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Effects of suction on steady boundary layer stagnation-point flow and heat transfer in porous medium towards a stretching/shrinking sheet with thermal radiation

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In this study, the effects of suction and thermal radiation on steady boundary layer stagnation point flow and heat transfer in porous medium towards a stretching and shrinking sheet is investigated. The governing partial differential equations (PDEs) are transformed into Nonlinear Ordinary Differential Equations (ODEs) utilizing similarity transformation. These equations are then solved numerically using the function BVP4C in MATLAB, with the numerical results presented through graphical illustrations. The investigation uncovers the existence of dual solutions and unique solutions within specific ranges of velocity ratio parameter ε, depending on the parameters involved. Furthermore, it is found that both the skin friction coefficient and the local Nusselt number increase as suction S and permeability parameter K increase. Also, higher values of Prandtl number Pr and thermal radiation parameter R result in a thinner thermal boundary layer.

Keywords: *suction; stagnation point flow; heat transfer; porous medium; stretching/shrinking sheet; thermal radiation.*

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1. Introduction

The problem of stagnation-point flow over a stretching and shrinking plate is significant in fluid mechanics. Initially investigated by Hiemenz [1], who studied the boundary layer on a straight circular immersed in the uniform flow of liquid. Since then, numerous researchers have explored and investigated the boundary layer stagnation-point flow encompassing different fluids, various geometries, and diverse conditions to acquire a deeper understanding of its characteristics and behavior. One of the researchers is Chiam [2], who conducted an investigation into the transfer of heat in a fluid flow over a flat sheet that is being stretched with a linear velocity. His findings indicated that when the stretching velocity of the plate and the stagnation flow velocity in the inviscid free stream are equal, no boundary layer is formed near the plate. Later, Mahapatra and Gupta [3] revisited the issue of stagnation-point flow towards a stretching with different stretching and velocities of flow. The research focused on the scenario where the sheet is stretched within its own plane at a velocity proportional to the distance from the stagnation point. They discovered that two different types of boundary layer are formed near the stretching plate, which were influenced by the ratio between the stretching and stagnation flow velocities. Other than Mahapatra and Gupta [3], the research on this type of problems has been studied by many researchers with various types of fluid such as micropolar fluid by Nazar et al. [4], oblique viscous flow by Lok et. al [5] along with Reza and Gupta [6] and oblique micropolar fluid by Lok et al. [7].

Recently, the study of boundary layer flow induced by a shrinking sheet has gained attention due to its various engineering applications. Unlike the stretching sheet, where the boundary velocity moves away from a fixed point, the velocity in the shrinking case converges towards a fixed point. Miklavčič and Wang [8] established the existence and uniqueness of steady viscous flow due to a shrinking sheet, considering the effects of suction. They discovered that dual solutions exist for specific suction values, but no boundary layer solution exists within certain suction ranges. In stagnation-point flow over a shrinking sheet, the vorticity induced by the shrinking effect often remains confined within the boundary layer due to the stagnation flow velocity in some cases. Wang [9] explored the stagnation flow problem related to a shrinking sheet and identified dual solutions for certain ratios of shrinking to stagnation flow rates. He noted that at higher rates of sheet shrinking, solutions may not be feasible, and in the two-dimensional scenario, solutions might be non-unique. Other than that, Fang [10] analyzed boundary layers forming over a sheet with a power-law surface velocity and continuous mass transfer. This study compared the continuously stretching surface with results obtained for the power-law shrinking sheet, revealing intriguing nonlinear behaviors and significantly enhancing our understanding of boundary layers.

Since the vorticity generated by the shrinking sheet is not confined within the boundary layer, an additional external force is needed to contain it and ensure steady flow. Suction at the sheet is the most effective external force for achieving this confinement. Numerous researchers have analyzed the problems of boundary layer with the effects of suction. Layek et al. [11] studied the form of boundary layer stagnation-point flow and heat transfer over a shrinking sheet situated in a porous medium, taking into account suction effects, as well as internal heat generation or absorption. They found that the horizontal velocity increases as the ratio of free stream velocity and stretching velocity increases. Moreover, they stated that the dimensionless temperature at a specific point on the sheet decreases due to suction. Other than that, another researcher example that considers the effects of suction in their study is Lin et al. [12]. They examined the flow of power law fluids over a flat surface, focusing on steady laminar boundary layers. The investigation reveals that the behavior of velocity and shear stress is significantly influenced by the parameter values explored in this study.

An additional aspect was introduced to the boundary layer flow and heat transfer by incorporating the thermal radiation effects. Including thermal radiation is crucial for achieving a more accurate and comprehensive representation of thermal behavior. One of the early studies by Viskanta and Grosh [13], who considered a problem of a boundary layer, focusing on how thermal radiation affects temperature distribution and heat transfer in a flowing medium that absorbs and emits radiation over a wedge. In their study, they stated that the approximation fails to accurately account for radiation leaving from the surface near the boundary, leading to inaccuracies in the temperature profiles in that region. Later, Hossain et al. [14] investigated the impact of radiation on the natural convection flow of a dense, viscous, and incompressible fluid passing over a heated vertical porous plate. Their findings reveal that the presence of suction leads to a thinner boundary layer at significant distances from the leading edge. While radiation effects do not alter this qualitative behavior, they do have an impact on the quantitative results. Furthermore, Abd El-Aziz [15] conducted research about the radiation effect on the flow and heat transfer over an unsteady stretching sheet. He found that the heat transfer rate increased with increasing radiation parameter R and other parameters which are unsteadiness parameter A and Prandtl number Pr. Other than Abd El-Aziz [15], the research on this type of problems which the effect of radiation on the unsteady flow and heat transfer over a stretching sheet has been studied by many researchers with various types of fluid such as mixed convection by Mukhopadhyay [16], magnetohydrodynamic by Hayat et al. [17] and unsteady mixed convection flow of the second grade fluid by Hayat et al. [18]. Other than that, Hayat et al. [19] conducted a research that focused on examining the mixed convection stagnation point flow of Carreau fluid considering thermal radiation and chemical reaction. The findings indicate that the material parameters are enhanced by the velocity field, while heat generation has an opposite effect on the thermal fields and local Nusselt number.

Rosali et al. [20] stated that the boundary layer flow and heat transfer over a stretching sheet in a porous medium holds great significance in various industrial applications. Fluid flow through a porous medium is encountered in numerous practical scenarios, including filtration processes, underground reservoirs, and heat exchangers. Since the influence of porous medium significantly affects the behavior of fluid flow, it becomes imperative to include the porous medium in this study. Irfan et al. [21] conducted research on the theoretical effects of unsteady MHD stagnation point flow of heat and mass transfer across a stretching and shrinking surface in a porous medium, considering internal heat generation/absorption, thermal radiation, and chemical reactions. The analysis revealed that the skin friction coefficient increases with higher values of the porosity parameter K_1 and other related parameters. Abbas et al. [22] studied heat transfer analysis on MHD stagnation point flow past a permeable shrinking/stretching sheet through a porous media, noting that increasing the porosity parameter K_1 enhances fluid velocity. Jamaludin et al. [23] numerically examined stagnation-point flow and heat transfer of mixed convection in an incompressible Cross fluid across a permeable shrinking sheet, with suction and thermal radiation effects. Their findings revealed dual solutions for both assisting and opposing flow regions. All these studies cover a variety of cases, using different fluids, mediums, and methods to address various curiosities and questions. Thus, this paper aims to modify and enhance previous research to expand knowledge and interest in the field. This paper studies the effects of suction on steady boundary layer stagnation-point flow and heat transfer in porous medium towards stretching/shrinking sheet with thermal radiation.

2. Mathematical formulation

We consider a steady and two-dimensional boundary layer flow near a stagnation-point of an incompressible Newtonian fluid over a stretching/shrinking sheet in porous medium as shown in Figure 1.

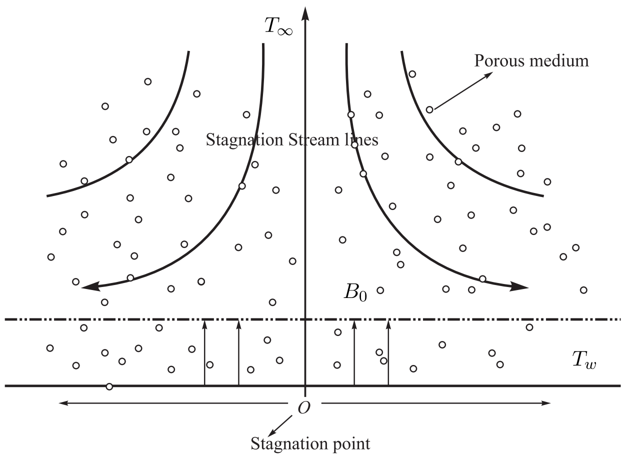

Fig. 1. Physical sketch of the problem.

A stream of fluid at temperature T_∞ moving with velocity $U(x)$ over the surface of the sheet with temperature T_w. Considering these assumptions, along with the utilization of Rosseland's approximation, the equations governing steady two-dimensional boundary layer flow, along with the equations accounting for heat radiation, can be expressed in conventional notation as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \nu\frac{\partial^2 v}{\partial y^2} + \frac{\nu}{K_1}(U-u), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

where $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, μ is the dynamic viscosity, ρ is the fluid density and $K = \frac{\nu}{aK_1}$ is the permeability parameter, K_1 is the porosity of the porous medium, a is the straining rate parameter and U is the stagnation flow velocity in the inviscid free stream. T is the temperature, T_w is the sheet's temperature and T_∞ is the free stream temperature that are assumed as constants with $T_w > T_\infty$. k is the thermal conductivity of fluid, c_p is the specific heat and q_r is the radiative heat flux. By using Rosseland's approximation for radiation, we obtain $q_r = -\left(\frac{4\sigma}{3k^*}\right)\frac{\partial T^4}{\partial y}$, where σ is the Stefan–Boltzmann constant and k^* is the absorption coefficient. Subsequently, we make the assumption that the temperature variation within the flow is such that T^4 about T_∞. We neglect the higher order terms and get, $T^4 = 4T_\infty^3 T - 3T_\infty^4$.

Now, equation (3) reduces to:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k^*\rho c_p}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The boundary conditions for the velocity components and temperature are provided by

$$\begin{aligned} &u = cx, \quad v = v_w, \quad T = T_w \quad \text{at} \quad y = 0; \\ &u \to U(x) = ax, \quad T \to T_\infty, \quad \text{as} \quad y \to \infty, \end{aligned} \tag{5}$$

where c is the stretching/shrinking rate of the sheet, $c > 0$ for stretching while $c < 0$ for shrinking, $a(> 0)$ is the stagnation flow rate parameters, v_w with $v_w < 0$ represents a predetermined suction velocity.

The governing equations (1) to (3) along with the boundary conditions (2) are transformed to ordinary differential equations (ODEs) using the similarity transformations:

$$\psi = \sqrt{a\nu}\,x f(\eta), \quad \eta = y\sqrt{\frac{a}{\nu}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{6}$$

$$v_w = -\sqrt{a\nu}\,S, \quad \mathrm{Pr} = \frac{\mu c_p}{k}, \quad R = \frac{kk^*}{4\sigma T_\infty^3}, \tag{7}$$

where ψ and η are the stream function and similarity variable, respectively. The stream function is denoted as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial y}$, which satisfy equation (1) equivalently. $S(= \frac{-v_w}{\sqrt{a\nu}})$ is the suction parameter where $S > 0$. After applying these transformations, the momentum equation (2) and energy equation (3) are reduced to the following ordinary differential equation:

$$f''' + f\,f'' - f'^2 + 1 + K(1 - f') = 0, \tag{8}$$

$$(3R + 4)\theta'' + 3R\,\mathrm{Pr}\,f\theta' = 0, \tag{9}$$

subjected to the conditions:

$$\begin{gathered} f(0) = S, \quad f^{'}(0) = \varepsilon, \quad \theta(0) = 1, \\ f'(\eta) \to 1, \quad \theta(\eta) \to 0, \quad \text{as} \quad \eta \to \infty. \end{gathered} \tag{10}$$

In this study, the physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which can respectively be denoted as

$$C_f = \frac{\tau_w}{\rho U^2}, \quad \mathrm{Nu}_x = \frac{x q_w}{k(T_w - T_\infty)}, \tag{11}$$

where τ_w is the surface shear stress and q_w is the surface heat flux which are respectively defined as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} + q_r, \tag{12}$$

μ represents the dynamic viscosity.

Substituting the similarity transformations (6) into equations (11) and (12), we obtain

$$C_f\sqrt{\mathrm{Re}_x} = f''(0), \quad \mathrm{Nu}_x\,\mathrm{Re}_x^{-\frac{1}{2}} = -\left(1 + \frac{4}{3R}\right)\theta'(0), \tag{13}$$

where $\mathrm{Re}_x = \frac{U_x}{\nu}$ represents the local Reynolds number and the thermal radiation parameter, $R = \frac{kk^*}{4\sigma T_\infty^3}$.

3. Results and discussion

The Nonlinear Ordinary Differential Equations (ODE) (8) and (9), under the specified boundary conditions (10) have been solved numerically for various parameter values. The parameters include suction S, permeability parameter K, Prandtl number Pr, thermal radiation R and velocity ratio parameter ε. These equations have been solved using the function BVP4C in MATLAB. The numerical computations were performed by inputting distinct values of parameters into the problem, and the results will be presented in both graph and table formats. The skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$, velocity profile $f'(\eta)$, temperature profile $\theta(\eta)$, for different parameters are obtained using this method. The results are discussed in detail with reference to the graphs and tabulated data. In addition, Table presents numerical comparative results of the present study with previous published for the skin friction coefficient $f''(0)$ in the absence of the suction parameter ($S = 0$) and permeability parameter ($K = 0$). The analysis indicates a strong concordance between the results of the current study and those previous published papers, thereby confirming our confidence in our numerical approach.

Table 1 presents the comparison of present study to Bhattacharyya and Layek [24] for skin friction coefficient $f''(0)$ with different values of velocity ratio parameter ε.

The variation of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ against ε for various values of parameter S is depicted in the Figures 2 and 3. From these figures, we observe

Table 1. Comparison values of $f^{''}(0)$ with different values of ε when $S = 0$ and $K = 0$.

ε	Wang [9] First solution	Wang [9] Second solution	Bhattacharyya and Layek [24] First solution	Bhattacharyya and Layek [24] Second solution	Present results First solution	Present results Second solution
-0.25	1.40224		1.402240		1.402240	
-0.50	1.49567		1.495669		1.495669	
-0.625			1.507156		1.507155	
-0.75	1.48930		1.489298		1.489298	
-1.00	1.32882	0	1.328816	0	1.328816	0
-1.15	1.08223	0.116702	1.082231	0.116702	1.082231	0.116702
-1.20			0.932472	0.233649	0.932473	0.233649
-1.2465	0.55430		0.584291	0.554285	0.584281	0.554296

that the solution exists up to critical value of ε_c. The dual solutions of the boundary layer flow over a shrinking sheet are obtained within the range of ε satisfying the inequality $\varepsilon_c \leqslant \varepsilon \leqslant -1.5$, where the critical points $\varepsilon_c \approx -1.9615, -2.1696, -2.3284$ for $S = 0.5, 0.8$ and 1, respectively. Additionally, it is observed that only one solution exists for all $\varepsilon > -1.5$ across all values of S, ensuring the uniqueness of the solution. Meanwhile, the solutions of the boundary layer flow over a stretching sheet are unique, where $\varepsilon > 0$. Moreover, no solutions are observed for $\varepsilon < \varepsilon_c$ which is attributed to the occurrence of boundary layer separation. We can interpret that as the suction parameter S increases, the generated vorticity remains confined for more negative values of magnitude of ε with $\varepsilon < 1.5$. This occurs in cases where the shrinking rates are much larger than the stagnation flow rates, thereby widening the range of conditions under which solutions exist. The results also suggest that the application of suction alters the boundary layer behaviour by reducing adverse pressure gradients and fostering the adhesion of flow to the surface. Consequently, this results in thinner boundary layers and the prevention of separation, thereby impacting both skin friction and heat transfer properties.

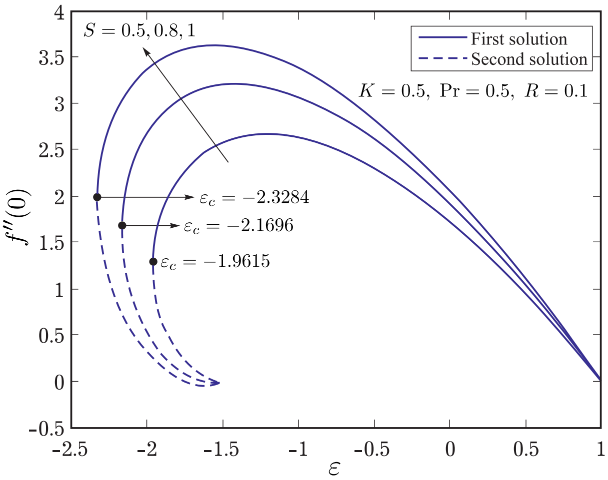

Fig. 2. Skin friction coefficient $f''(0)$ for different values of S against ε and fixed $K = 0.5$, $\mathrm{Pr} = 0.5$, and $R = 0.1$.

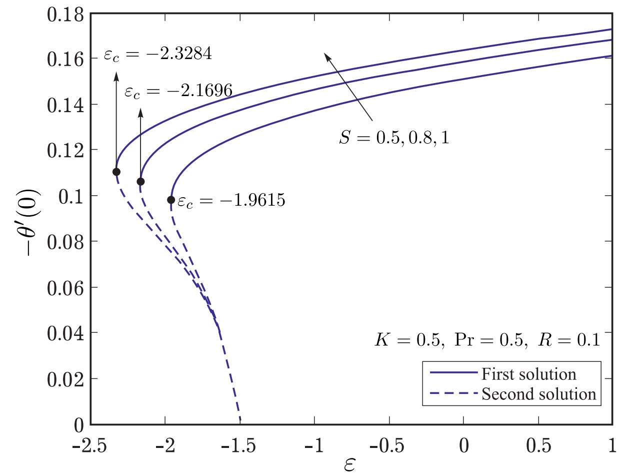

Fig. 3. Local Nusselt number $-\theta'(0)$ for different values of S against ε and fixed $K = 0.5$, $\mathrm{Pr} = 0.5$, and $R = 0.1$.

Figures 4 and 5 show the graphical results of skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ for different values of permeability parameter $K = 0, 0.5, 0.8$. The figures indicate that the dual solutions of the boundary layer flow over a shrinking sheet are attained within distinct ranges of ε, depending on the value of the permeability parameter K. When $K = 0$, the range of ε satisfies the inequality $\varepsilon_c \leqslant \varepsilon \leqslant -1$, where $\varepsilon_c \approx -1.8925$. Conversely, for $K = 0.5$, the range of ε fulfills the inequality $\varepsilon_c \leqslant \varepsilon \leqslant -1.5$, where $\varepsilon_c \approx -2.3284$. While $\varepsilon_c \leqslant \varepsilon \leqslant -1.8$, where $\varepsilon_c \approx -2.5955$ for $K = 0.8$. Observations reveal that only one solution exists for $\varepsilon > -1$ when $K = 0$, $\varepsilon > -1.5$, when $K = 0.5$ and $\varepsilon > -1.8$, when $K = 0.8$. Furthermore, in cases where the shrinking rate is large ($\varepsilon < \varepsilon_c$), no solution exists. In contrast, there are unique solutions for the boundary layer flow over a stretching sheet where $\varepsilon > 0$. From the results, it can be inferred that the skin friction coefficient increases with increasing K due to the adjustments in boundary layer behavior caused by porous medium. Similarly, the presence of a porous medium affects the temperature distribution within the boundary layer. As shown on figure, the local Nusselt number $-\theta'(0)$ increases as K increases, showing an effect of porous medium on heat transfer within the boundary layer.

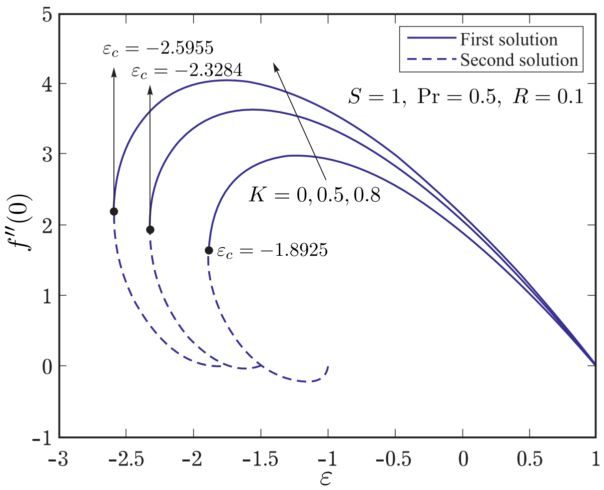

Fig. 4. Skin friction coefficient $f''(0)$ for different values of K against ε and fixed $S = 1$, $\mathrm{Pr} = 0.5$, and $R = 0.1$.

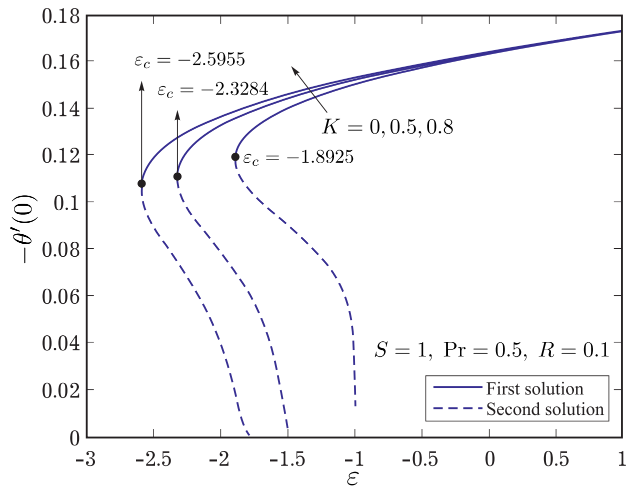

Fig. 5. Local Nusselt number $-\theta'(0)$ for different values of K against ε and fixed $S = 1$, $\mathrm{Pr} = 0.5$, and $R = 0.1$.

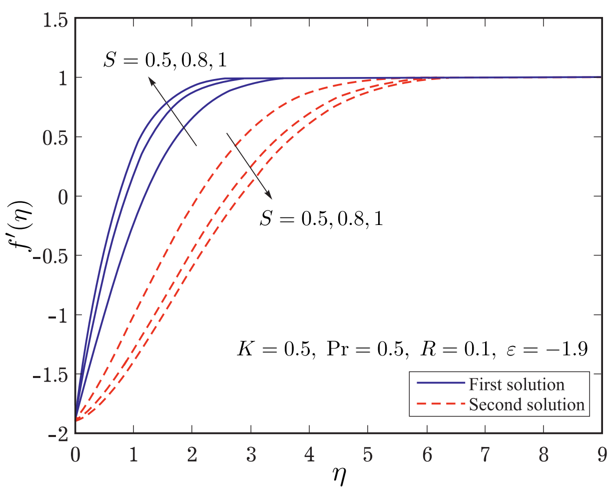

Fig. 6. Velocity profiles for different values of S for fixed $K = 0.5$, $\mathrm{Pr} = 0.5$, $R = 0.1$ and $\varepsilon = -1.9$.

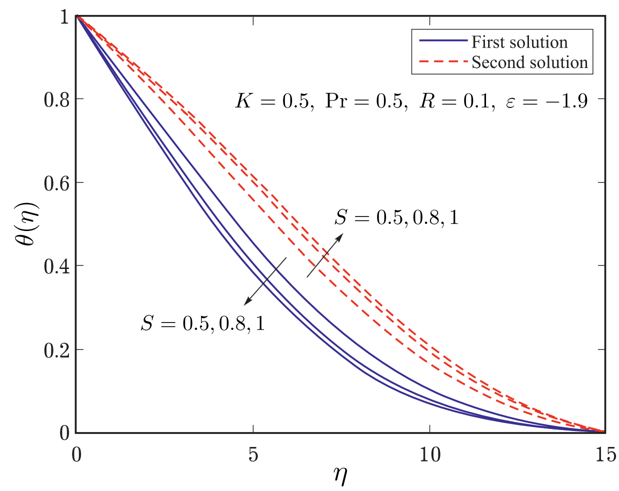

Fig. 7. Temperature profiles for different values of S for fixed $K = 0.5$, $\mathrm{Pr} = 0.5$, $R = 0.1$ and $\varepsilon = -1.9$.

Figures 6–9 present the velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$ for various value of suction parameter S and permeability parameter K, where the dual solution exists. These figures focus on the boundary layer flow over a shrinking sheet where the velocity ratio parameter $\varepsilon < 0$, specifically set to $\varepsilon = -1.9$. Figure 6 shows the impact of suction parameter S on the velocity profile $f'(\eta)$. For the first solution, the fluid velocity increases with the increasing values of S whereas decreases with increasing S in the second solution. Next, Figure 7 illustrates the temperature profile at various S values, where the temperature distribution decreases with increasing S for the first solution while increases with increasing S for the second solution. Meanwhile, Figure 8 presents the effect of permeability parameter K on the velocity profile $f'(\eta)$, showing an increase in the fluid velocity as the K values increases for the first solution while decreases with higher values of K for the second solution. Then, Figure 9 demonstrates the effect of K on the temperature profile, showing a decrease in temperature for the first solution and an increase for the second solution as K increases.

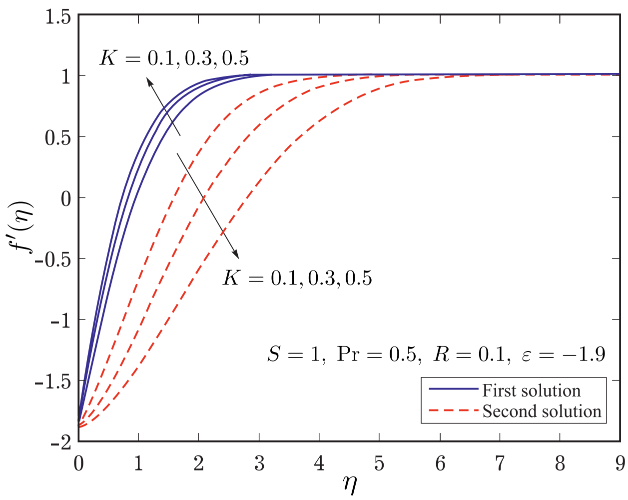

Fig. 8. Velocity profiles for different values of K for fixed $S = 1$, $\mathrm{Pr} = 0.5$, $R = 0.1$ and $\varepsilon = -1.9$.

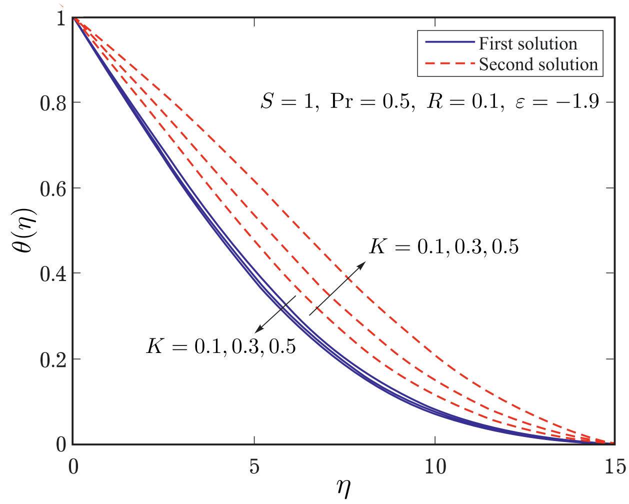

Fig. 9. Temperature profiles for different values of K for fixed $S = 1$, $\mathrm{Pr} = 0.5$, $R = 0.1$ and $\varepsilon = -1.9$.

In Figures 6–9, solid lines represent the first solution, while dashed lines indicate the second solution. Both solution profiles asymptotically satisfy the far-field boundary conditions, validating the numerical results. This also confirms the dual nature of the solutions shown in these figures.

4. Conclusion

In this study, we studied the effect of suction on the steady boundary layer stagnation-point flow and heat transfer in a porous medium near a stretching/shrinking sheet with thermal radiation. The various parameters that affect the flow characteristics, including suction parameter S, permeability parameter K, Prandtl number Pr and thermal radiation parameter R are taken into consideration. The results were compared with the previous studies to verify the validity of the method used, showing consistency in the skin friction coefficient $f''(0)$. We conclude that dual solutions exist for both skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$ for shrinking cases whereas the solution is unique for stretching cases. It was observed, that an increase in the value of S and K leads to higher skin friction coefficient and the heat transfer rates at the surface. The obtained velocity profiles and temperature profiles asymptotically satisfy the far-field boundary conditions for both solutions, confirming the validity of the numerical results and the existence of dual solutions.

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[1] Hiemenz K. Die Grenzschicht an einem in den gleichförmigen Flussigkeitsstrom eingetauchten geraden Kreiszylinde. Dingler's Polytechnical Journal. **326**, 321–324 (1911).

[2] Chiam T. C. Stagnation-point flow towards a stretching plate. Journal of the Physical Society of Japan. **63** (6), 2443–2444 (1994).

[3] Mahapatra T. R., Gupta A. S. Magnetohydrodynamic stagnation-point flow towards a stretching sheet. Acta Mechanica. **152**, 191–196 (2001).

[4] Nazar R., Amin N., Filip D., Pop I. Stagnation point flow of a micropolar fluid towards a stretching sheet. International Journal of Non-Linear Mechanics. **39** (7), 1227–1235 (2004).

[5] Lok Y. Y., Amin N., Pop I. Non-orthogonal stagnation point flow towards a stretching sheet. International Journal of Non-Linear Mechanics. **41** (4), 622–627 (2006).

[6] Reza M., Gupta A. S. Steady two-dimensional oblique stagnation-point flow towards a stretching surface. Fluid Dynamics Research. **37**, 334 (2005).

[7] Lok Y. Y., Pop I., Ingham D. B., Amin N. Mixed convection flow of a micropolar fluid near a non-orthogonal stagnation-point on a stretching vertical sheet. International Journal of Numerical Methods for Heat & Fluid Flow. **19**, 459–483 (2006).

[8] Miklavčič M., Wang C. Y. Viscous flow due to a shrinking sheet. Quarterly of Applied Mathematics. **64**, 283–290 (2006).

[9] Wang C. Y. Stagnation flow towards a shrinking sheet. International Journal of Non-Linear Mechanics. **43** (5), 377–382 (2008).

[10] Fang T. Boundary layer flow over a shrinking sheet with power-law velocity. International Communications of Heat and Mass Transfer. **51** (25–26), 5838–5543 (2009).

[11] Layek G. C., Mukhopadhyay S., Samad Sk. A. Heat and mass transfer analysis for boundary layer stagnation-point flow towards a heated porous stretching sheet with heat absorption/generation and suction/blowing. International communications in Heat and Mass Transfer. **34** (3), 347–56 (2007).

[12] Lin Y., Zheng L., Li B., Ma L. A new diffusion for laminar boundary layer flow of power law fluids past a flat surface with magnetic effect and suction or injection. International Journal of Heat and Mass Transfer. **90**, 1090–1097 (2015).

[13] Viskanta R., Grosh R. J. Boundary layer in thermal radiation absorbing and emitting media. International Journal of Heat and Mass Transfer. **5** (9), 795–806 (1962).

[14] Hossain M. A., Alim M. A., Rees D. A. S. The effect of radiation on free convection from a porous vertical plate. International Journal of Heat and Mass Transfer. **42** (1), 181–191 (1999).

[15] Abd El-Aziz M. Radiation effect on the flow and heat transfer over an unsteady stretching sheet. International Communications in Heat and Mass Transfer. **36** (5), 521–524 (2009).

[16] Mukhopadhyay S. Effect of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. International Journal of Heat and Mass Transfer. **52** (13–14), 3261–3265 (2009).

[17] Hayat T., Qasim M., Abbas Z. Radiation and mass transfer effects on the magnetohydrodynamic unsteady flow induced by a stretching sheet. Zeitschrift für Naturforschung A. **65** (3), 231–9 (2010).

[18] Hayat T., Qasim M. Radiation and magnetic field effects on the unsteady mixed convection flow of a second grade fluid over a vertical stretching sheet. International Journal for Numerical Methods in Fluids. **66** (7), 820–32 (2011).

[19] Hayat T., Qayyum S., Waqas M., Ahmad B. Influence of thermal radiation and chemical reaction in mixed convection stagnation point flow of Carreau fluid. Results in Physics. **7**, 4058–4064 (2017).

[20] Rosali H., Badlilshah M. N., Johari M. A. M., Bachok N. Unsteady boundary layer stagnation point flow and heat transfer over a stretching sheet in a porous medium with slip effects. CFP Letters. **12** (10), 52–61 (2020).

[21] Irfan M., Farooq M. A., Mushtaq A., Shamsi Z. H. Unsteady MHD bionanofluid flow in a porous medium with thermal radiation near a stretching/shrinking sheet. Mathematical Problems in Engineering. **2020** (1), 8822999 (2020).

[22] Abbas M. A., Bhatti M. M., Rashidi M. M. Heat transfer on Magnetohydrodynamic stagnation point flow through a porous shrinking/stretching sheet: a numerical study. Thermal Science. **24** (2 Part B), 1335–1344 (2020).

[23] Jamaludin A., Nazar R., Pop I. Mixed convection stagnation-point flow of cross fluid over a shrinking sheet with suction and thermal radiation. Physica A: Statistical Mechanics and its Applications. **585**, 126398 (2022).

[24] Bhattacharyya K. Boundary layer flow and heat transfer over an exponentially shrinking sheet. Chinese Physics Letters. **28** (7), 074701 (2011).

Вплив всмоктування на стаціонарний потік у точці застою прикордонного шару та передачу тепла в пористому середовищі до листа, що розтягується/стискається, з тепловим випромінюванням

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У цьому дослідженні вивчається вплив всмоктування та теплового випромінювання на постійний потік в точці застою прикордонного шару та передачу тепла в пористому середовищі до листа, що розтягується та стискається. Основні диференціальні рівняння в частинних похідних (PDE) перетворюються на нелінійні звичайні диференціальні рівняння (ODE) за допомогою перетворення подібності. Потім ці рівняння розв'язуються чисельно за допомогою функції BVP4C у MATLAB, а числові результати представлені у вигляді графічних ілюстрацій. Дослідження виявляє існування дуальних розв'язків та єдиних розв'язків у певних діапазонах параметра відношення швидкостей ε залежно від задіяних параметрів. Крім того, виявлено, що як коефіцієнт поверхневого тертя, так і локальне число Нуссельта зростають із збільшенням всмоктування S та параметра проникності K. Крім того, вищі значення числа Прандтля Pr та параметра теплового випромінювання R призводять до тоншого теплового прикордонного шару.

Ключові слова: *всмоктування; потік в точці застою; теплообмін; пористе середовище; розтягування/стискання листа; теплове випромінювання.*