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COMPARATIVE CHARACTERISTICS AND SELECTION OF SPEED BEARINGS

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Abstract. The characteristics of bearing supports have been considered. Improving the efficiency of mechanical processing, the quality of operation of mechanisms that use sliding bearing supports, and ensuring stable operation are always important tasks. Solving these tasks contributes to reducing labor costs, reducing operating costs, and increasing the productivity of individual operations. The following main criteria are proposed for the selection of supports: the magnitude of the loads that supports can withstand; the range of allowable shaft rotation frequencies; accuracy in maintaining the position of the axis of shaft rotation; stability of shaft rotation (possibility of auto-oscillations and undesirable transient processes); energy costs and economic indicators of manufacturing and operation; vibroacoustic characteristics (noise level, sound level).

The study of the movement of the working body that separates the friction pairs in the bearing is based on two fundamental laws of hydrodynamic lubrication theory: the law of mass conservation and the law of momentum conservation. Mathematical models of supports with fluid lubrication, based on the Navier-Stokes equations, were used. The requirements for bearing supports are formulated on the basis of the tasks solved by the entire mechanism. The flow parameters of the working body affect the load-bearing capacity of radial bearings, and the proposed evaluation dependencies can also be used for tapered supports. The calculation results indicate a significant influence of the flow parameters of the working body on the expansion of the areas of rarefaction and the range of their values, as well as on the reduction of the area and range of increased pressures. It has been established that with small shaft eccentricities rotating at speeds of 60–70 m/s and with a radial clearance of 80 μm, the increase in load capacity can reach 20 %. An important qualitative feature has been identified: with an increase in the Reynolds number Re*, the load capacity of the bearing increases. The greatest intensity of changes in load capacity due to the influence of flow parameters of the working body is observed at a relative eccentricity of e = 0.2–0.4. The terms in the Navier-Stokes equation that take into account the parameters of the working fluid flow can have values that are comparable to other terms, so ignoring them is not always permissible.

Keywords: bearing support, high-speed spindle, flow, working fluid, lubrication, surfaces, friction pairs.

Introduction

For machine-building factories, improving the efficiency of mechanical processing, the quality indicators of the mechanism using a given type of bearings, and stable operation are always relevant tasks. Solving these tasks contributes to reducing labor costs, reducing operating costs, and increasing the productivity of individual operations. Optimal metalworking equipment includes bearings that allow for increased accuracy and productivity, reduced finishing operations, and decreased cost of manufacturing parts. High-speed processing is advisable to use. To ensure this, changes in the design of metal-cutting machines, spindles, and bearings are necessary to enable working at rotational and linear speeds that exceed the traditional machining modes by many times.

Rolling bearing supports are the most common. Despite the high precision requirements for manufacturing all parts of these bearings, their characteristics limit their range of application due to:

- limitations of the possible rotational frequencies range;
- tendency to forced and self-oscillations due to the actual difference in the sizes of the rolling elements in the bearings, their uneven wear, different contact stiffness between the rings and individual balls or rollers, and so on;
- increased noise of the rolling bearing assembly.

These drawbacks limit the use of rolling bearings in high-precision mechanisms with high rotational speeds. An alternative to rolling bearings are sliding bearings: hydrostatic, hydrodynamic, hydrostatic-dynamic, gas-static, gas-dynamic, and magnetic-based bearings.

Hydrodynamic sliding bearings are divided into two main categories based on their working principle: those with a hydrostatic principle of load capacity creation and those with a hydrodynamic principle. The hybrid hydrostatic-dynamic principle is also used, which uses constant pressure from a power source and the pressure created by hydrodynamic effects in the bearing structure.

In a hydrodynamic bearing, the main load is carried by the working fluid, which creates pressure in the thin film that arises between the bearing surfaces during rotation. Modern hydrodynamic bearings are used in various precision mechanisms such as high-speed spindle assemblies, turbines, pumps, generators, deaerators, engines, and compressors when ordinary roller or ball bearings do not meet their requirements [1].

Typically, the design consists of an inner and outer ring, with seals at the contact points. Due to improved seal design and composite materials used, the hydrodynamic bearing has virtually no fluid losses during operation (or they are minimal). Such a mechanism is characterized by a long service life.

Hydrodynamic bearings are often used in manufacturing due to their low precision requirements, low noise level, minimal vibrations, and high damping capability, which allows them to work well in a wide range of operational conditions. However, they are not commonly used in high-speed spindles due to limitations such as wear during start-up and shutdown, limited stiffness, the need for constant lubrication, unstable spindle position during changes in rotation frequency, and technological difficulties in manufacturing and installing bearings.

Spindles with hydrostatic lubrication are widely used in metalworking machines due to their high precision, load-bearing capacity, damping capability, and the absence of wear in such bearings, which provides them with practically unlimited durability [2, 3].

However, the increased requirements for high-speed and high-precision machines have revealed a significant disadvantage of fluid-lubricated sliding bearings (both hydrodynamic and hydrostatic), which is the significant heat generation resulting from the relative sliding of the lubricant layers. The power dissipated due to friction is proportional to the viscosity of the lubricant and the square of the rotation speed, resulting in a complicated cooling system. For example, high-precision round grinding machines suffer from thermal deformation caused by heat generation in hydrostatic bearings of grinding wheels. Therefore, the use of such spindles requires thermal calculations and the provision of an appropriate thermal regime.

Since the movement of forming tools is carried out by a spindle and spindle bearings, they make a decisive contribution to the output characteristics of machine tools [4]. Therefore, the task of increasing the efficiency of mechanical processing is very relevant, and its solution contributes to reducing labor costs, reducing operating costs, increasing the productivity of individual operations, and automating the processing of complex parts. The most acceptable way to increase accuracy and productivity, and reduce the amount of reference work and the cost of producing parts is to use high-speed machining, which allows optimizing the process of mechanical processing. The use of high-speed machining requires changes in the design of spindles that will provide operation at rotation and linear speeds that are several times higher than those in traditional processing, as well as the use of numerical control systems with higher speed calculation of trajectory and modern tool designs [3].

Problem Statement

The smoothness and high-speed rotation of shafts, especially spindles, are determined by the precision and type of bearing supports used in them. Rolling and sliding bearings are predominantly used in modern spindle designs. To compare bearings when selecting them for use in given conditions, it is advisable to use the following main criteria:

1) the load capacity of the bearings;
2) the range of permissible rotational frequencies of the shaft;
3) the accuracy of maintaining the position of the axis of rotation of the shaft;
4) the stability of shaft rotation in the given type of bearings (the possibility of auto-oscillations and unwanted transient processes);
5) energy costs and economic indicators of manufacturing and operating this type of bearings;
6) vibroacoustic characteristics (noise level, sound level).

Review of Modern Information Sources on the Subject of the Paper

Hydrodynamic lubrication is often considered to be a condition of perfectly lubricated contact, as lubricating films are typically much thicker (usually 5–500 micrometers) than the roughness height on the bearing surface, thus preventing hard contact. The friction coefficient in the hydrodynamic regime can be very small (around 0.001). Friction slightly increases with sliding velocity due to resistance to motion caused by the working fluids, whose viscosity is practically independent of temperature. The disadvantage of sliding hydrodynamic bearings is the need to take measures to prevent physical contact between supporting parts, which may occur during start-up and low-speed rotation. This contact is determined by the properties of the working fluid, especially its viscosity and interaction with the friction pair materials. The occurrence of dry contact in the sliding pair is particularly dangerous.

On these regimes, adhesive wear may occur during start-ups and stops, while corrosive (chemical) wear of bearing surfaces may also occur as a result of interaction with the lubricating material.

Shafts (spindles) on hydrostatic supports start rotating only after pressure is applied to special pockets and the shaft is lifted to create a gap between the friction surfaces. This prevents them from touching and possible seizing processes.

Requirements for bearing supports are formulated based on the tasks that the entire mechanism solves. In addition to specific requirements, there are also several mandatory requirements: ensuring reliable operation at all operating modes; providing basic performance characteristics with a guaranteed resource in conditions of repeated starts and the presence of aggressive environments (liquid oxygen, hydrogen, ammonia); ensuring guaranteed delivery of lubricant material at all operating modes of bearing supports with the necessary flow rate, pressure and acceptable efficiency; minimum weight and size dimensions; maximum achievable speed of shaft rotation; high resistance to shock and vibration loads; absence of significant heating in bearing supports; absence of cavitation processes in supports; minimum cost of production and operation [5, 6].

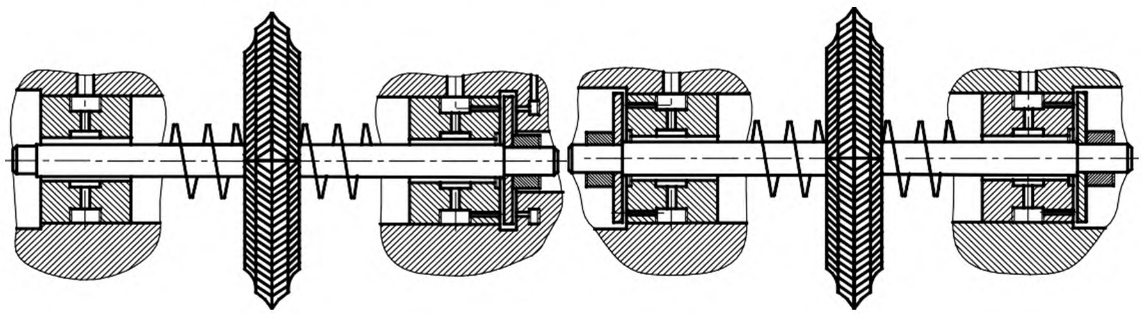
Each type of bearing support has its advantages and disadvantages, which in turn determine the area of their rational use. Table 1 shows the main characteristics for determining the required type of bearings.

Table 1

Comparative characteristics of bearing supports

Type of bearing supports:	Model / Idle power N, kVt	Speed parameter $d \times n$, mm×min^{-1}	Radial and axial runout of the spindle Δ, mkm
Rolling bearings	SH 24/15 0,7 kVt	$0 \ldots 1 \times 10^6$	0,5
Hydrodynamic bearings	SH 24/15 3,5 kVt	$0,1 \ldots 1 \times 10^6$	0,05
Hydrostatic bearings	EGS 24/25 4,5 kVt	$0 \ldots 1,5 \times 10^6$	0,05
Gas bearings	A 24/25 1,9 kVt	$0 \ldots 2,5 \times 10^6$	0,1…0,5
Electromagnetic bearings		$0 \ldots 4 \times 10^6$	

For bearing units that experience loads oriented differently in space, traditional design layouts of supports involve the presence of a thrust bearing (Fig. 1).



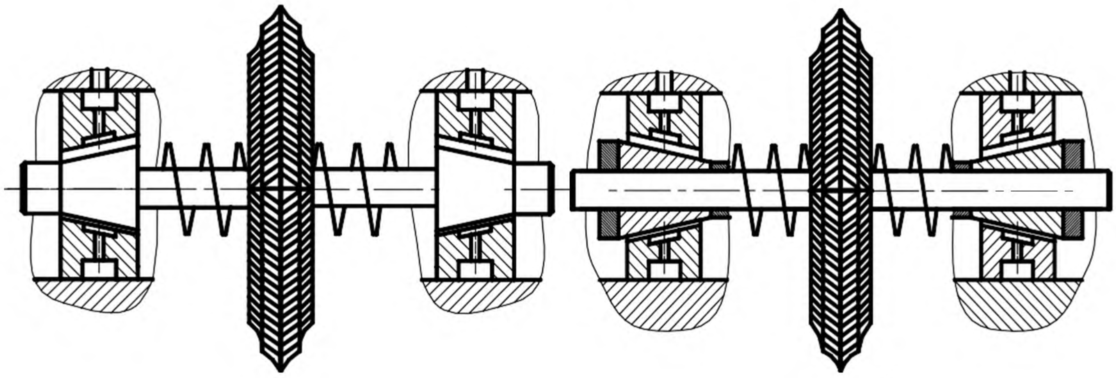
a *b*

Fig. 1. Schemes of radial-thrust bearing supports:
a – with one thrust bearing; b – with two thrust bearings

These support schemes have drawbacks: the use of two types of bearings (radial and thrust) leads to an increase in the number of throttling elements and, as a result, to the complication of the design and an increase in the axial dimensions of the unit. Considering that most mechanisms are loaded with both radial and axial forces, it is necessary to use single and double-sided supports along with bearing supports (Fig. 4). Ensuring the necessary accuracy of the relative arrangement of the radial and axial working surfaces of the friction pairs with such supports is a difficult task, reducing the coefficient of useful action and increasing the costs of lubricating material and support dimensions [3]. Conical working surface bearings partially allow for meeting these requirements [3, 6].

The main advantages of conical supports are their ability to simultaneously withstand both radial and axial loads. It has been established [7] that with an increase in the conicity of the support, the axial stiffness and the ability to withstand axial loads increase. The conical sliding bearing increases the reliability and durability of the support, as well as increases the technical and economic efficiency of its manufacture and operation [8–10]. These studies indicate the prospect of using this type of support in high-speed mechanisms. Possible disadvantages include the complexity of ensuring the accuracy of the manufacture of conical surfaces, an increase in dimensions in the radial direction, and the possibility of jamming during stops and starts, which requires additional measures.

It arrangement of rotor-thrust units on tapered sliding bearings involves two options (Fig. 2): a) the shaft and bearing are made tapered; b) the shaft is cylindrical with a rigidly fixed tapered bushing that rests on a tapered support.

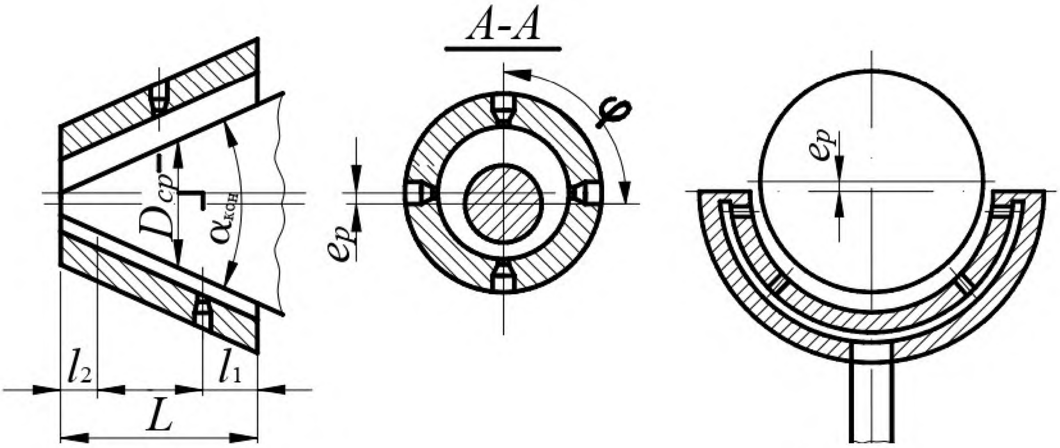


a *b*

Fig. 2. Schematic diagrams of radial-thrust bearing assemblies:
a – with a tapered surface for support; b – with tapered sleeves on a cylindrical shaft

The lubricating working fluid is supplied radially and then, entering the bearing's working zone, flows in the axial direction, thus lubricating the bearing's supporting tapered surfaces. Lubricant can also be supplied to the tapered bearing in the axial direction, either from the larger or smaller diameter of the bearing, but mainly from the smaller diameter, as centrifugal forces will throw the lubricant supplied to the gap towards the periphery. Therefore, supplying lubricant from the larger diameter is impractical since it will force lubricating material out of the working clearance.

By the phase state of the working fluid, sliding bearings are divided into two main groups: liquid lubrication bearings and gas lubrication bearings. However, when working with liquid lubrication, a portion of the lubricating material volume in the lubricating layer of the bearings can transition from a liquid to a gas phase and vice versa. Therefore, for a more complete and accurate description of the processes that occur in the lubricating layer of sliding bearings, the two-phase state of the lubricating material must be considered. By the direction of the load applied to the sliding bearings, they are divided into radial, axial, and radial-axial bearings. Radial sliding bearings are used as supports in high-speed turbine machines when axial loads are insignificant. When significant axial loads are present, axial (thrust) bearings are used. If significant radial and axial loads act simultaneously, radial-axial bearings are used, based on the design of two supports (radial and thrust) or with one working surface (such as tapered or spherical) (Fig. 3) [5].



a *b*

Fig. 3. Radial-thrust sliding bearings:
a – single and double-row tapered with oil grooves; b – semi-spherical

Significant research has been dedicated to sliding bearing supports of various geometries, such as multi-wedge, segmented, self-aligning with pads, and with spiral grooves. The bearing is characterized by high axial stiffness in both directions due to hydrodynamic wedges created by the previous load. The results of studies on self-aligning bearing supports are presented in the works of scientists [1, 11]. A series of studies are dedicated to the investigation of tapered sliding bearings, which include spiral grooves to improve their characteristics [4, 5, 12]. In works [13–15], the characteristics of spherical and tapered supports with spiral grooves lubricated by a non-compressible fluid are studied for cases of complete and incomplete filling of the bearing gap with the fluid. The description of static characteristics of tapered bearings with spiral grooves designed for high-speed spindles is given in works by Japanese scientists [16–19]. In addition to spiral grooves, as mentioned above, the internal surface of tapered bearings can be profiled with grooves of different geometric shapes. For example, in the work of Japanese scientists [18], a theoretical study of the thickness of the gas lubrication film closed in the working gap of a tapered bearing, with the internal surface profiled with straight grooves, is carried out. An analytical expression for determining the thickness of the lubrication film is obtained for three possible displacements: axial, radial, and nutation; the geometric condition of the operability of the studied gas static support – a tapered bearing with grooves along the generator, is determined.

Hybrid bearings, which are constructions consisting of both sliding and rolling bearings, occupy a special place [2]. An attempt has been made to optimize the size of a tapered bearing to reduce friction [16] under different operating conditions. Speed regimes and flow turbulence are taken into account, but the problem is solved in a hydraulic formulation, which does not allow the pressure in the lubricating layer to be determined taking into account the variable thermophysical properties.

Objectives and Problems of Research

The aim of this work is to develop criteria for comparing and selecting sliding bearing supports based on their main characteristics, as well as to develop analytical formulas for calculating these characteristics.

Main Material Presentation

There are three main types of working surfaces for sliding bearings: smooth, with longitudinal grooves, and hybrid. In modern high-speed turbomachinery and equipment, conical smooth sliding bearings (Fig. 4) are used as rotor bearings, primarily due to their simple design resulting from the absence of throttles, cost-effectiveness, and simplified operation [17].

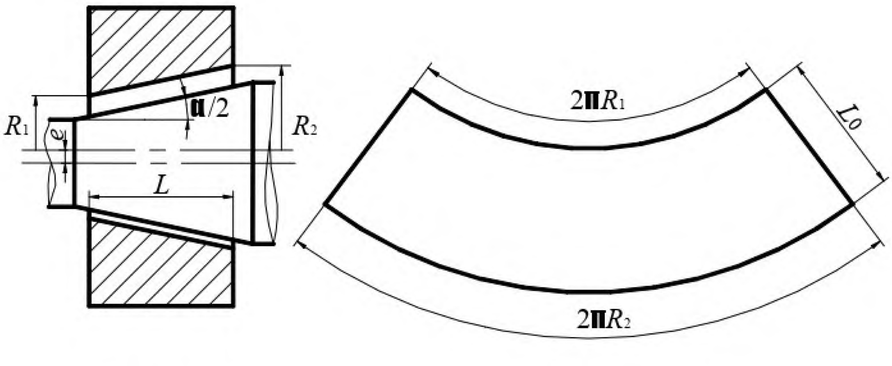
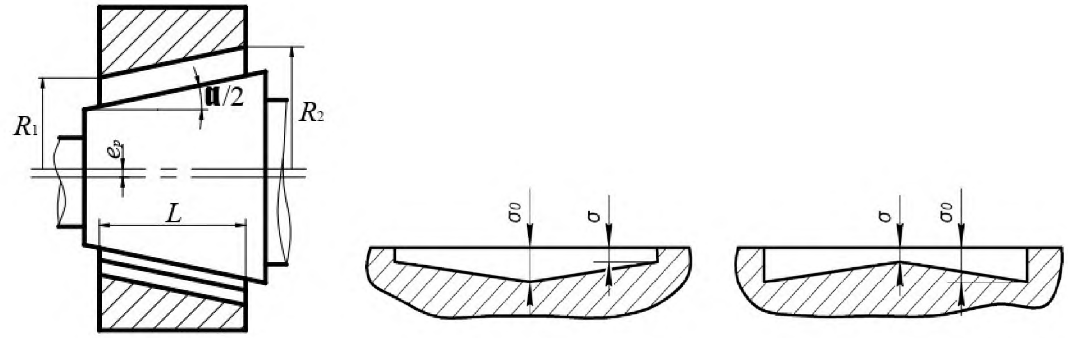


Fig. 4. Conical plain sliding bearing: a – calculation scheme; b – unfolded bearing surface

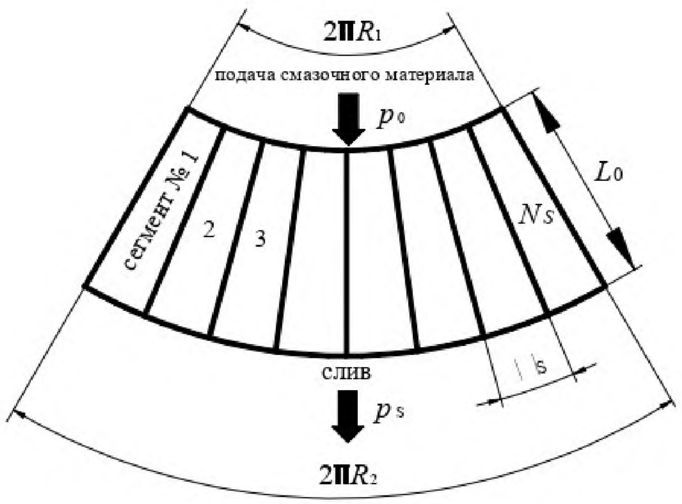
One of the disadvantages of classic sliding bearings is limited vibration resistance. By creating a smooth surface with longitudinal grooves, it is possible to significantly change the characteristics of the bearing: the load-bearing capacity decreases, but at the same time, the stability of the rotor movement increases due to the emergence of additional effects.

With the correct selection of the lubricant pressure and geometric characteristics, bearings with longitudinal grooves are capable of reducing the negative impact of vortex formation and existing rotor vibration in high-speed turbomachinery.

The conical sliding bearing with longitudinal grooves (Fig. 5), the unwrapped supporting surface of which is a trapezoidal surface divided by NS equal segments with grooves along the circumference, allows not only to increase the vibration resistance but also to withstand axial loads that arise in modern high-speed turbomachinery [15, 18, 19].



a



b

Fig. 5. Sliding tapered bearing with longitudinal grooves:
a – computational scheme; b – unfolding of the bearing surface

Combining two different types of sliding bearings in one design allows for a new type of hybrid conical bearing, which combines a smooth surface with a surface profiled with longitudinal grooves (Fig. 6).

The length L of the hybrid bearing is determined by the sum of the lengths of the smooth L_s and profiled L_p parts. The minimum and maximum radii of the bearing surface of the hybrid bearing are R_0 and R, respectively, and the angle corresponding to one segment is φ. The advantage of this type of bearing is as follows: the smooth part of the bearing provides greater load capacity, while the profiled part provides greater resistance. These types of bearings are necessary when it is necessary to provide greater load capacity while maintaining a stable position of the rotors in high-speed turbomachinery. This is particularly relevant for various high-speed turbo generators used in aviation and space technology [11, 13, 16].

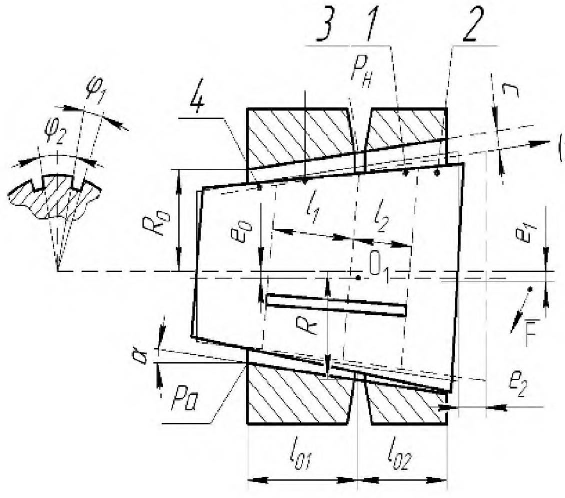


Fig. 6. Conical hybrid bearing

The thickness *h* of the lubricating layer has a significant impact on the pressure distribution in the lubricating layer and is a function of the position of the center of the shaft and the angular coordinate, which is included in the Reynolds equation. Therefore, it is necessary to more closely examine the determination of the thickness of the lubricating layer as a function of the total (common radial and axial) clearance [7, 15–19].

The thickness of the lubricating layer as a function of the total clearance is determined by considering the geometry of the bearing assembly. For the eccentric position of the rotor shaft in the conical bearing, in the absence of skewness of the rotor axis with respect to the bearing axis, the clearance function h does not depend on the coordinate r and is only a function of the angular coordinate φ. The determination of the function $h(\varphi)$ is carried out initially in the *XOY* plane (Fig. 7), where the radial clearance function h_{XY} has the form:

$$h_{XY} = h_{0_{XY}} - Xsin\beta - Ycos\beta, \tag{1}$$

where h_{0XY} – average radial clearance in the plane *XOY*.

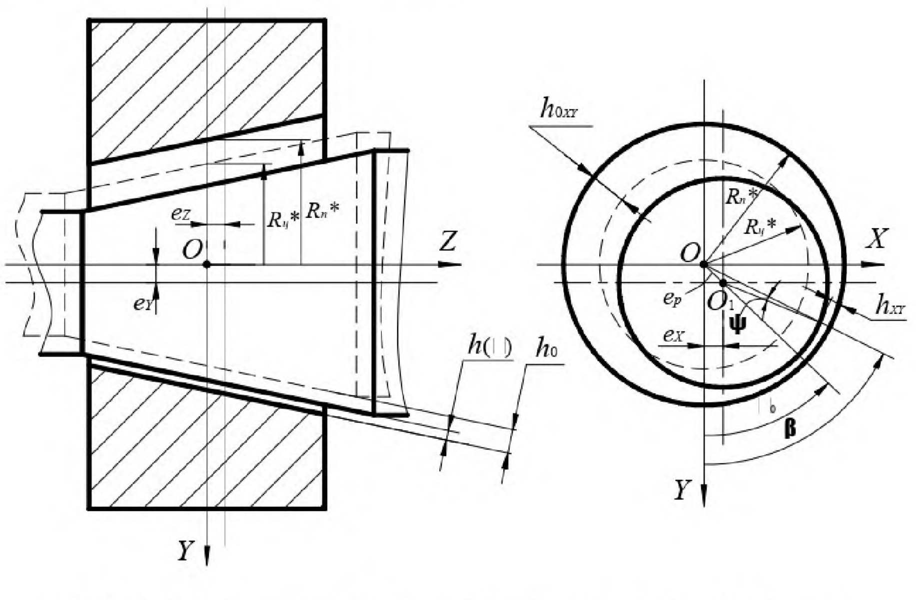


Fig. 7. Calculation scheme for determining the clearance h in the absence of rotor axis tilt relative to the bearing axis

In the presence of misalignment between the axis of the rotor and the axis of the tapered bearing, the total clearance h is a function of two variables, the coordinates r and φ, and is given by the following expression (Fig. 8):

$$h_{(r,\varphi)} = h_0 - (Xsin\beta + Ycos\beta)\cos\left(\frac{\alpha}{2}\right) + Zsin\left(\frac{\alpha}{2}\right) - (r - r_1)tg\gamma. \qquad (2)$$

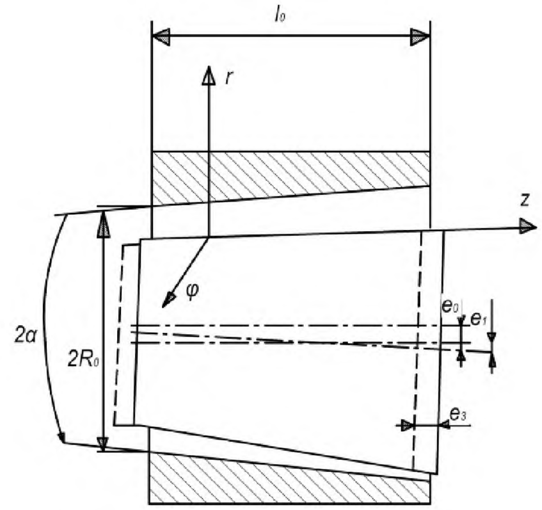


Fig. 8. Calculation scheme for determining the clearance h in the presence of misalignment between the axis of the rotor and the axis of the bearing

Let's consider the flow of lubricant material in the radial clearance of a sliding tapered bearing in a cylindrical coordinate system [10, 15–19]. Taking into account the small thickness of the lubricant film, we will align the rφ plane with the plane that relates to the bearing's supporting surface at the point where the movement is being considered. The r-axis will be directed parallel to the bearing's formation, and the y-axis coincides with the normal to its surface (Fig. 9).

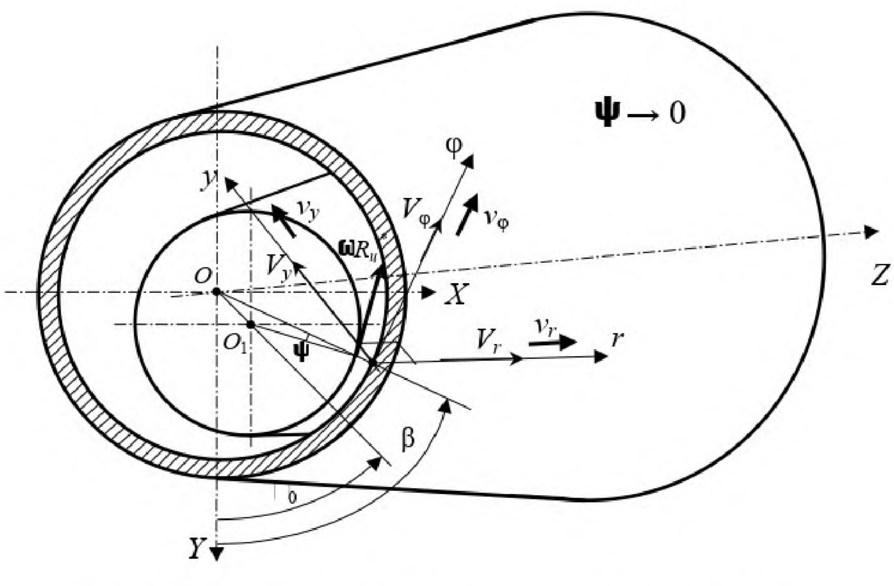


Fig. 9. Calculation scheme of the working fluid flow in a tapered bearing

The study of the motion of the working fluid in the bearing is based on two fundamental laws of hydrodynamic lubrication theory: the law of conservation of mass and the law of conservation of momentum [4, 11]. Mathematical models of fluid lubricated bearings are based on the Navier-Stokes equations. We make the following assumptions:

- the clearances between the surfaces are much smaller than the dimensions of the surfaces;
- the motion of the working fluid in the clearances is stationary and relatively slow;
- the motion of the working fluid is assumed to be isothermal, i.e., the temperature in the lubricating film is constant. The Reynolds equation is the main equation of hydrodynamic lubrication theory. By performing differential operations of vector analysis [12, 19], the laws (2.13)–(2.14), taking into account the compressibility of the working fluid and the smallness of the gravitational forces, will look like this:

the continuity equation:

$$\frac{\partial\bar{\rho}}{\partial\bar{t}}+\frac{\partial(\bar{\rho}\bar{v_r})}{\partial\bar{r}}+\frac{\bar{\rho}\bar{v_r}}{\bar{r}}+\frac{\partial(\overline{\rho v_\varphi})}{\bar{r}\partial\varphi}+\frac{\partial(\overline{\rho v_y})}{\partial\bar{y}}=0, \quad (3)$$

Navier-Stokes equations:

$$ReSh\frac{\partial\bar{v_r}}{\partial\bar{t}}+Re\left(\bar{v_r}\frac{\partial\bar{v_r}}{\partial\bar{r}}+\bar{v_\varphi}\frac{\partial\bar{v_r}}{\bar{r}\partial\varphi}+\bar{v_y}\frac{\partial\bar{v_r}}{\partial\bar{y}}-\frac{\overline{v_\varphi^2}}{\bar{r}}\right)=-\frac{\partial\bar{p}}{\partial\bar{r}}+\bar{\mu}\frac{\partial^2\bar{v_r}}{\partial\overline{y^2}}+\frac{\partial\bar{v_r}}{\partial\bar{y}}\frac{\partial\bar{\mu}}{\partial\bar{y}};$$

$$ReSh\frac{\partial\overline{v_\varphi}}{\partial\bar{t}}+Re\left(\bar{v_r}\frac{\partial\overline{v_\varphi}}{\partial\bar{r}}+\bar{v_\varphi}\frac{\partial\overline{v_\varphi}}{\bar{r}\partial\varphi}+\bar{v_y}\frac{\partial\overline{v_\varphi}}{\partial\bar{y}}-\frac{\overline{v_r v_\varphi}}{\bar{r}}\right)=-\frac{\partial\bar{p}}{\bar{r}\partial\varphi}+\bar{\mu}\frac{\partial^2\overline{v_\varphi}}{\partial\overline{y^2}}+\frac{\partial\overline{v_\varphi}}{\partial\bar{y}}\frac{\partial\bar{\mu}}{\partial\bar{y}}; \quad (4)$$

$$\frac{\partial\bar{p}}{\partial\bar{y}}=0,$$

where dimensionless parameters are defined as:

$$\bar{v_r}=\frac{v_r}{v_{r0}};\ \ \overline{v_\varphi}=\frac{v_\varphi}{v_{\varphi 0}};\ \ \overline{v_y}=\frac{v_y}{v_{y0}};\ \ \bar{r}=\frac{r}{r_0};\ \ \bar{y}=\frac{y}{y_0};\ \ \bar{p}=\frac{p}{p_0};\ \ \bar{\rho}=\frac{\rho}{\rho_0};\ \ \bar{\mu}=\frac{\mu}{\mu_0};$$

$$v_{r0}=\omega_0 r_0;\ \ v_{\varphi 0}=\omega_0\varphi_0;\ \ v_{y0}=\omega_0 h_0;\ \ y_0=h_0;\ \ r_0=\frac{R_2}{\sin(\alpha/2)};$$

$$R_e=\frac{\omega_0 h_0^2\rho_0}{\mu_0};\ \ Sh=\frac{1}{\omega_0 t_0};\ \ \psi=\frac{h_0}{r_0}.$$

When considering the working fluid in such bearings, local inertial terms in the system (4) can be neglected. Therefore, the motion of the lubricant in these bearings is quasi-stationary. The non-stationarity of the lubrication process is expressed through boundary conditions or time-dependent forces. When solving problems with the inclusion of heat dissipation in the walls, it is necessary to take into account the terms containing $\frac{\partial\bar{\mu}}{\partial\bar{y}}$. in the simplified Navier-Stokes equations (4). In cases where heat dissipation in the walls can be neglected, taking into account the small Reynolds number and the influence of local terms, systems of equations (5–6) can be obtained, the joint solution of which gives the Reynolds equation for pressures. If the variation of viscosity coefficients with thickness is ignored, then taking into account the above-mentioned considerations and adding turbulence coefficients to the equations, the system of equations (3–4) in dimensional form is expressed as:

$$\frac{\partial\rho}{\partial\bar{t}}+Re\left(\frac{\partial(\rho v_r)}{\partial r}+\frac{\rho v_r}{r}+\frac{\partial(\rho v_\varphi)}{r\partial\varphi}+\frac{\partial(\rho v_y)}{\partial y}\right)=0; \quad (5)$$

$$\frac{\partial p}{\partial r}=\mu K_r\frac{\partial^2 v_r}{\partial y^2};\ \ \frac{\partial p}{r\partial\varphi}=\mu K_\varphi\frac{\partial^2 v_\varphi}{\partial y^2};\ \ \frac{\partial p}{\partial y}=0. \quad (6)$$

Taking into account that the thickness of the lubricating layer h_0 is approximately three orders of magnitude smaller than the other two dimensions of the bearing (*Rn** and *L*), the pressure in the lubricating layer, up to the value of ψ, does not change along the y coordinate but depends only on the axial r and circumferential φ coordinates. Hence, it is possible to integrate the first and second equations of the system (6):

$$\frac{\partial p}{\partial r}\frac{y^2}{2} = \mu K_r v_r + f_1(r,\varphi)y + f_2(r,\varphi);$$

$$\frac{\partial p}{r\partial \varphi}\frac{y^2}{2} = \mu K_\varphi v_\varphi + f_3(r,\varphi)y + f_4(r,\varphi). \tag{7}$$

Taking into account the assumption of no-slip conditions for the working lubricant material on the bearing surfaces, we can express the boundary conditions for the velocities V_r, V_φ and V_y as follows:

$$y = 0:\ \ v_r = 0;\ \ v_\varphi = 0;\ \ v_y = 0;$$

$$y = h:\ \ v_r = V_r;\ \ v_\varphi = V_\varphi;\ \ v_y = V_y. \tag{8}$$

The values of the flow velocities of the lubricant in the radial and axial directions, taking into account the boundary conditions (8), will be equal to:

$$v_r = \frac{1}{2\mu K_r}\frac{\partial p}{\partial r}y(y-h) + V_r\frac{y}{h};\quad v_\varphi = \frac{1}{2\mu K_\varphi}\frac{\partial p}{r\partial \varphi}y(y-h) + V_\varphi\frac{y}{h}. \tag{9}$$

Substituting the obtained velocity values into the continuity equation (5) and integrating it over the coordinate y within the radial gap from *0* to *h*, we get:

$$h\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}\left(\rho\int_0^h v_r\,dy\right) + \frac{\rho}{r}\int_0^h v_r\,dy + \frac{\partial}{r\partial\varphi}\left(\rho\int_0^h v_\varphi\,dy\right) + \rho v_y = 0. \tag{10}$$

The volumetric flow rate of the lubricant material in the circumferential and axial directions per unit time, taking into account (9), will be equal to:

$$\int_0^h v_r\,dy = -\frac{h^3}{12\mu K_r}\frac{\partial p}{\partial r} + \frac{V_r h}{2};\quad \int_0^h v_\varphi\,dy = -\frac{h^3}{12\mu K_\varphi}\frac{\partial p}{r\partial\varphi} + \frac{V_\varphi h}{2}. \tag{11}$$

If we denote the radial convergence velocity of the bearing and shaft support surfaces as V_y and substitute the expression (10) into equation (11), we obtain the main equation for determining the pressure field – the Reynolds equation, generalized for the case of two-dimensional turbulent flow of a viscous working fluid [13–19]:

$$\frac{\partial\rho}{r\partial r}\left[\frac{\rho r h^3}{\mu K_r}\frac{\partial p}{\partial r}\right] + \frac{\partial}{r\partial\varphi}\left[\frac{\rho h^3}{\mu K_\varphi}\frac{\partial p}{r\partial\varphi}\right] = 12h\frac{\partial\rho}{\partial t} + 6\frac{\partial}{r\partial r}(\rho r h V_r) + 6\frac{\partial}{r\partial\varphi}(\rho h V_\varphi) + 12\rho V_y, \tag{12}$$

where the velocity values at points on the surface of the shaft are determined by:

$$V_r = (\dot X\sin\beta + \dot Y\cos\beta)\sin\left(\frac{\alpha}{2}\right) + \dot Z\cos\left(\frac{\alpha}{2}\right);\quad V_\varphi = \omega r\sin\left(\frac{\alpha}{2}\right) + \dot X\cos\beta + \dot Y\sin\beta;$$

$$V_y = (\dot X\sin\beta + \dot Y cos\beta)\cos\left(\frac{\alpha}{2}\right) - \dot Z\sin\left(\frac{\alpha}{2}\right). \tag{13}$$

The determination of the pressure field $p(r,\varphi)$ at a particular moment in time is a boundary problem of solving the Reynolds equation (12) – a nonlinear partial differential equation – with the following boundary conditions:

1) given drain pressure ps (on the ends of the bearing) and pressure in the chambers p_H:

$$p(r_1,\varphi) = p_{s1};\quad p(r_2,\varphi) = p_{s2};\quad p(r_{Hn},\varphi_{Hn}) = p_{Hn}, \tag{14}$$

where r_H, φ_H, n – the coordinates and number of the groove;

2) the conditions for the coordinate φ are written based on Sommerfeld's hypothesis, according to which the rotor support surface is completely covered with a lubricant layer. The validity of this approach is demonstrated, for example, in [15]. Then, the conditions for the junction can be written:

$$p(r,0) = p(r, 2\pi\sin(\alpha/2));\quad \frac{\partial p}{\partial\varphi}(r,0) = \frac{\partial p}{\partial\varphi}(r, 2\pi\sin(\alpha/2)). \tag{15}$$

After determining the value of the total gap h, one can directly proceed to consider issues related to the flow of lubricant material in an eccentric annular gap. When modeling the flow of lubricant material, two peculiarities need to be taken into account: the small (tens of micrometers) thickness of the lubricant layer and the rather complex geometry of the radial gap.

The reference data on the thermophysical properties of lubricant materials [6, 10] used in sliding bearings turn out to be inconvenient for numerical implementation. Therefore, these data are approximated using the least squares method [12, 16], which allows finding analytical dependencies for the properties of a single-phase material in the form of functions of pressure and temperature for water. However, unlike water and liquid hydrogen, for which phase transitions are possible, oils can only exist in the liquid phase and begin to burn at further increases in temperature.

The increase in rotating speeds leads to the possibility of laminar as well as turbulent flows of the working fluid in the bearing gap. The transition from laminar to turbulent flow is characterized by the critical value of the Reynolds number Re* [15]:

$$Re > Re^*;\quad Re = \frac{\rho v_m L}{\mu};\quad Re^* \approx (1{,}2 \ldots 2)\cdot 10^3,$$

where v_m – the average flow velocity of the working fluid; L – characteristic size.

The flow of the working fluid in the lubricating medium under turbulent conditions is characterized by mixing of elementary volumes, the presence of chaotic pulsations of velocities and pressures in all directions, and the appearance of turbulent ("eddy") viscosity. The influence of eddy viscosity on determining the pressure field in the lubricating layer can be taken into account by introducing turbulence coefficients K_r and K_φ along the respective directions. In single-phase flows of lubricating material, turbulence coefficients are determined using the Constantinides methodology [17, 19]:

$$K_\varphi = 1 + 0{,}044\cdot(k^{*2}\cdot Re)^{0{,}725};\quad K_r = 1 + 0{,}0247\cdot(k^{*2}\cdot Re)^{0{,}65},$$

where k^* – the pocket coefficient determines the mixing path and depends on the radial clearance.

The coefficient k^* can take such values: $k^* \approx 0.2...0.4$; small values of k^* correspond to small radial clearances from 10 to 100 microns. The pocket coefficient is often calculated using an empirical relationship: $k^* \approx 0{,}125\cdot Re^{0{,}07}$. The Reynolds number value for a tapered bearing is determined as:

$$Re = \frac{\omega R^*_{ц}\rho h}{\mu}.$$

When calculating numerical values of pressure fields in the lubricating layer, the following main assumptions are made:

1) the lubricating material is a continuous medium that fills the entire radial gap;
2) the lubricating material is considered to be a Newtonian fluid, meaning that the stress tensor in the lubricating layer is linearly dependent on the tensor of deformation velocities;

3) the lubricating medium is isotropic, meaning that physical properties at each point are the same in all directions;

4) changes in thermodynamic parameters across the lubricating layer due to its small size are neglected;

5) velocity gradients are only taken into account in the direction normal to the rubbing surfaces, and the velocity of the lubricating material in this direction is considered to be small;

6) the curvature of the lubricating layer due to its small thickness (compared to the length and radii of the bearing) is neglected;

7) the effects of surface tension, inertia, and the mass of the lubricating material, as well as changes in the volume of the lubricating material associated with temperature changes, are neglected;

8) sliding of the lubricating material relative to the bearing and rotor surfaces is absent (due to the phenomenon of adsorption), and the velocity of the boundary layers of the lubricating material is equal to the velocity of the adjacent supporting surfaces;

9) the working surfaces of the shaft and bearing are assumed to be perfectly smooth, and the shape of their cross-section does not change along the axis of the bearing, and manufacturing and assembly inaccuracies of the support assembly are insignificant.

In addition to the above assumptions, the following factors are taken into account when determining the pressure field:

- the flow of the lubricating material is not established – a non-stationary problem is considered;
- the working fluid is a compressible fluid;
- the thermal regime of the flow is non-isothermal;
- the flow regime can be either laminar or turbulent.

Conclusions

1. The flow parameters of the working fluid have an impact on the load-bearing capacity of radial bearings, and the proposed evaluation dependencies can also be used for tapered bearings.
2. Analysis of the Navier-Stokes equation has been carried out using the method of small perturbations for small eccentricities. The results of calculations indicate a significant influence of the flow parameters of the working fluid on the expansion of the area of rarefactions and the range of their values, as well as on the reduction of the area and range of elevated pressures. It has been established that at small eccentricities, the bearing load capacity can be significantly increased due to the flow parameters of the working fluid; at rotational speeds of 60–70 m/s and a radial clearance of 80 μm, the increase can reach 20 %.
3. An important qualitative feature has been established: as the Reynolds number Re* increases, the bearing load capacity increases. The most intense change in load capacity due to the influence of flow parameters of the working fluid is observed at a relative eccentricity of e = 0.2–0.4.
4. The terms of the Navier-Stokes equation that take into account the flow parameters of the working fluid can have a value that is comparable to other components, so ignoring them is not always permissible.

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