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MONTE CARLO METHODS: A FEATURES REVIEW IN TERMS OF USE FOR ASSESSING THE RELIABILITY OF RC STRUCTURES

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Recently, the optimization issue relevance of reinforced concrete (RC) structures design solutions through the maximum use of their bearing capacity resource has increased significantly; in turn, solving this issue depends on a fundamental understanding of the reliability and durability concepts. Because any loads, impacts, or bearing capacity reserve parameters are random variables, there is a need to build stochastic models, which can become the "reliability design" concept base shortly.

Among other things, this review article is devoted to the Monte Carlo methods features analysis in terms of their use in the RC members' reliability assessment tasks. Based on a modern literary sources review, recommendations for further studies of the RC structures' reliability and durability (including damaged ones) under the conditions of the combined action of loads and a corrosive environment (using Monte Carlo methods) were also formulated.

Keywords: Monte Carlo methods, distribution function, stochastic model, probabilistic analysis, reliability assessment, reinforced concrete (RC) member.

Introduction

According to the Structural Reliability Handbook (2015), there are different levels of performance specifications (see Fig. 1): from prescriptive (that includes a detailed description of process completion) to pure performance (that permits greater freedom degree in achieving the same goals). In turn, verification methods of reliability of any building structures, in particular, reinforced concrete (RC) ones, are predominantly performance-based solutions (Structural Reliability Handbook, 2015); nevertheless, they are prescriptive in determining actions to provide comparable indicators and ensure a safety level, for example, according to the requirements of current Ukrainian DBN V.2.6-98:2009 and American (ACI 318-19) regulations. Moreover, taking into account the reliability assessment problem complexity (Tytarenko et al., 2023), even the profile Ukrainian and international normative documents such as DBN V.1.2-14:2018, EN 1990:2002, ISO 13823:2008, and ISO 2394:2015 contain only recommendations, without a clear methodological base for various operational cases.

Based on Fig. 1 (see below), it becomes explicit that the reliability probabilistic analysis (including non-failure, durability, and residual life) is a highly relevant issue nowadays due to the global need to achieve maximum efficiency and economy in both new construction and reconstruction. In turn, the probabilistic calculation of RC members consists of finding the probabilistic characteristics of displacements, forces, or stresses from the probabilistic characteristics of loads and impacts, the strength (deformability) of materials, and the geometric parameters (Ditlevsen et al., 2005; Raizer, 1998; Tytarenko et al., 2023). In sum, the probabilistic approach to the calculation (probabilistic analysis) is that all characteristics of structures, which determine the so-called "load-bearing capacity reserve", as well as all actions on them, are random variables (processes).

Therefore, the main goal of our theoretical research is a complete analysis of the statistical modeling methods features (so-called "Monte Carlo methods") according to well-known theories in terms of their use in the RC members' reliability assessment tasks. Based on a modern literary sources review, an additional goal of the work is the development of some recommendations for future studies of the reliability and durability of RC structures (including damaged ones) under the conditions of the combined action of mechanical loads and a corrosive-aggressive environment.

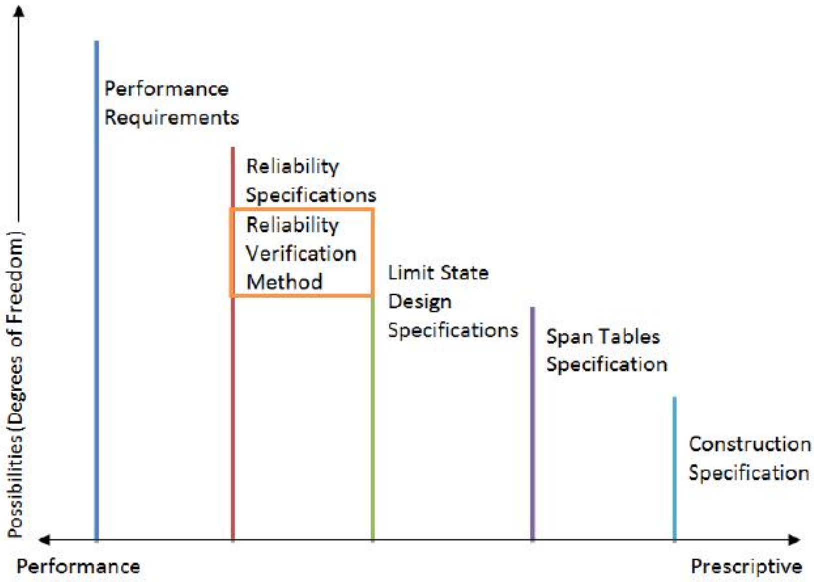

Fig. 1. Levels of performance specifications

Materials and Methods

The simplest is a probabilistic calculation of structures whose operation can be described by a linear function; however, in practice, there are frequent cases when the stochastic variables function is not strictly linear but differs little from it – and while solving the specific problem, it can be approximately considered linear (Raizer, 1998). This assumption is possible when input parameter values random changes – X_1, X_2, …, X_n – are insignificant (in the range of 20 %); in turn, the statistical variability of values of RC members' operational parameters often meets these requirements (Khmil et al., 2021). To calculate the statistical characteristics of such functions, they are linearized by expanding in a Taylor series around the random arguments distribution center – at the point of function mathematical expectation (Raizer, 1998).

Nonetheless, no stochastic variables function is strictly linear – and when it comes to minimizing the values of the structures' future failure probabilities – the assumption given in the above paragraph may not work. Furthermore, some parameters that must be considered during the construction of reliable stochastic models of the operation of RC members have random changes in values, which highly exceed the limits of 20 % (for example, the variability of some types of climatic loads and aggressive environmental impacts can be up to two times). At the same time, due to increasing values of input parameter variation coefficients, the probability of structure failure also increases, and, as a result, their overall reliability decreases. This problem is especially significant for RC members of the CC3 consequence class buildings by DBN V.1.2-14:2018 and EN 1990:2002, for which reliability analysis is required in many cases.

In addition to the above, since the error permissible value in the nonlinear systems failures calculation depends not only on the number of input variables but also on the volumes of their statistical samples – the search for an analytical solution using the statistical linearization method will be problematic. Thus, there is a need to use universal methods for calculating probabilistic problems – Monte Carlo methods (Ditlevsen et al., 2005; Raizer, 1998).

The mechanism of solving integration tasks by Monte Carlo methods is based on the fact that the probability theory (among its few interpretations) is interpreted as a mathematical theory of the sample means behavior with the tendency for their stabilization as the sample volume increases (Ditlevsen et al., 2005). In turn, the reliability of RC structures, in the general case, can be quantified – by the failure probability or the probability of non-failure operation (P_f), which are also connected with the reliability index (β) through the well-known error function (Khmil et al., 2021).

The reliability index (β) takes actions and resistances (Structural Reliability Handbook, 2015). Then, it represents these as random variables in stochastic models (see Fig. 2), in which: Q_m, V_Q, Q_n – the average action, the variation coefficient relative to the action, and the nominal design action in accordance; R_m, V_R, R_n – the average resistance, the variation coefficient relative to the resistance, and the nominal design resistance in accordance. The distance between the two curves in Fig. 2 (see below) is the under-question parameter performance (Structural Reliability Handbook, 2015).

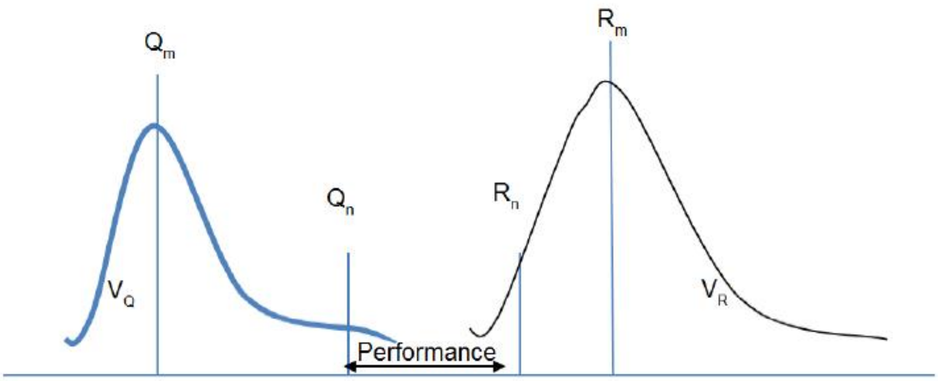

Fig. 2. Action and resistance models

The distribution curves of the models described above are assumed to follow a log-normal (or often normal) distribution (Structural Reliability Handbook, 2015). However, the assumption about the random variables log-normality (or normality) is relevant only if they are statistically independent. Moreover, some weighty parameters of RC members' operation often do not obey the normal distribution law; for example, according to Schiessl (2005) and Van Coile et al. (2014), the concrete cover (c, mm) is beta-distributed.

Thus, in our opinion, the disadvantages of almost all methods for calculating the failure probability (the two-moments method, the hot spots method, etc.), based on the normal distribution law only (Khmil et al., 2021; Raizer, 1998; Structural Reliability Handbook, 2015), are the following:

- significant calculation errors due to an approximate description of stochastic variables distribution;
- the functions that define failure (or non-failure operation) areas must be differentiable everywhere.

Unlike others, the Monte Carlo methods are more efficient since they give a smaller spread in assessing the failure (or non-failure operation) probability (Ditlevsen et al., 2005; Raizer, 1998). The central idea of these methods consists of the sample (based on the statistical distribution) construction for each stochastic variable involved in the task, and as these methods deal with the simulation of the limit state function, the larger the sample is taken, the more accurate the probability of failure (or non-failure operation) (Ditlevsen et al., 2005; Nogueira et al., 2012).

The Monte Carlo methods term describes a widely used class of approaches (Ditlevsen et al., 2005), but these approaches mostly use a single template:

- the area of possible input data is determined;
- the input data from the area defined above using some given probability distribution are randomly generated (the so-called "pseudo-random number generator" is used);
- deterministic calculations on the input data are performed;
- intermediate results of individual calculations are reduced to the final result.

The general expression for finding the failure probability (P_f), according to Ditlevsen et al. (2005) and Raizer (1998), can be represented as Eq. (1):

$$P_f \approx m^{-1} \sum F_R(Q_i), \tag{1}$$

in which: m is the number of tests; $F_R(Q_i)$ is the distribution function value of the R parameter (load-bearing capacity) at the Q argument (loading effect); Q_i is the simulated realization of the Q parameter on the i-test.

The Monte Carlo methods' main disadvantages are the following (Ditlevsen et al., 2005; Nogueira et al., 2012; Raizer, 1998):

- one of the distribution functions of the R and Q parameter values must be predetermined in the multidimensional case;
- many simulations m are required to calculate the failure probability accurately.

We consider the last point in more detail. Usually, to estimate the failure probability of 10^{-n} accurately, the number of simulations must be higher than 10^{n+2} or 10^{n+3} – it means that for civil engineering structures (including RC), where the failure probability is within $10^{-3}...10^{-6}$ (refers to P_f established values by DBN V.1.2-14:2018), from 10^5 to 10^9 realizations of the limit state function are required. In the case when complex numerical models are involved (which leads to high computational work), these methods may not be reliable – theoretically, it leads to an actual failure probability by a sampling range, that approaches infinity (Nogueira et al., 2012).

Several method modifications are also known, increasing its efficiency by reducing the assessment variance (Raizer, 1998). One way to reduce the variance is to stratify the simulated sample (only on the required class intervals and with the given volumes of class samples) – for example, the Q_i value sample, when using Eq. (1).

Results and discussion

The issues of probabilistic analysis of non-failure, durability, or residual life of structures (as the main parameters of their reliability both at the design stage and during operation) have always been the least studied and, therefore, the most relevant. Thus, in recent years, they have become sufficiently widely covered in the studies of Ukrainian and foreign scientists. However, in not many works, Monte Carlo methods are the main ones in terms of their use in reliability probabilistic analysis of RC members. For example, according to Khmil et al. (2021), the statistical linearization method is used for probability evaluation of the failure-free operation of RC beams strengthened under load; Schiessl's (2005) research summarized the design algorithm and identified the information (that is required to realize a calculation of probability-based service life), but only based on the generalized normal distribution of all input variables; in turn, the study of Van Coile et al. (2014) is devoted to a review of improved technique examples for modeling the flexural resistance of a concrete slab during fire via a mixed log-normal distribution (including one of the Monte Carlo methods used).

In many works (Nogueira et al., 2012; Conciatori et al., 2009; Guo et al., 2023; Pellizzer et al., 2015; Şengül, 2011; Yuan et al., 2023), among others, the following issues were studied using the Monte Carlo simulation: 1) corrosion occurrence and influence on RC members; 2) reliability algorithms during probabilistic durability assessment of RC structures in aggressive (chloride-containing) environments; 3) time-dependent seismic reliability; etc. In addition to the reliability analysis in the above works, Conciatori et al. (2009) also compared the Monte Carlo and Rosenblueth methods in simulating the penetration of chloride ions into RC and predicting the development of corrosion.

The RC (including prestressed) member strengths variabilities have been based on the results interpretation of the Monte Carlo simulations in the research of MacGregor et al. (1983); the structural reliability (durability) concepts of RC structures (including taking into account a seismic safety, an environmental action, etc.), in turn, were proposed in the works of Hosseini et al. (2023), Jitao et al. (2019), Vořechovská et al. (2017), and Wang et al. (2016). The eco-friendly design optimization of intermediate RC moment frames, the concrete failure analysis, and optimization of the RC beam life cycle support strategy, based on the Monte Carlo simulation, were carried out in the studies of Akhavan Kazemi et al. (2023), Huang et al. (2016), and Wang et al. (2022). Finally, according to the results of Badal et al. (2023), Wang et al. (2023), and Zhang et al. (2023), several applied (seismic resilience) and new approaches (based on the third-moment method, etc.) for RC structures reliability assessment were presented.

But nowadays, despite some basic studies described above, issues of constant data monitoring (representativeness and reliability of samples), deployment of Internet of Things (IoT) systems, and taking into account all factors of the environment RC members' operation have not been sufficiently studied.

Conclusions

- Nowadays, the issues of assessing the reliability (including non-failure, durability, and residual life) of RC structures have become especially relevant due to the global need to achieve maximum efficiency and economy of both new construction (design stage) and reconstruction (operational stage). In turn, the probabilistic approach to these issues allows us to get the failure (or non-failure operation) indices with high accuracy and prognosis of the durability (residual life) of different RC members.
- Among the methods for assessing the failure probability, the statistical modeling methods (so-called "Monte Carlo methods") show themselves in the best way due to the possible work with any sample ranges and distribution laws of stochastic variables distribution.

Prospects for further research

- Based on the review of several new and improved approaches to the RC members' reliability and durability assessment, which are based on the use of Monte Carlo methods, it becomes clear that in the future, to create objective calculation methods and increase the reliability of results, besides the warranty of stochastic data samples' necessary volumes, the urgent task also will be to ensure continuous monitoring of these data.
- So, we recommend using the Monte Carlo methods for future studies in the field of RC structures' reliability and durability assessment (including damaged ones) under the conditions of the combined action of mechanical loads and a corrosive-aggressive environment.

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ОГЛЯД ОСОБЛИВОСТЕЙ ВИКОРИСТАННЯ МЕТОДІВ МОНТЕ-КАРЛО ПРИ ОЦІНЮВАННІ НАДІЙНОСТІ ЗАЛІЗОБЕТОННИХ КОНСТРУКЦІЙ

Останнім часом значно зросла актуальність проблеми оптимізації проектних рішень залізобетонних елементів за рахунок максимального використання ресурсу їх несучої здатності; своєю чергою, вирішення цієї проблеми залежить від фундаментального розуміння таких понять як «надійність» та "довговічність", а оскільки параметри будь-яких навантажень, впливів або резерву несучої здатності є випадковими величинами, побудова максимально достовірних стохастичних моделей роботи конструкцій незабаром має стати основою широкої концепції “надійнісного проектування».

Разом із тим найбільш простим (при незначній мінливості даних) є імовірнісний аналіз конструкцій, роботу яких можна наближено описати за допомогою лінійної функції; однак насправді жодна функція випадкових величин не є строго лінійною. Більше того, зі збільшенням значень коефіцієнтів варіації вхідних змінних, підвищується і ймовірність відмови елементів, що виключає можливість застосування більшості методів її оцінювання; таким чином, виникає необхідність використання універсальних методів розрахунку нелінійних систем – так званих “методів Монте-Карло”.

Серед іншого, метою даної оглядової статті було проведення аналізу особливостей методів Монте-Карло з точки зору їх використання в задачах оцінювання безвідмовності, довговічності та залишкового ресурсу (як ключових параметрів забезпечення надійності) залізобетонних елементів в умовах експлуатації; крім того, в роботі були викладені основні переваги та недоліки цих методів відповідно до загальновідомих теорій.

Насамкінець, на основі огляду сучасних літературних джерел були сформульовані рекомендації щодо подальших досліджень надійності та довговічності залізобетонних конструкцій (в т.ч. пошкоджених) в умовах сумісної дії на них механічних навантажень й корозійно-агресивного середовища із використанням методів Монте-Карло.

Ключові слова: методи Монте-Карло, функція розподілу, стохастична модель, імовірнісний аналіз, оцінка надійності, залізобетонний елемент.