing power, the algorithm does not use the entry of a cycle into a cycle and recursive functions. It is quite optimized. Document verification includes a double check of the document number, and it is a fairly simple verification mechanism. This algorithm is interesting that when even one digit is changed, the fractal image changes radically. The use of the algorithm quickly and qualitatively makes it possible to protect documents and carry out their verification.

Keywords: *document protection, fractal, randomized system of iterative functions (RSIF), document number verification, Cantor's set.*
UDC 621.126

1. Introduction

Protecting documents in today's digital world is quite complex and not an easy task. It requires more and more complex protection elements. Constructing fractal images is not a simple task as it needs to store large arrays of data in memory and to do complex mathematical calculations. Checking, whether a figure is a fractal, is not a problem for it is enough to compare the first and second iterations of the fractal image. And if the fractal image is submitted in the form of a digit cipher, it will make it possible to double-check the document number. When using RSIF, there is no need to store large arrays of data in memory, the initial set is only one point [1, p. 102–103]. The construction algorithm does not require significant calculations; at each step, the coordinates of one point are calculated to display it on the screen [2, p. 68–71]. Validating a document involves dividing the document into parts which is not a difficult task. It also involves comparing the parts to templates. This algorithm allows you to check the first and the second iterations of the fractal with the template. This will make it possible to verify the document number.

2. Basic material presentation

Algorithm for constructing a fractal image using RSIF based on a given document number (Fig. 1).

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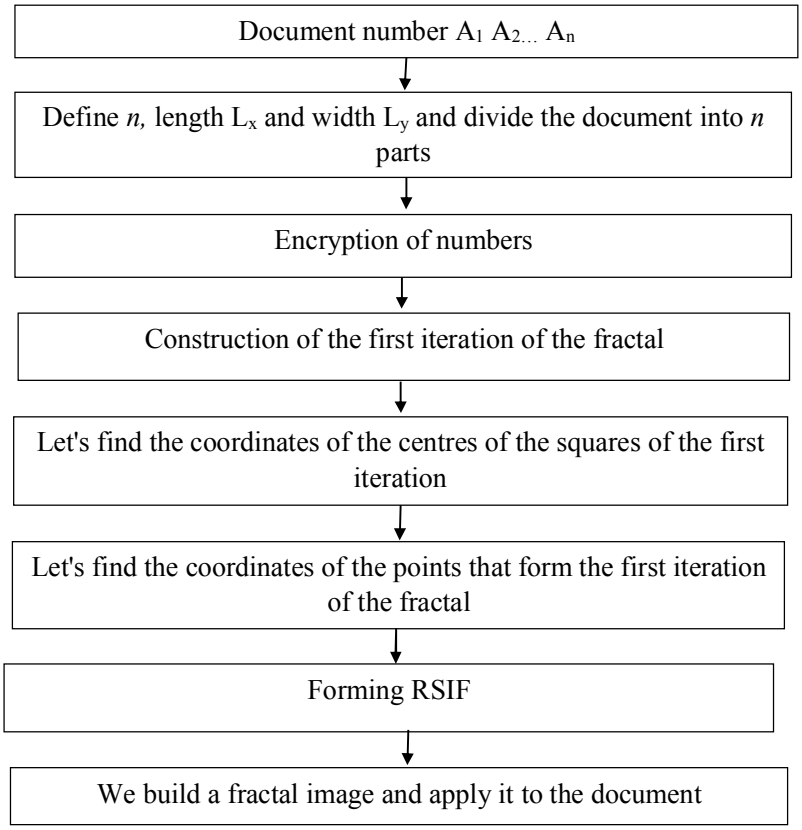


Fig. 1. Algorithm for constructing a fractal image using RSIF based on a given document number

To construct a fractal image, we will use a series of documents. Our example we will use 9-digit digital numbers (the number of digits can be different).

We will select a rectangular area on the document in which we will place as many squares as there are numbers in the document series so that it is completely filled (Fig. 2):

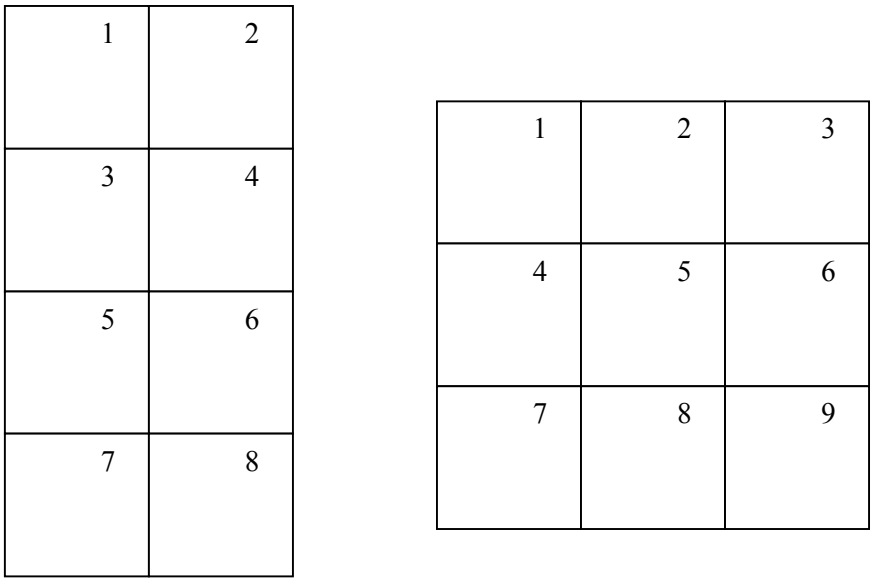


Fig. 2. Examples of a selected rectangular area divided into squares (eight-digit and nine-digit document numbers)

In the next action, we will divide each square into 9 identical squares (Fig. 3):

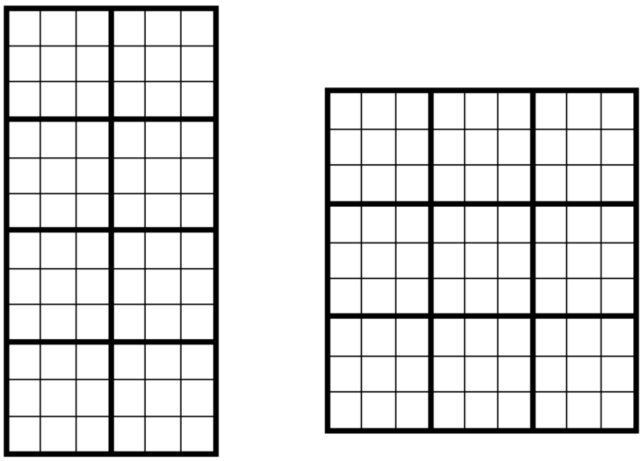


Fig. 3. Division of squares into 9parts

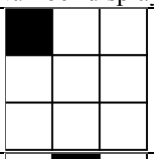
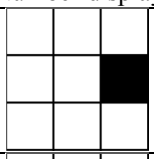
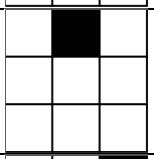
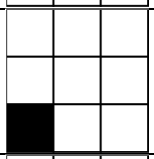
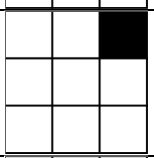
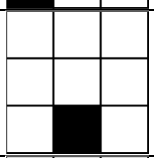
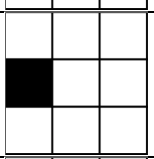
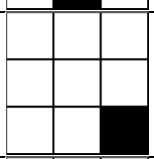
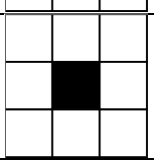
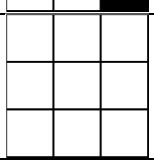
Each number from 1 to 9 will be presented in the form of a sketched square (Table 1) according to the scheme (Fig. 4).

1	2	3
4	5	6
7	8	9

Fig. 4. Scheme of correspondence of a number to a square

Table 1

Display of numbers

Number display	Number	Number display	Number
	1		6
	2		7
	3		8
	4		9
	5		0

The display of numbers can be changed as desired, for example, for different types of documents.

To construct a fractal, we will use a randomized system of iterative functions of RSIFfor each sketched square. This will be the first iteration of the fractal [3]:

$$x_n = x_a - \frac{x_a - x_{n-1}}{k}, \quad (1)$$

$$y_n = y_a - \frac{y_a - y_{n-1}}{k}, \quad (2)$$

where x_{n-1}, y_{n-1} – the initial coordinate of the point; x_n, y_n –the next coordinate of point **C**; x_a, y_a – the coordinate of the point that forms the first iteration of the fractal of the corresponding square; k –the proportionality factor (the ratio of the segment length to the segment length of the first iteration).

To find the coordinates of points $\mathbf{x_a}$ and $\mathbf{y_a}$, we use the following formula [3]:

$$x_a = \frac{kx_b - \frac{L_x}{2}}{k-1}, \quad (3)$$

$$y_a = \frac{ky_b - \frac{L_y}{2}}{k-1}, \quad (4)$$

where L_x, L_y – the length and width of the space of the selected rectangular area divided into squares; x_b, y_b – the geometric centre of the first iteration of the fractal of the corresponding square; k – the proportionality factor (the ratio of the segment length to the segment length of the first iteration).

The proportionality coefficient is determined by the following formula[3]:

$$K = \frac{L}{\Delta L1}, \quad (5)$$

where L – the total length of the span; $\Delta L1$ – the length of the segment of the first iteration.

Let's look at an example how to implement this method of document protection:

1. To do this, we will randomly select a nine-digit document number:

157702869.

2. In the next step, build the first iteration of the fractal according to table 1, and document series 157702869, (0 – not displayed by a square):

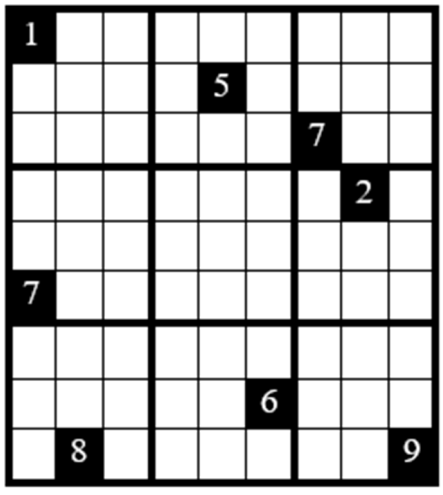


Fig. 5. The display of the first iteration of the fractal, respectivelyto series 157702869

3. Choose the length L_x and width L_y of the square area of 500 pixels each, the x and y coordinates will be determined according to the pixels of the image.

4. Find the coordinates of the centres of the squares of the first iteration (Table 2):

Table 2

Centres of squares of the first iteration according to series 157702869

Series number	1	5	7	7	0	2	8	6	9
Coordinates center	(28;28)	(83;250)	(139;361)	(306;28)	–	(194;417)	(472;83)	(417;306)	(472;472)

5. To find the coordinates of points $\mathbf{x_a}$ and $\mathbf{y_a}$, we will use formulas **(3), (4)**, **(5)** and summarize the results in Table 3:

Table 3

Centres of squares of the first iteration according to the series 157702869

Series number	1	5	7	7	0	2	8	6	9
The coordinates of the points that form the first iterations of the fractal	(0;0)	(55;222)	(111;333)	(278;0)	–	(166;389)	(444;55)	(389;278)	(444;444)

6. Form a table of iterative functions using **(1), (2)**,**(5)**,thensummarize the results in Table 4:

Table 4

RSIF for constructing a fractal according to the series 157702869

Series number	RSIF
1	x = 0 – (0 – x)/9 y = 0 – (0 – y)/9
5	x = 55 – (55 – x)/9 y = 222 – (222 – y)/9
7	x = 111 – (111 – x)/9 y = 333 – (333 – y)/9
7	x = 278 – (278 – x)/9 y = 0 – (0 – y)/9
0	-
2	x = 166 – (166 – x)/9 y = 389 – (389 – y)/9
8	x = 444 – (444 – x)/9 y = 55 – (55 – y)/9
6	x = 389 – (389 – x)/9 y = 278 – (278 – y)/9
9	x = 444 – (444 – x)/9 y = 444 – (444 – y)/9

7. Perform RSIF and get the result (Fig. 6):

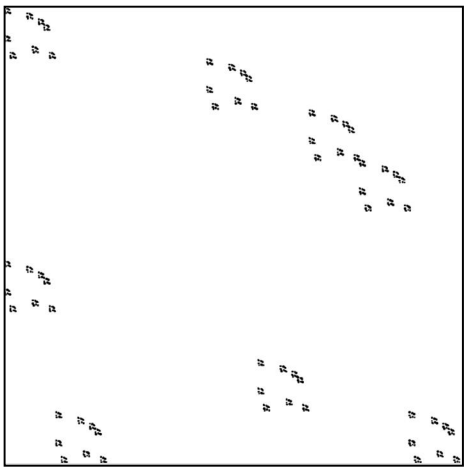


Fig. 6. Constructed fractal according to series 157702869

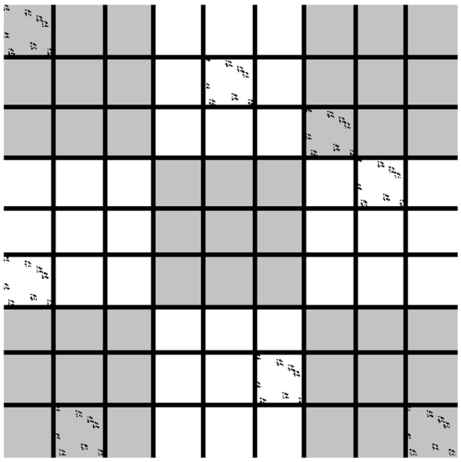
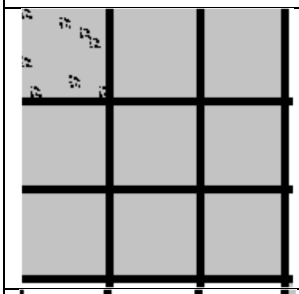
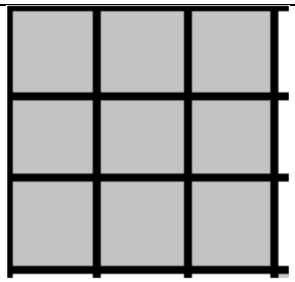
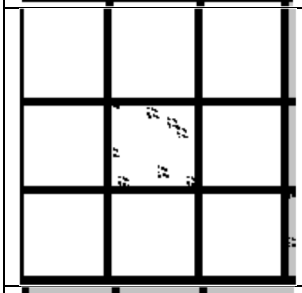
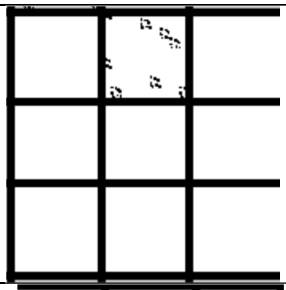
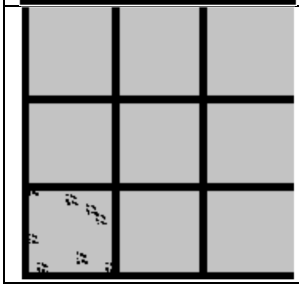
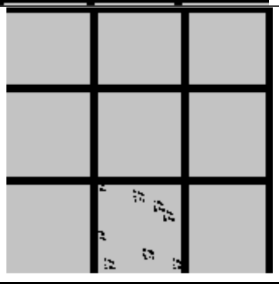


Fig. 7. Dividing the fractal image into parts

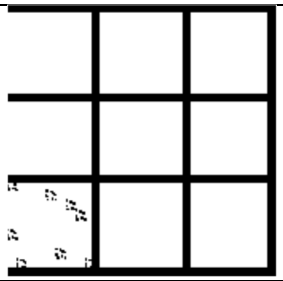
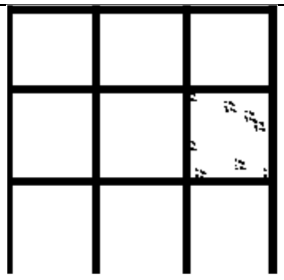
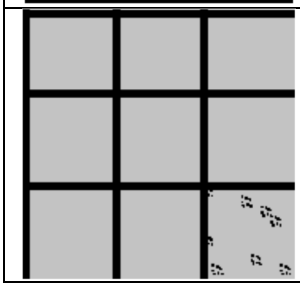
To decipher the image and determine its series, it is necessary to divide the image into 81 parts according to Fig. 4. As a result, we can redistribute the document series using Table 1 (Fig. 7), the result will be summarized in Table 5:

Table 5

Summary results

Image	The corresponding unit of the series	Image	The corresponding unit of the series
1	2	3	4
	1		0
	5		2
	7		8

Continueted Table 5

1	2	3	4
	7		6
	9		

As we see on Fig. 7, the decoded value corresponds to the initial value**157702869**.

A similarly performed algorithm for the second iteration should give the same result(Fig. 8):

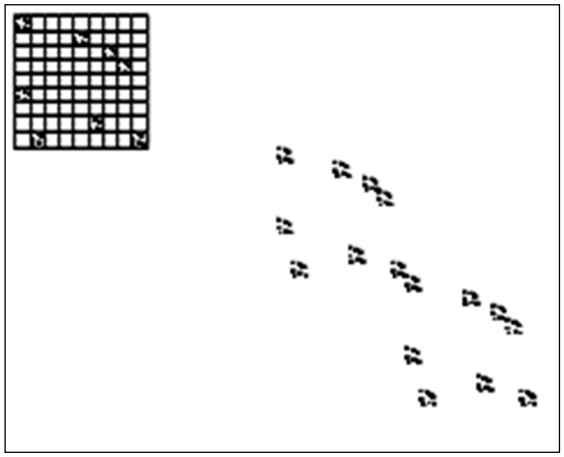


Fig. 8. Division of the segment of the second iteration of the fractal image into parts

As we see onFig. 8, the decoded value corresponds to the initial value **157702869**.Thus double verification of the document series takes place.

Conclusions

Graphic constructions in the form of fractal images will increase the effectiveness of document protection. They will allow you to combine a series of documents with an image on it. The given algorithm is flexible. This allows it to change according to the size and the types of the document. Double verification and a simple method of analysis make it difficult to forge the document. This approach is practical for the production of a large number of blanks since the speed of image formation using the RSIF system is quite significant as concerns deterministic systems of iterative functions for constructing the ractal images. When the resolution is increased, it is possible to perform triple verification of the document on the third iteration.

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ЗАХИСТ ДОКУМЕНТІВ ЗА ДОПОМОГОЮ ФРАКТАЛЬНИХ ЗОБРАЖЕНЬ, СФОРМОВАНИХ РАНДОМІЗОВАНОЮ СИСТЕМОЮ ІТЕРАЦІЙНИХ ФУНКЦІЙ

О. Юнак

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В цій статті розглянуто результати розробки алгоритму захисту документів за допомогою фрактальних зображень, сформованих рандомізованою системою ітераційних функцій РСІФ. Цей алгоритм полягає у побудові фрактального зображення, яке наноситься на документ і побудоване на основі номера документа. Кожна цифра номера документа буде добавляти до РСІФ дві ітераційні функції, крім цифри 0. Фрактальне зображення, побудоване за допомогою алгоритму,залежить від від кількості цифр, порядка-знаходження цифри та її значення. Алгортм побудови фрактального зображення в розрахунках не потребує великих обчислювальних потужностей, алгоритм не використовує входження циклу в цикл та рекурсивних функцій, і є досить оптимізованим. Верифікація документа включає в себе подвійну перевірку номера документа, та являє собою досить простий механізм перевірки. Цей алгоримт цікавий тим, що зазмінихоча б однієї цифри фрактальне зображення кардинально змінюється. Використання алгоритму швидко і якісно дає змогу захищати документи та проводити їхню верифікацію.

Ключові слова: *захист документів,фрактал, рандомізована система ітераційних функцій (РСІФ), верифікація номера документа, набір Кантора.*