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ENHANCEMENT OF MEDICAL MRI IMAGES BASED ON FRACTAL OPERATORS

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Abstract. This article presents the research of texture enhancement algorithms on medical images. Medical MRI brain scans contain large areas with low level grey colors that carry useful information for doctors. Texture improvement allow to highlight large grey areas on images for future detailed recognition. Based on the study of existing texture enhancement methods, it was determined that fractal operators are effective for processing medical images. The mathematical framework of fractal operators is presented based on the approximation equation of the Grünwald-Letnikov fractional derivatives. The creation of fractal differential masks and the algorithm of masks usage for image enhancement are described based on this equation. The approximation error of the Grunwald-Letnikov derivative is calculated in comparison with the analytical value of the Grunwald-Letnikov derivative. The algorithm based on the fractal derivative shows improvements in image parameters such as contrast, correlation, energy, and homogeneity compared to the original image parameters. A comparison of the results of the algorithm based on the fractal differential with other algorithms for improving the texture of images is also given. It is concluded that the fractal differential algorithm is well-suited for MRI image enhancement tasks, unlike other algorithms, both in visual comparisons and numerical metrics, and thus can be applied to solve real-world problems.

Keywords: Medical images, Magnetic Resonance Imaging (MRI), fractal operators, algorithms, Python, image enhancement.

Introduction

Medical images play a crucial role in modern medicine, allowing doctors to obtain important information about patients' health. They provide the ability not only to visualize internal or external structures of the body, such as organs and tissues, but also to perform more accurate diagnoses of diseases and abnormalities. With medical imaging, doctors can timely identify issues and develop individualized treatment plans, which is critical for improving the quality of life and treatment outcomes for patients.

Magnetic Resonance Imaging (MRI) is one of the most common medical imaging methods that provides detailed images of internal organs and tissues. The high resolution and contrast of MRI images allow for the detection of pathological changes at early stages. However, the quality of MRI images can bereduced by various artifacts and noise, which requires the use of image enhancement techniques to increase their diagnostic value.

There are various methods for enhancing MRI images to obtain clearer, more detailed, and informative images. Improvements can be made to equipment and signal acquisition methods to ensure better sensitivity and reduced noise in images. Additionally, minimizing artifacts on MRI scans can be achieved by better stabilizing the patient in the MRI machine during scanning. However, these enhancement methods are not always applicable. In practice, images of different resolutions are often analyzed, frequently displaying visible artifacts, noise, and blurred contours. Considering such image shortcomings, it is better to use computer processing algorithms to improve image quality [1].

Problem statement

The object of the study is the textures of medical MRI images.

The subject of the study is the methods and algorithms for improving MRI image textures using fractional derivatives.

The goal of the work is to apply mathematical tools, develop algorithmic and software solutions for improving medical MRI image textures using fractional derivatives, and enhance image processing and analysis quality. To achieve this goal, the following sub-tasks can be identified:

- Review theoretical aspects of fractional differential and their usage in signal and image processing.
- Develop an algorithm for enhancing image textures using fractional derivatives.
- Develop a software implementation of the texture enhancement algorithm.
- Conduct experimental studies to evaluate the effectiveness of the developed algorithm.
- Compare the results of improving image textures using the fractal derivative with other methods.
- Analyze the obtained results and provide conclusions about the feasibility of using the fractal derivative to improve image textures.

Scientific novelity of the work lies in the development of an algorithm and software for improving the textures of medical MRI images using fractional derivatives. The developed algorithmic and software tools will enhance the quality of medical MRI images for further analysis by doctor.

The practical significance of the work lies in applying the software and algorithmic tools for enhancing the textures of medical MRI images using fractional derivatives in various fields of medicine. This method improves MRI image quality, which contributes to more accurate diagnosis, better visual representation, and enhancement of automated systems for recognizing and classifying pathologies in MRI images.

Review of Modern Information Sources on the Subject of the Paper

It can be observed that most existing methods for texture enhancement are based on integer-order derivatives. Such algorithms include the Laplacian filter, the Sobel algorithm, and the Pruitt algorithm [2]-[4]. These algorithms rely on classical differential operators and enhance images by strengthening edges, contours, and other details. Edge and contour enhancement is achieved by applying filters to the original image. The filter calculates the change in brightness between neighboring pixels — the gradient, the higher gradient values indicating the presence of edges. However, these algorithms have certain drawbacks: insufficient flexibility to highlight details of different scales, which may lead to the loss of some structures in the image; potential ineffectiveness in processing thin and complex structures; and possible orientation sensitivity, resulting in incorrect edge detection depending on their orientation. There is also a method for image enhancement based on wavelet transformation [13]. This method involves decomposing the image into different frequency components. Such decomposition allows isolating details of different scales at different frequency levels and subsequently processing each frequency separately, thus improving image quality. The main drawbacks of the wavelet method for image enhancement include the complexity of configuration, computational intensity, potential information loss, and insufficient effectiveness for very fine details. Work [16] implemented one of the fractal methods and investigated its impact on automatic segmentation of intracerebral hemorrhages in CT scans using artificial neural networks.

Returning specifically to the specifics of medical images, it is important to note that MRI images are characterized by relatively small, thin structures of various scales, as well as large smooth areas where the color does not change significantly. Additionally, the structures on the MRI image are usually self-similar, they have the properties of self-repetition on different scales, self-similarity means that there is similarity between certain pixels and areas of pixels in terms of gray color. Given the above-described characteristics, it is fractal operators that are suitable for processing MRI images.

Fractional differential operators (fractals derivatives) are generalizations of classical differential operators. They allow differentiation of non-integer orders, which opens new possibilities for signal and image analysis and processing. The use of fractional differential operators allows better control over the enhancement of different frequency components of an image, which is particularly useful for improving textures and highlighting details. Different degrees of enhancement allow to open various details of the structure by adjusting the enhancement level. This article uses fractional differential operators to improve image quality and increase contrast.

Objectives and Problems of Research

The fractal differential mask save low frequencies in smooth regions of the image and high frequencies where there are significant changes in intensity, thereby enhancing texture details in areas where intensity changes are less noticeable. Since noise is characterized by high frequency, applying a high-frequency filter to an image not only enhances the elements of interest but also significantly increases the level of noise. The proposed by Yui-Fei, Pu, and others [5] allow to bypass this shortcoming, based on fractional-order derivatives (also known as YuiFeiPU operators). The article suggests several methods for constructing such masks based on Riemann-Liouville and Grünwald-Letnikov derivatives. However, the literature review did not reveal studies on the impact of such enhancement using fractal operators for MRI images in the context of comparison with other algorithms. This work is also dedicated to implementing algorithms for processing MRI images using fractal operators.

Main Material Presentation

The fractional differentiation framework

This chapter describe the necessary theoretical background for fractional differential [17] usage in signal processing, also the approaches approaches underlying fractional differentiation are explained.

There are several ways to determine the derivative of fractional order. Common generalized derivatives include the Riemann-Liouville, Grünwald-Letnikov, and Riesz derivatives.

The Riemann-Liouville fractional derivative of order v (0≤v≤n) can be expressed by the formula:

$$\mathrm{D}^{\mathrm{V}}_{\mathrm{R\text{-}L}}S(x)=\frac{d^n}{dx^n}\frac{d^v}{[d(x-a)]^v}s(x)_{G-L}=\sum_{k=0}^{N-1}\frac{(x-a)^{k-v}S^{(k)}*(a)}{\Gamma(k-v+1)}+\frac{1}{\Gamma(n-v)}\int_a^x\frac{s^{(n-}(\zeta)}{(x-\zeta)^{v-n+1}}d\zeta\,, \tag{1}$$

where n is the smallest digital number bigger than v.

Fractional derivative of order v based on Grünwald-Letnikov fractional differential is expressed by the formula:

$$\mathrm{D}^{\mathrm{V}}_{\mathrm{G\text{-}L}}S(x)=\frac{d^v}{[d(x-a)]^v}s(x)_{G-L}=\lim_{N\to\infty}\left\{\frac{\frac{x-a}{N}}{\Gamma(-v)}\sum_{k=0}^{N-1}\frac{\Gamma(k-v)}{\Gamma(k+1)}*s\left(x-k\left(\frac{x-a}{N}\right)\right)\right\}, \tag{2}$$

Where signal length s(x) is in [a,x], v-any real numbers (including fractal). $\mathrm{D}^{\mathrm{V}}_{\mathrm{R\text{-}L}}$ - the Grünwald-Letnikov fractional differential operator. Γ - the gamma function, an extension of the factorial concept to real numbers.

From formula (2) fractional differential can be expressed as follows:

$$\Delta^v s(x)=\frac{1}{\Gamma(-v)}\sum_{k=0}^{N-1}\frac{\Gamma(k-v)}{\Gamma(k+1)}*s\left(x-k\left(\frac{x-a}{N}\right)\right), \tag{3}$$

Formula (3) represents the fractional differential for the signal s(x), specifically the numerical approximation of the fractional differential.

The mathematical framework for constructing the mask overview

Based on the above definitions, the main steps for applying the fractional operator to construct fractional differential masks, as described by the authors in [5], and the algorithm for their use are presented.

According to formula (3) assume that a=0, let's divide signal s(x) on N equal parts, the duration belongs to interval [0, x]. N+1 nodes obtained, which in context of image processing allows the processing of N+1 pixels.

$$\begin{cases} S_N = S(0) \\ \quad \dots \\ S_k = S(x - ks/N), \\ \quad \dots \\ S_0 = S(x) \end{cases} \tag{4}$$

For large enough N, formula (3) can be simplified to the form:

$$\frac{d^v}{dx^v} = \frac{x^{-v}N^{-v}}{\Gamma(-v)} \sum_{k=0}^{N-1} \frac{\Gamma(k-v)}{\Gamma(k+1)} * s\left(x - \frac{kx}{N}\right) = \frac{x^{-v}N^{-v}}{\Gamma(-v)} \sum_{k=0}^{N-1} \frac{\Gamma(k-v)}{\Gamma(k+1)} * s_k \,, \tag{5}$$

The signal value can be rewritten as following s(x+(vx/2N)-(kx/N)), according to formula (5) it will take the form:

$$\frac{d^v}{dx^v} = \frac{x^{-v}N^{-v}}{\Gamma(-v)} \sum_{k=0}^{N-1} \frac{\Gamma(k-v)}{\Gamma(k+1)} * s\left(x + \frac{vx}{2N} - \frac{kx}{N}\right), \tag{6}$$

Comparing formula (5) with formula (6) we can say that in (6) the signal value s(x) is introduced at non-node points, except for v=0, +-2, +-4… Substituting the values v=-2, 0, 2 into signal s(x+(vx/2N)-(kx/N)), neighboring signal points obtained, such as: s(x+(x/N)-(kx/N)), s(x-(kx/N)), s(x-(x/N)-(kx/N)). With these 3 neighboring signal points, Lagrange interpolation can be used to create polynomial interpolation.

The interpolation polynomial for the signal S(x) has the following form:

$$\begin{aligned} S(\zeta) = {} & \frac{(\zeta - x + \frac{kx}{N})(\zeta - x + \frac{x}{N} + \frac{kx}{N})}{\frac{2x^2}{N^2}} s(x + \frac{x}{N} - \frac{kx}{N}) \\ & - \frac{(\zeta - x + \frac{x}{N} + \frac{kx}{N})(\zeta - x + \frac{x}{N} + \frac{kx}{N})}{\frac{2x^2}{N^2}} s(x - \frac{kx}{N}) \qquad , \\ & + \frac{(\zeta - x + \frac{x}{N} + \frac{kx}{N})(\zeta - x + \frac{kx}{N})}{\frac{2x^2}{N^2}} s(x - \frac{x}{N} - \frac{kx}{N}) \end{aligned} \tag{7}$$

Next, considering that ξ=x+(vx/2N)-(kx/N), the signal value will take the following gorm:

$$S\left(x+\frac{vx}{2N}-\frac{kx}{N}\right)=\left(\frac{v}{4}+\frac{v^2}{8}\right)s\left(x+\frac{x}{N}-\frac{kx}{N}\right)+\left(1-\frac{v^2}{4}\right)s\left(x-\frac{kx}{N}\right)+\left(\frac{v^2}{8}-\frac{v}{4}\right)s\left(x-\frac{x}{N}-\frac{kx}{N}\right)$$
$$=\left(\frac{v}{4}+\frac{v^2}{8}\right)s_{k-1}+\left(1-\frac{v^2}{4}\right)s_k+\left(\frac{v^2}{8}-\frac{v}{4}\right)s_{k+1} \quad (8)$$

,

Substituting formula (8) into (6) the following expression received:

$$\frac{d^v}{dx^v}=\frac{x^{-v}N^{-v}}{\Gamma(-v)}\sum_{k=0}^{N-1}\frac{\Gamma(k-v)}{\Gamma(k+1)}*\left[s_k+\frac{v}{4}(s_{k-1}-s_{k+1})+\frac{v^2}{8}(s_{k-1}-s_k+s_{k+1})\right], \quad (9)$$

In computer vision, objects and filters have limited values. Also a pixel in an image has s specific color value that is limited, the smallest distance in the two dimensional image between two pixels for x and y coordinate is one pixel, Therefore, the length of the signal can be defined as the size of the image matrix, where x is in range [0,x], for y is in range [0,y], it can be said that the distance in the x and y coordinates: $h_x=(x/N)=1$ and $h_y=(x/N)=1$ respectively, and the biggest value to which the distance can be divided being $N_x=(x/h_x)=[x]$, $N_y=(x/h_y)=[y]$, [5].

For k=n<=N-1, from formula (9) the partial approximation for n+2 for negative values along x and y coordinates can be expressed as:

$$\frac{d^v s(x,y)}{dx^v}=\left(\frac{v}{4}+\frac{v^2}{8}\right)s(x+1,y)+\left(1-\frac{v^2}{4}=\frac{v^3}{8}\right)s(x,y)+\frac{1}{\Gamma(-v)}$$
$$*\sum_{k=1}^{n-2}\left[\frac{\Gamma(k-v+1)}{(k+1)!}*\left(\frac{v^2}{8}-\frac{v}{4}\right)+\frac{\Gamma(k-v)}{k!}*\left(1-\frac{v^2}{4}\right)+\frac{\Gamma(k-v-1)}{(k-1)!}*\left(\frac{v^2}{8}-\frac{v}{4}\right)\right]$$
$$*s(x-k,y)+\left[\frac{\Gamma(n-v=1)}{(n=1)!\Gamma(-v)}*\left(1-\frac{v^2}{4}\right)+\frac{\Gamma(n-v-2)}{(n-2)!\Gamma(-v)}*\left(\frac{v}{4}+\frac{v^2}{8}\right)\right]$$
$$*s(x-n+1,y)+\frac{\Gamma(k-v+1)}{(k+1)!}*\left(\frac{v^2}{8}-\frac{v}{4}\right)*s(x-n,y) \quad , \quad (10)$$

$$\frac{d^v s(x,y)}{dy^v}=\left(\frac{v}{4}+\frac{v^2}{8}\right)s(x,y+1)+\left(1-\frac{v^2}{4}=\frac{v^3}{8}\right)s(x,y)+\frac{1}{\Gamma(-v)}$$
$$*\sum_{k=1}^{n-2}\left[\frac{\Gamma(k-v+1)}{(k+1)!}*\left(\frac{v^2}{8}-\frac{v}{4}\right)+\frac{\Gamma(k-v)}{k!}*\left(1-\frac{v^2}{4}\right)+\frac{\Gamma(k-v-1)}{(k-1)!}*\left(\frac{v^2}{8}-\frac{v}{4}\right)\right]$$
$$*s(x,y-k)+\left[\frac{\Gamma(n-v=1)}{(n=1)!\Gamma(-v)}*\left(1-\frac{v^2}{4}\right)+\frac{\Gamma(n-v-2)}{(n-2)!\Gamma(-v)}*\left(\frac{v}{4}+\frac{v^2}{8}\right)\right]$$
$$*s(x,y-n+1)+\frac{\Gamma(k-v+1)}{(k+1)!}*\left(\frac{v^2}{8}-\frac{v}{4}\right)*s(x,y-n) \quad , \quad (11)$$

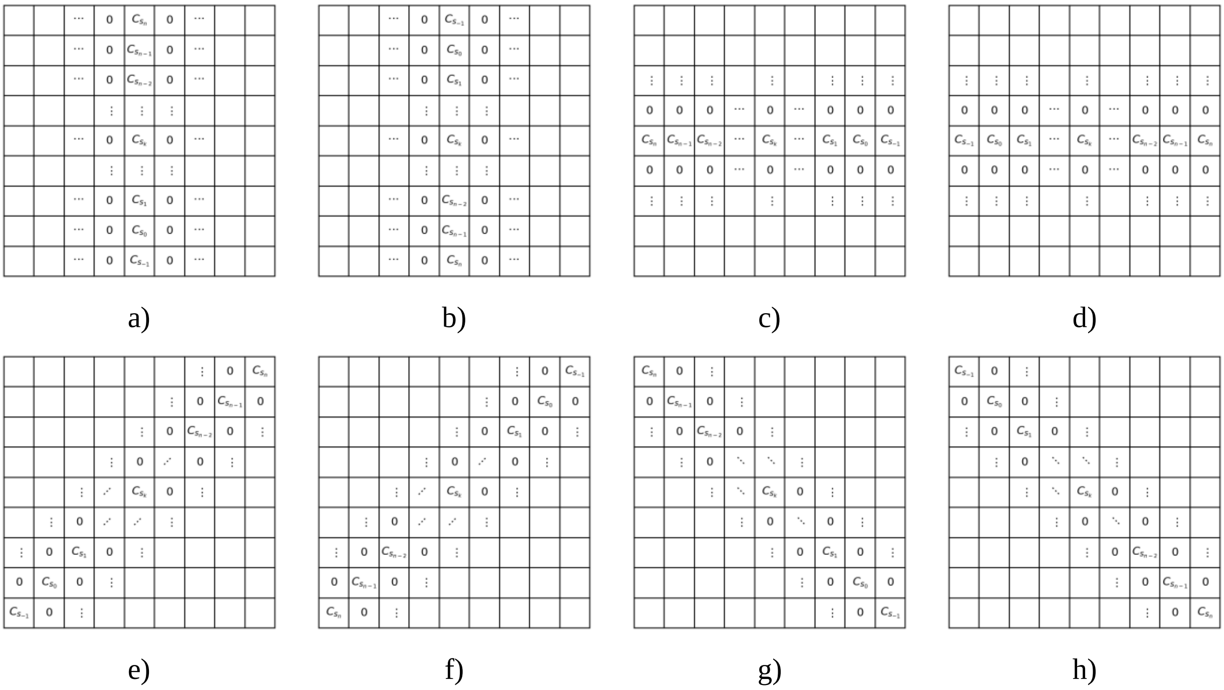


Fig. 1. Masks for the eight directions: (a) W1 negative along the x-axis; (b) W2 positive along the x-axis; (c) W3 negative along the y-axis; (d) W4 positive along the y-axis; (e) W5 left-upward diagonal; (f) W6 right-downward diagonal; (g) W7 right-upward diagonal; (h) W8 left-downward diagonal [5]

In Fig.1, masks corresponding to the eight symmetrical directions are shown(a) W1 negative along the x-axis; (b) W2 positive along the x-axis; (c) W3 negative along the y-axis; (d) W4 positive along the y-axis; (e) W5 left-upward diagonal; (f) W6 right-downward diagonal; (g) W7 right-upward diagonal; (h) W8 left-downward diagonal. These masks allow to calculate the fractional differential in eight symmetrical directions. The presence of these eight masks also creates anti-rotation capability, enabling same processing results regardless of the texture's position on the image. At Fig.1 $C_{s_{-1}}$ is the pixel mask coefficient $S_{-1} = s(s + s/N)$ i C_{s_0} is the pixel coefficient $S_0 = s(x)$. Formula (10) and (11) also show that when k->n=1 then a mask size of 3×3 is obtained, when k->n=3 then mask of size 5×5, when k->n=2m-1, this can be expressed by the formula n = (2m+1) × (2m+1), where n usually add numbers.

According to formulas (10) and (11) mask coefficients W1-W8 are calculated using the formulas:

$$\begin{cases} C_{s_{-1}} = \dfrac{v}{4} + \dfrac{v^2}{8} \\ C_{s_0} = 1 - \dfrac{v^2}{2} + \dfrac{v^3}{8} \\ C_{s_k} = \dfrac{1}{\Gamma(-v)}\left[\dfrac{\Gamma(k-v+1)}{(k+1)!}\left(\dfrac{v}{4}+\dfrac{v^2}{8}\right) + \dfrac{\Gamma(k-v)}{k!}\left(1-\dfrac{v^2}{4}\right) + \dfrac{\Gamma(k-v-1)}{(k-1)!}\left(-\dfrac{v}{4}+\dfrac{v^2}{8}\right)\right] \end{cases}, \quad (12)$$

The algorithm error estimation

The mathematical framework presented in the previous section is based on the approximate calculation of the fractal differential value. The differential value for the approximate calculation and the analytical value will differ. This section presents the calculations of the relative error for the approximation described in formula (9). The relative error from fractional derivatives can be calculated using the formula:

$$\partial = \left| \frac{\left.\frac{d^v s(x)}{dx^v}\right|_{algo} - \frac{d^v s(x)}{dx^v}}{\frac{d^v s(x)}{dx^v}} \right|, \tag{13}$$

In formula (13) $\left.\frac{d^v s(x)}{dx^v}\right|_{algo}$ algorithmic value and $\frac{d^v s(x)}{dx^v}$ the fractional differential analytical value. Algorithmic value can be expressed by the formula from [5]:

$$\left.\frac{d^v x^p}{dx^v}\right|_{algo} = \frac{x^{-v} N^{-v}}{\Gamma(-v)} \sum_{k=0}^{N-1} \frac{\Gamma(k-v)}{\Gamma(k+1)} \left[\begin{aligned} &\left(x - \frac{kx}{N}\right)^p + \frac{v}{4}\left(\left(x + \frac{x}{N} - \frac{kx}{N}\right)^p - \left(x - \frac{x}{N} - \frac{kx}{N}\right)^p\right) \\ &+ \frac{v^2}{8}\left(\left(x + \frac{x}{N} - \frac{kx}{N}\right)^p - 2\left(x - \frac{kx}{N}\right)^p + \left(x - \frac{x}{N} - \frac{kx}{N}\right)^p\right) \end{aligned} \right], \tag{14}$$

The fractional differential analytical value can be expressed by the formula:

$$\frac{d^v x^p}{dx^v} = \frac{\Gamma(p+1) x^{p-v}}{\Gamma(p-v+1)}, p > -1, \tag{15}$$

Substituting the values x=1, p=1.5, N=[0;10], v=[0;1] the relative error σ for fractals of different orders is calculated:

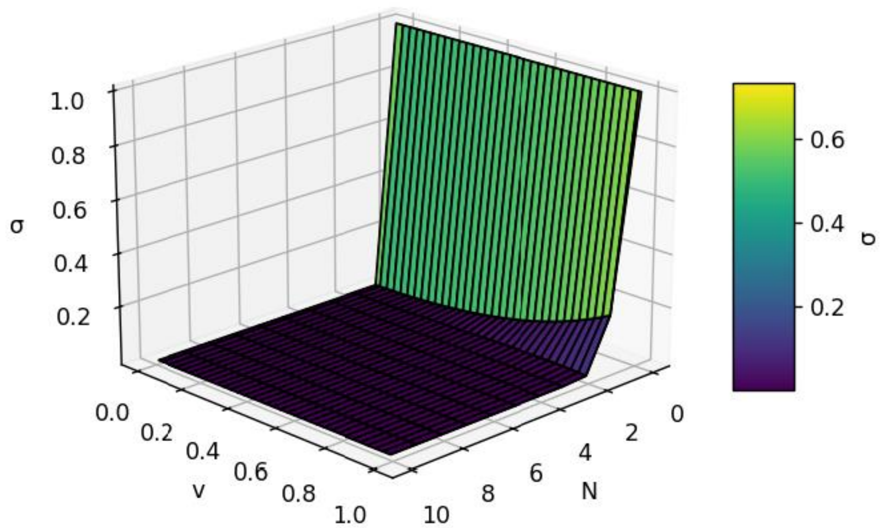


Fig. 2. The graph of the dependence of the error σ on the parameters v and N.

On Fig. 2 displayed the dependence of the error σ to the fractional order value v, and value N. The error value σ is big for small N values and almost consistently small for bigger N values, also the changes of the value v have little impact on the error value. If n-mask size and assuming that N=n the error calculated for different mask values:

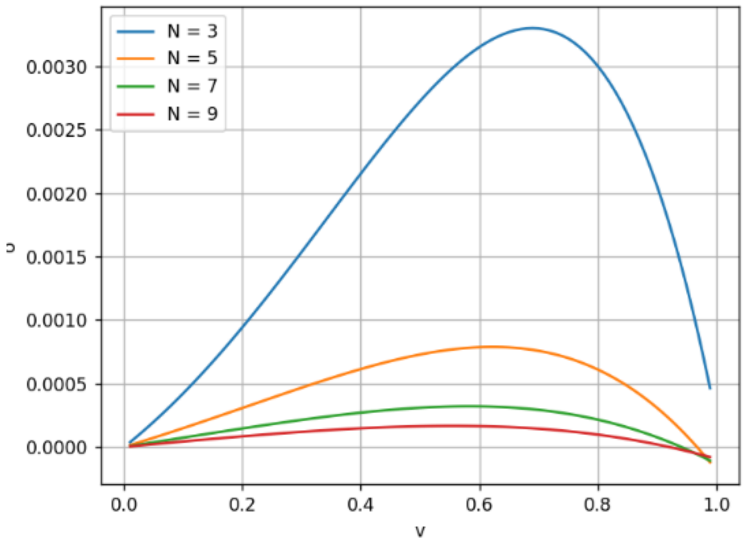


Fig. 3. The graph of the dependence of the relative error σ to the fractional order v

On Fig. 3 displayed the dependence between σ and v, small masks have the worst precision. A large value of N allow to decrease error value, but not significantly. A mask size N=7 is good enough for use in calculations and allow relatively quick computation.

Software implementation

The software implementation of the algorithm for improving the textures of medical images was performed in the Python programming language[6]. OpenCV[8] as well as scipy[7] and numpy libraries were used for image processing.

The sequence of the algorithm is as follows: The first step is to read the input image in greyscale. As a result of reading, the two-dimensional array of pixels received, where the color of each pixel is coded by 256 bits. Next, based on Fig.1 eight matrices W1-W8 created. The matrix dimensions was selected as 7x7 according to the arguments given in the section above and estimates according to Fig. 3. The coefficients of the mask are calculated according to the formula (12), where k is the corresponding index of the coefficient and depends on the size of the matrix, v is the value of the fractal order. It is important to note that coefficient values $C_{s_{-1}}$ and C_{s_0} are calculated according to partial formulas, all other C_{s_k} are calculated according to general formula for calculating coefficients. The next step is convolution [10] operations on the images. The operation of convolution of the original image with each of the eight masks W1-W8 is performed by using the convolution function from scipy library. As a result of these operations, the eight images, respectively, in eight directions received, each image showing improvements in each separate direction. The next step is to combine all eight matrices into one, summing all the matrices and then normalizing the combined matrix, since when summing the gray values can go beyond [0,255] values. Save the resulting image into file.

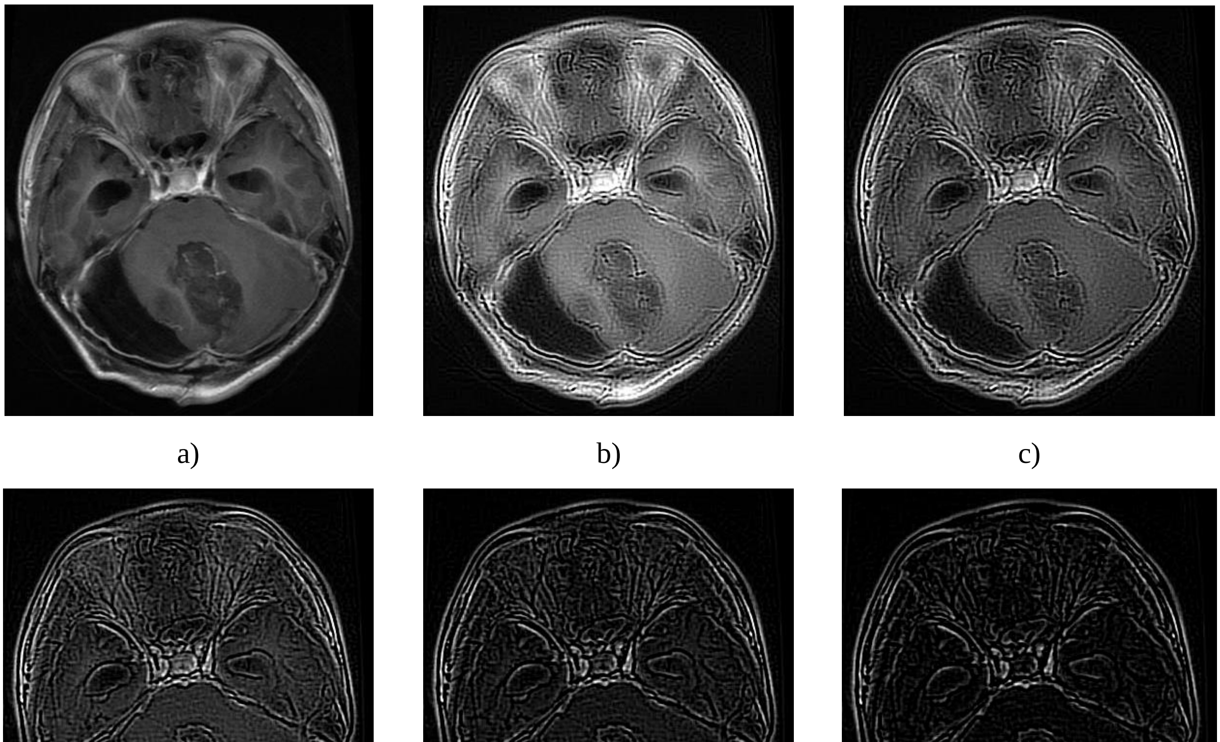
An important element of research is the selection of a set of input data, the main problems are: the data accessibility, as medical data contain confidential information; the data representability; image quality; format compatibility. An anonymous (depersonalized) set from the resource[10] containing hundreds of brain MRI images in jpg format was chosen for our research, but our algorithm can read images of any extensions and various sizes. Most of the images have a size of 512x512 pixels, which is more than enough to analyze the details. It is also worth noting that the described algorithm can process images of any size; however, this will affect the algorithm's performance, as the convolution operation is quite resource-intensive.

Results and Discussion

Visual comparison

Applying the software implementation of the algorithm for improving medical images for brain MRI, in the interval from v[0.5;0.9] with a mask of size 7x7, improved MRI images were obtained Fig. 4. (b), (c), (d), (e) , (f). The images shows that the value of the order of differentiation v acts as the degree of image improvement, the more the value of v increases, the more gray colors disappear from the image, and as a result, the actual loss of gray areas of the image is obtained, while the edge areas become more visible. For the value v=0.5, on the contrary, the gray areas are strengthened, they became more visible, at the same time, the edges of the areas are quite strongly defined.

According to the visual comparison, it can be concluded that the fractal differential mask is suitable for improving images. Changing the value of v allows to change the degree of visibility of certain areas. Additionaly, it is also observed that neighboring levels of order v look very similar, which means that there is a continuous interpolation of the fractional differential for derivatives of neighboring orders.



a) b) c)

d) e) f)

Fig. 4. Brain MRI (a) original image, (b) improved by fractional derivative of order 0.5, (c) improved by fractional derivative of order 0.6, (d) fractal derivative of order 0.7, (e) fractal derivative of order 0.8, (f) fractal derivative of order 0.9

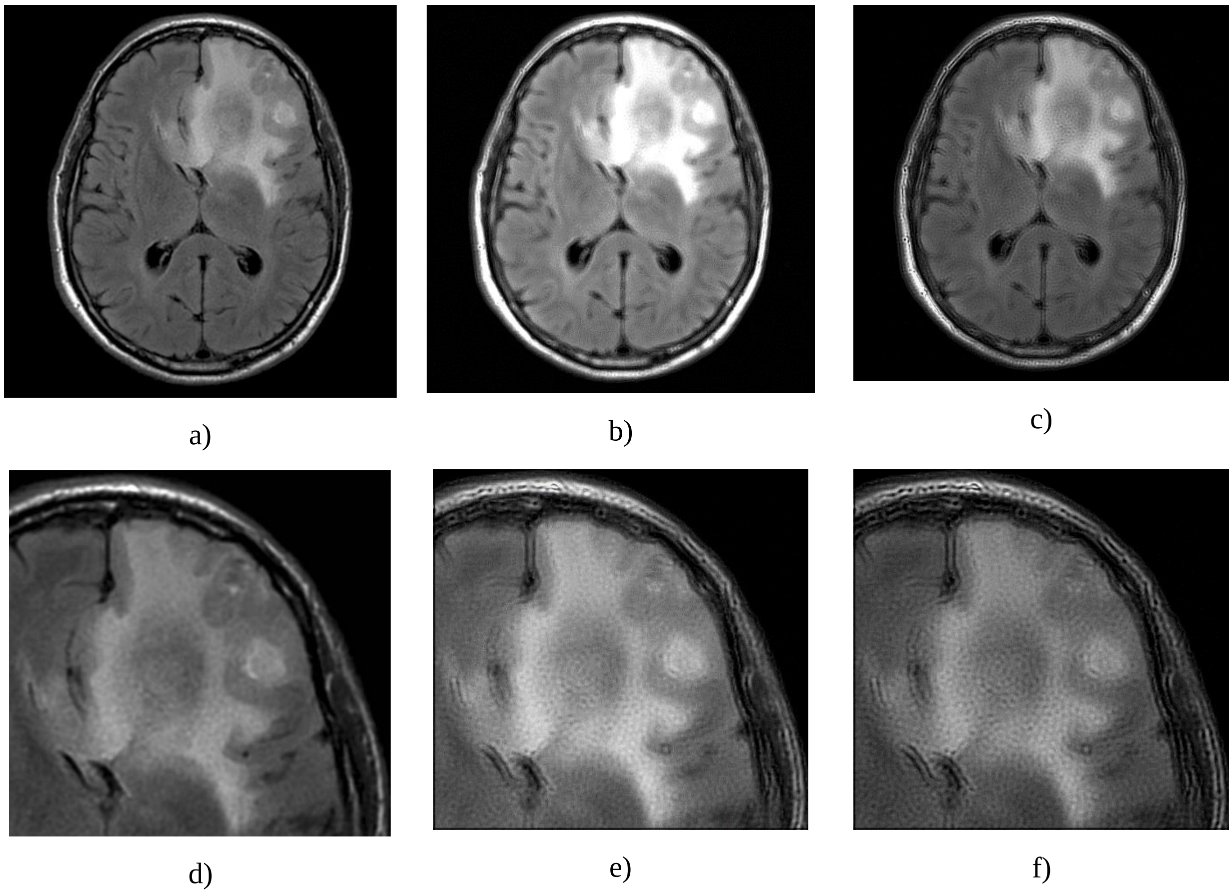


Fig. 5. Brain MRI (a) original image, (b) enhanced with fractional derivative of order 0.5, (c) enhanced with fractional derivative of order 0.6, (d) upper right part of image (a), (e) enhanced image (d) with fractal derivative of order 0.5 , (f) crossed image (d) fractal derivative of order 0.6

Let's also consider Fig.5 for comparison, it shows both the full MRI image (a) and its enhancements (b), (c), and the upper right part of the image (d) and its enhancements (e), (a). Reviewing the enhanced images using the fractional derivative, it can be concluded that the enhancement occurs regardless of the size and structure of the image. This implies that the fractional differential algorithm effectively handles enhancement at different image scales.

Image parameters comparation

To compare the parameters of images, the Gray-Level Co-Occurrence Matrix (GLCM) [11] will be used. This mathematical tool is used for texture analysis and represents a table describing how frequently pixel pairs occur in the image at a certain distance and in a particular direction. By constructing this matrix for the original image Fig. 4 (a) as well as for the enhanced images based on the fractional mask algorithm, analytical parameters such as contrast, correlation, energy, and homogeneity of the image can be obtained.

Contrast – measure difference of intensity between pixel pair, reflecting the degree of local variations within the image.

$$contrast = \sum_{i,j} (i-j)^2 P(i,j) , \tag{16}$$

Formula (16) allow to calculate contrast, P(i,j) GLCM matricx coefficient, i,j – indices of matrix P.

Correlation – measure how the value of pixels linked with their neighbors across whole image.

$$correlation = \sum_{i,j} \frac{(i-\mu_i)(j-\mu_j)p(i,j)}{\sigma_i \sigma_j}, \quad (17)$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(p(i,j)-\mu)^2}, \quad (18)$$

Formula (17) used for calculation correlation value between pixels, P(i,j) GLCM matrix coefficient, i,j –matrix indices, μ_i i μ_j - average value in matrix row and column, $\sigma_i \sigma_j$ – standard deviations thatcalculated by the formula (18).

Energy - measures the texture of an image by showing the sum of the squares of the GLCM elements. High energy values indicate the presence of regular textures (19).

$$energy = \sum_{i,j} P(i,j)^2, \quad (19)$$

Homogeneity – determines the closeness of the GLCM element distribution to the GLCM diagonal. High homogeneity values indicate textures with little variation in gray levels (20).

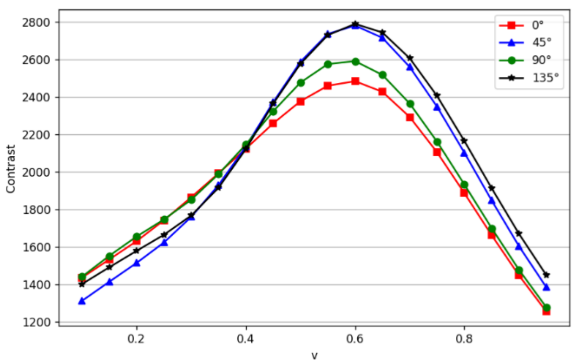
$$h = \sum_{i,j} \frac{p(i,j)}{\sigma_i \sigma}, \quad (20)$$

Let's construct a matrix for the original image with the following input parameters: a pixel distance of 5 and angles of 0, 45, 90, and 135 degrees. Table 1 shows the gray-level matrix parameters for different angles of the original image.

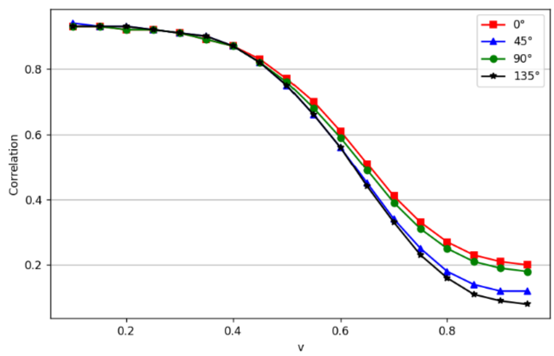
Table 1

Analytical Data of Gray-Level Matrix for Original Image

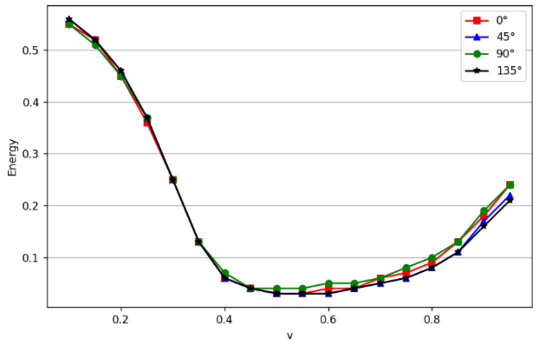
Angle	0	45	90	135
Contrast	781.13	721.47	778.21	763.37
Correlation	0.76	0.78	0.76	0.77
Energy	0.06	0.06	0.06	0.06
Homogeneity	0.22	0.22	0.23	0.22



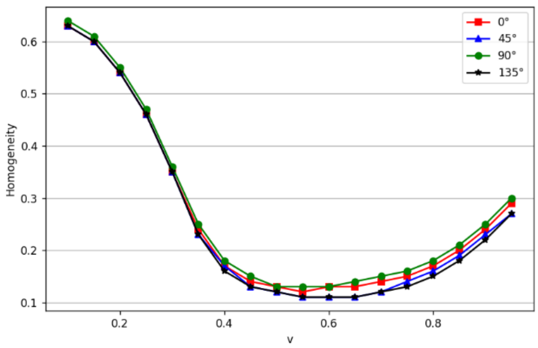
a)ii



b)



c)



d)

Fig. 6. The graphs for contrast (a), correlation(b), energy(c), homogeneity(d) for four angles in dependence from parameter v

Figure 5 shows the dependence of the value of: contrast (a), correlation (b), energy (c) and homogeneity (d) from the value of the v-fractional derivative. The values of contrast, correlation, energy and homogeneity are given for four angles of 0, 45, 90, 135 degrees. All four curves show the same interdependence from the value of v, which means that the change in the value of contrast, correlation, energy, and homogeneity occurred uniformly in all directions.

Let's review each graph separately. In Fig. 5, graph (a) shows that the contrast of the image increases and reaches its peak between the values of v = [0.4, 0.7], a high contrast value indicates that the image becomes clearer and the textures are more visible. Comparing the top contrast value with the contrast value in Table 1 for the original image, there is a significant improvement, approximately 3-4 times. Moving on to correlation in Fig. 5, graph (b), the correlation significantly decreases as the value of v increases, this means that pixel intensities at a certain distance are less likely to have a linear dependency, indicating a nonlinear enhancement of pixel color values. Compared to the original image, the correlation also decreases. The energy values in Fig. 5, graph (c) show the lowest values also in the range of v = [0.4, 0.7], which means that the texture changes in the image become more irregular and inhomogeneous, making them sharper. Comparing the energy of the fractal differential images with the original image, we also note that it decreases for the enhanced image. The fourth indicator is the homogeneity of the image, shown in Fig. 5, graph (d). Homogeneity shows the lowest values in the range of v = [0.4, 0.7], indicating that the image has a less uniform structure. A less uniform structure implies increased sharpness.

According to the above analysis of contrast, correlation, energy and homogeneity values, it can be concluded that the values of the fractional derivative parameter v in the range v = [0.4,0.7] are the most optimal for use, which coincides with the visual comparisons above.

Comparison of result analysis with other algorithms

This chapter presents a comparison of the fractal differential algorithm with other image enhancement algorithms. The comparison includes algorithms commonly used in the field of computer vision, such as the nonlinear image enhancement algorithm based on Laplacian pyramid transformation [12], the wavelet algorithm [13], and the Sobel algorithm [2].

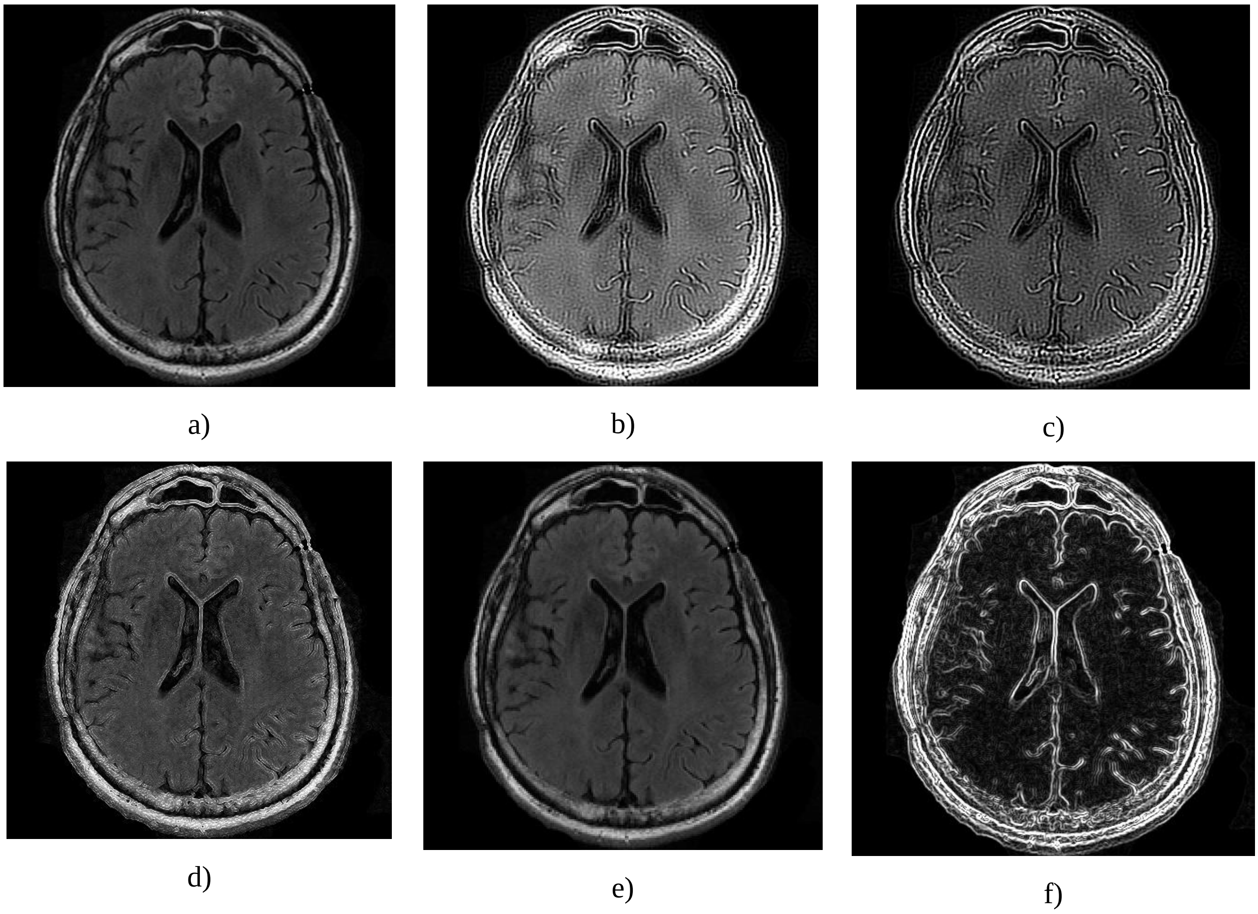


Fig. 7. MRI of the brain: (a) original image, (b) enhanced with fractional derivative of order 0.5, (c) enhanced with fractional derivative of order 0.6, (d) enhanced with the nonlinear Laplacian pyramid transformation algorithm, (e) enhanced using the wavelet algorithm, (f) enhanced using the Sobel algorithm.

From Fig.7. it is evident that the fractional differential algorithm (b) and (c) significantly improved the original texture (a), similar to the Sobel algorithm (f) and the nonlinear Laplacian pyramid transformation algorithm (d). The wavelet algorithm (e), however, did not provide as significant improvement as the beforementioned methods. The Sobel algorithm enhanced edge areas well, but large relatively homogeneous regions, which also contains a lot of information just become black. Data loss in large, relatively homogeneous areas is unacceptable for image enhancement algorithms. The nonlinear Laplacian pyramid transformation algorithm has a serious drawback: it introduced some artifacts (noise) into the enhanced image. The fractional differential algorithm, on the other hand, improved all areas of the image without introducing noise. It can be concluded that among the examined algorithms, the fractional differential algorithm performed the best in enhancing texture.

The next step in the analysis is to compare the characteristics of image enhancement algorithms using such parameters as: information entropy and average image gradient.

Information entropy [14] is calculated by the formula:

$$H(x) = \sum_{i=1}^{n} p(x_i) \log_2 p(x_i), \tag{21}$$

From formula (21) H(X) –information entropy value, $p(x_i)$–імовірність виникнення події x_i, n – the number of unique pixel values. The value of information entropy corresponds to the complexity and randomness of the image.

Average gradient [14] it is calculated as the average value of the sum of all image pixels. It represents the degree of change in pixel intensity, which corresponds to the sharpness and contrast of the image. [6].

Table 2

Numerical Comparison of Algorithms

Image	Information entropy	Average gradient
(a)	5.46	35.42
(b)	5.93	67.73
(c)	7.02	48.14
(d)	5.6	47.4
(e)	5.47	35.48
(f)	5.93	46.73

In Table 2, the parameters of information entropy and average gradient for the images from Fig. 7 are shown. It can be seen that the Laplacian pyramid transformation algorithm (d) has a higher average gradient value than the original image, but it almost does not improve the information entropy. The wavelet algorithm (e) shows a better average gradient value, but the information entropy remains nearly the same as in the original image. The Sobel algorithm (f) slightly improves the information entropy but worsens the average gradient value. The fractional differential algorithm improves both information entropy (b), (c), indicating increased complexity and thus enhancement of textures, and significantly improves the average gradient value, almost doubling it, which in turn indicates improved image sharpness.

Summarizing the above comparisons of the algorithms, it can be stated that the algorithm based on fractional differential masks effectively enhances both relatively smooth areas, where changes in gray color values are not visually noticeable, as well as the contours in the image.

Conclusions

According to the results of the conducted research, namely: a review of the mathematical apparatus, the creation of an algorithm, and the software implementation for improving the textures of medical MRI images using the fractal derivative, the following conclusions can be drawn:

- The review of the theoretical aspects of fractal operators confirmed their potential for medical image processing. This approach allows for the effective processing of complex self-similar structures characteristic of medical images.
- The developed algorithm for constructing masks, calculating coefficients for masks, and applying masks for image processing demonstrated its effectiveness. The choice of eight masks allowed for image enhancement regardless of the position of the element in the image. The selected mask size of 7x7 was optimal and provided a sufficiently low error in calculations.
- The implementation of the software in the Python language made it possible to use partially ready-made functions for mathematical processing and ensured a sufficiently high execution speed. Also, this algorithm can be easily integrated independently or comprehensively into a medical image processing system.
- Experimental studies showed a significant improvement of textures for the order of the fractional derivative in the range v = [0.4;0.7] compared to the original image, which confirms the effectiveness of the fractal derivative in this context.
- Conducted comparisons of improved image textures using the fractal derivative with other methods indicate the advantages of the fractal approach. On improved textures, details are saved and contrast is increased, especially in large, relatively homogeneous areas of the image. With the help of

numerical parameters, it has been proved that the value of the average gradient of the image has increased by two times.

The findings confirm the feasibility of using the fractal derivative to improve the textures of medical MRI images, especially in cases where detailing of structures is required. Thus, the application of the fractal derivative is a promising direction for further research and development in the field of medical image processing in order to improve their quality and analysis.

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ПОКРАЩЕННЯ МЕДИЧНИХ МРТ ЗОБРАЖЕНЬ НА ПІДСТАВІ ФРАКТАЛЬНИХ ОПЕРАТОРІВ

Enhancement of Medical MRI Images Based on Fractal Operators

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Анотація. В даній статті було проведено дослідження алгоритмів покращення текстур на медичних зображеннях. Медичні МРТ знімки мозку містять великі області з низьким рівнем сірого кольору, що несуть важливу інформацію для лікарів. Покращення текстури дозволяє виділити великі сірі області на зображеннях для подальшого детального розпізнавання. На основі проведеного дослідження наявних методів покращення текстур визначено, що саме фрактальні оператори є ефективними для обробки медичних зображень. Наведено математичний апарат фрактальних операторів на основі рівняння апроксимації фрактальних похідних Грюнвальда-Лєтнікова. Базуючись на рівнянні описується створення фрактальних диференційних масок, та алгоритму застосування даних масок для покращення зображень. Проводиться дослідження похибки апроксимації похідної Грюнвальда-Лєтнікова в порівнянні з аналітичним значенням похідної Грюнвальда-Лєтнікова. Алгоритм на основі фрактальної похідної показує покращення по таких параметрах зображення як: контраст, кореляція, енергія та гомогенність в порівнянні з параметрами оригінального зображення. Також наведено порівняння результатів алгоритму ну основі фрактального диференціала з іншими алгоритмами для покращення текстури зображень. Приходимо до висновку що фрактальний диференціальний алгоритм добре підходить для задач покращення МРТ зображень на відміну від інших алгоритмів як по візуальних порівняннях так і за числовими показниками, а отже може бути застосований для вирішення реальних задач.

Ключові слова: Медичні зображення, Магнітно резонансна томографія (МРТ), фрактальні оператори, алгоритми, Python, покращення зображень.